Optimal Pump Control of Broadband Raman Amplifiers via Linear Programming

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Abstract

An algorithm for controlling the pump powers of broadband Raman amplifiers is proposed. The algorithm is based on a variant of the standard Raman model for pump-channel interactions, and determines pump settings that minimize peak-to-peak ripple of the channel powers with respect to any given per-channel target. It is shown that such optimal pump settings can be computed via the solution of linear programs. Some examples are presented.

1 Introduction

It has long been known that stimulated Raman scattering can be employed to build amplifiers for compensating fiber loss in optical transmission systems. We refer the reader to [5, 11, 12, 13], [1, Chapter 8], [18, Chapter 3], and the references given there. Raman amplifiers use the fiber itself as the amplification medium. Raman pump power is launched into the fiber and then transferred to the signals via stimulated Raman scattering.

In recent years, there has been a lot of interest in Raman amplification; see, e.g., [2, 16, 17, 20, 21], the survey paper [13], and the references given there.

There are two main reasons for this renewed interest in Raman amplification. First, the Raman effect has a broad gain curve, which makes it very attractive for today's broadband DWDM (dense wavelength-division multiplexing) systems [7]. By employing a small number of Raman pumps, operating at different frequencies, it is possible to provide sufficient gain throughout the whole signal band. In Figure 1 we display the typical Raman gain curves for a state-of-the-art DWDM system with six Raman pumps. Note that the gain curves are normalized so that the maximal gain for each curve is one. Also shown in Figure 1 is a typical set of channel frequencies in such a DWDM system. There are 128 channels from 186.50 THz to 192.85 THz, with 50 GHz spacing.

Second, Raman amplifiers require pumps with output of several hundreds of milliwatts. Semiconductor pump lasers with such power outputs have now become commercially available at an appropriate cost, and thus Raman amplification has become feasible.

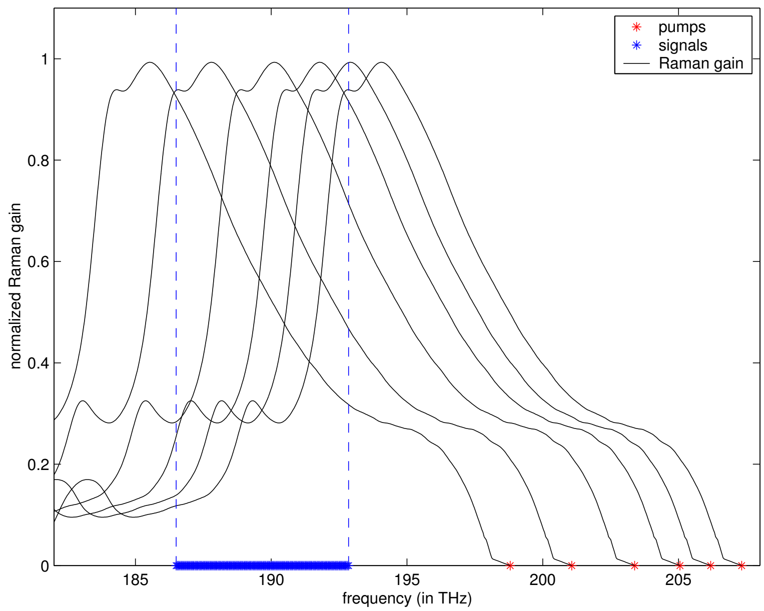

Figure 1: Raman gain of six pumps operating at different frequencies

As indicated above, Raman amplifiers for broadband systems employ multiple pumps. The powers of these pumps need to be adjusted dynamically, so that signal powers are as flat as possible, relative to some given power target.

For flat power targets, some approaches for controlling the pump powers of broadband Raman amplifiers have appeared in the literature. In [20], simulated annealing is used to flatten signal powers. In [15, 17], a genetic algorithm is employed to flatten signal powers. Both these approaches are based on the standard simplified Raman model that expresses signal powers in a log scale as linear functions of the linear pump powers. However, simulated annealing and genetic algorithms are very generic optimization methods that do not take advantage of the structure of the Raman control problem. Moreover, they are not guaranteed to find optimal pump settings that minimize signal ripple. Even when these techniques converge to close-to-optimal pump settings, convergence is too slow to control the Raman pumps of an actual system

In this paper, we propose a different approach for controlling the pump powers of broadband Raman amplifiers. The algorithm is based on a new variant of the standard simplified Raman model, and determines pump settings that minimize peak-to-peak ripple of the channel powers with respect to any given per-channel target. Such optimal pump settings can be computed via the solution of certain linear programming problems.

The paper is organized as follows. In Section 2, we state the standard simplified Raman model and present our new variant thereof. In Section 3, we present a first formulation of the control problem. In Section 4, we give a precise formulation of the Raman control problem as a linear program. In Section 5, we present some examples. Finally, in Section 6, we make some concluding remarks.

2 The underlying model

The equations that describe the power evolution of Raman amplification are well known; see, e.g., [6, 8, 9, 10, 16, 21]. If only pump-signal interactions are taken into account, then these equations result in a simple Raman model that expresses signal powers in a log scale as linear functions of the linear pump powers. This simplified model is the basis for the approaches described in [15, 16, 17, 20] for the control and design of Raman amplifiers. Next, we state the simplified model and present a new variant thereof.

We consider the propagation of n signal channels along a fiber of length L. The signals are amplified by m backward Raman pumps, and possibly a number of forward pumps. We denote by $S_i(z)$, $i = 1, 2, \dots, n$, and $P_j(z)$, $j = 1, 2, \dots, m$, the power (in mW) at position z of the i-th channel, and the j-th backward pump, respectively. Moreover, r_{ij} is the Raman gain between the i-th channel and the j-th backward pump. With this notation, the simplified Raman model can be stated as follows:

$$y_i = c_i + \sum_{j=1}^{m} r_{ij}\,\overline{P_j}, \quad i = 1, 2, \dots, n. \tag{1}$$

Here,

$$y_i := 10 \log 10\,(S_i(L)) \tag{2}$$

is the i-th channel power (in dBm) at the end of the fiber, and

$$\overline{P_j} := \int_0^L P_j(z)\,\mathrm{d}z. \tag{3}$$

Moreover, the term c_i in (1) represents the i-th channel power at the beginning of the fiber, fiber loss, and Raman gain provided by possible forward pump. As well see below, the term c_i actually drops out in our formulation of the Raman model.

The Raman control problem is to adjust the initial values

$$\rho_j := P_j(L), \quad j = 1, 2, \dots, m, \quad m, \tag{4}$$

of the m backward pumps such that the signal powers (1), y_i, $i = 1, 2, \dots, n$, at the end of the fiber are as "flat" as possible in some yet to be specified sense. Next, we introduce the vectors

$$\rho := \begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_m \end{bmatrix}, \quad \overline{P} := \overline{P}(\rho) = \begin{bmatrix} \overline{P_1} \\ \overline{P_2} \\ \vdots \\ \overline{P_m} \end{bmatrix}, \quad y := y(\rho) = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \quad c := \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}, \tag{5}$$

and the $n \times m$ matrix

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \vdots & \vdots & & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nm} \end{bmatrix}. \tag{6}$$

With (5) and (6), the system (1) can be stated in the following compact form:

$$y(\rho) = c + R\,\overline{P}(\rho). \tag{7}$$

Recall that the control variables are the initial values (4) of the backward pumps, which, by (5), are just the entries of the vector ρ in (7). Moreover, in (7), R is the Raman gain matrix for which nominal values are known, and $y(\rho)$ is the vector of channel powers (in dBm) at the end of the fiber. In a simulation environment, values for $y(\rho)$ are generated by a suitable Raman simulator. In an actual system, values for $y(\rho)$, up to some noise level, are measured by the system's optical monitor (OMON). In view of (3), the constant term, c, in (7) represents fiber loss and the Raman gain generated by the forward pumps. Values for c are not available in general. Fortunately, the vector c can be eliminated easily. Indeed, let ρ^{old} be the current powers of the backward pumps and let $y(\rho^{\text{old}})$ be the corresponding channel powers as generated by a Raman simulator or measured by an OMON. Then, by considering (7) for both ρ and ρ^{old}, and by taking differences, we obtain the following relation:

$$y(\rho) = y\big(\rho^{\text{old}}\big) + R\,\Big(\overline{P}(\rho) - \overline{P}\big(\rho^{\text{old}}\big)\Big). \tag{8}$$

By linearizing $\overline{P}\big(\rho\big)$ about ρ^{old}, we get

$$\overline{P}\big(\rho\big) - \overline{P}\big(\rho^{\text{old}}\big) \approx J\,\big(\rho - \rho^{\text{old}}\big), \tag{9}$$

where

$$J = J\big(\rho^{\text{old}}\big) := \mathrm{D}\,\overline{P}\big(\rho^{\text{old}}\big) \tag{10}$$

is the Jacobian of the function

$$\rho \mapsto \overline{P}\big(\rho\big) \tag{11}$$

at the pump settings ρ^{old}. Note that, in view of (3) and (5), the entries of the vector $\overline{P}\big(\rho\big)$ are just the integrals of the backward pump powers along the fiber.

Finally, by inserting the approximation (9) into (8), we obtain the linear relation

$$y(\rho) = y\big(\rho^{\text{old}}\big) + R\,J\big(\rho - \rho^{\text{old}}\big) \tag{12}$$

between the initial values ρ of the backward pump powers (in mW) and the corresponding signal powers $y(\rho)$ (in dBm) at the end of the fiber. Equation (12) is the basis of our Raman control algorithm.

Note that equation (12) still involves the Jacobian matrix (10), J, of the mapping (11). A simple approximation of this backward pump Jacobian is as follows:

$$\mathrm{D}\,\overline{P}\big(\rho\big) \approx J_d := \big(1 - \exp\left(-\overline{\alpha}\right)\big) \begin{bmatrix} 1/\alpha_1 & 0 & \cdots & 0 \\ 0 & 1/\alpha_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1/\alpha_m \end{bmatrix}. \tag{13}$$

Here,

$$\overline{\alpha} \approx \alpha_j\,L, \quad j = 1, 2, \ldots, m,$$

is a nominal value for the netloss along the fiber. Note that J_d is a constant diagonal matrix that does not depend on ρ.

More accurate approximations are possible and will be described elsewhere.

3 The control problem

We are now in a position to present a precise mathematical formulation of the Raman control problem.

Let ρ^{old} be the current powers (in mW) of the m backward pumps, and let $y^{\mathrm{old}} := y\big(\rho^{\mathrm{old}}\big)$ be the corresponding powers (in dBm) of the n signal channels, as measured by an optical monitoring device (OMON) or provided by a Raman simulator. Note that ρ^{old} and y^{old} are vectors of length m and n, respectively. Let y^{target} be a given vector (of length n) of target powers (in dBm) for the n channels. We stress that the entries of y^{target} are allowed to be arbitrary per-channel target powers, and thus y^{target} is not restricted to constant or tilted target powers.

The Raman control problem then is to compute new pump settings ρ such that the corresponding channel powers $y(\rho)$ have minimal possible peak-to-peak ripple (relative to the target y^{target}) and are at the same time close enough to the target y^{target}. The computation of such new pump settings is based on the linear relation (12) between ρ and $y(\rho)$. Moreover, we use the simple diagonal approximation (13), J_d, in (12). Thus, the relation (12) becomes

$$y(\rho) = y^{\mathrm{old}} + M\big(\rho - \rho^{\mathrm{old}}\big), \quad \text{where} \quad M = R\,J_d. \tag{14}$$

Recall from (3) and (6) that $R = \big[R_{ij}\big]$ is the $n \times m$ Raman gain matrix whose entries R_{ij} are nominal values for the Raman gain provided for channel i by pump j. Moreover, by (13), the matrix J_d only involves nominal values of the fiber netloss and of the fiber loss at the pump wavelengths.

Next, we formally define what we mean by ripple (relative to the target). Let y and y^{target} be the channel powers and their target values, respectively, for n channels. We distinguish between two cases: multiple channels ($n > 1$) and a single channel only ($n = 1$). In the first case ($n > 1$), we set

$$\mathrm{ripple}\big(y - y^{\mathrm{target}}\big) := \max_{i=1,2,\dots,n}\left(y_i - y_i^{\mathrm{target}}\right) - \min_{i=1,2,\dots,n}\left(y_i - y_i^{\mathrm{target}}\right).$$

In the second case ($n = 1$), we set

$$\mathrm{ripple}\big(y - y^{\mathrm{target}}\big) := 2\left|y - y^{\mathrm{target}}\right|.$$

The reason for this distinction is that the first definition makes no sense for a single channel, since the "ripple" would be always zero.

With this above notation, the basic problem of determining new pump settings ρ can be stated as the following optimization problem:

$$\begin{aligned} &\text{minimize} \quad \mathrm{ripple}\big(y(\rho) - y^{\mathrm{target}}\big) \\ \text{over all} \quad \rho \in \mathbb{R}^m \quad &\text{with} \quad y(\rho) = y^{\mathrm{old}} + M\big(\rho - \rho^{\mathrm{old}}\big), \\ &\qquad\qquad \rho^{\min} \le \rho \le \rho^{\max}. \end{aligned} \tag{15}$$

Here, $\rho^{\min}$ and $\rho^{\max}$ are vectors of minimal and maximal possible pump powers.

We remark that the optimization problem (15) is linear in the variables ρ. This is important since, as we will describe in Section 4 below, it allows us to rewrite (15) as a linear programming problem. This is a well-understood and computationally easy problem, and we can employ one of the standard algorithms, such as the simplex method, for its solution.

4 Formulation as a linear program

In this section, we rewrite the optimization problem (15) as a linear program.

First, instead of ρ, we introduce the transformed variable

$$d = \rho^{\text{old}} - \rho. \tag{16}$$

We call d the suggested pump change. Once we commit to that change, we obtain the new pump settings as $\rho = \rho^{\text{old}} - d$. With (16), the model (14) underlying our Raman control algorithm becomes

$$y = y(d) = y^{\text{old}} - Md.$$

To express $\text{ripple}\big(y(d) - y^{\text{target}}\big)$ in terms of linear variables, we introduce the additional unknowns σ_1 and σ_2, and require that they satisfy the following constraints:

$$\sigma_1\, e^{(n)} \geq y^{\text{old}} - Md - y^{\text{target}}, \tag{17}$$

$$\sigma_2\, e^{(n)} \leq y^{\text{old}} - Md - y^{\text{target}}. \tag{18}$$

Here, $e^{(n)}$ denotes the vector of all ones:

$$e^{(n)} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \in \mathbb{R}^n.$$

The inequalities (17) and (18) imply that the objective function in (15) is bounded as follows:

$$\text{ripple}\big(y - y^{\text{target}}\big) \;\leq\; \sigma_1 - \sigma_2. \tag{19}$$

We now replace the objective function in (15) by the right-hand side of (19). Moreover, we add the constraints (17) and (18) to the original problem (15). The resulting new version of (15) is the following linear program:

$$\begin{aligned} &\text{minimize} \quad \sigma_1 - \sigma_2 \\ &\text{over all} \quad \sigma_1,\, \sigma_2 \in \mathbb{R}, \quad d \in \mathbb{R}^m \quad \text{with} \\ &-\sigma_1\, e^{(n)} - Md \leq y^{\text{target}} - y^{\text{old}}, \\ &\sigma_2\, e^{(n)} + Md \leq y^{\text{old}} - y^{\text{target}}, \\ &\rho^{\text{old}} - \rho^{\text{max}} \leq d \leq \rho^{\text{old}} - \rho^{\text{min}}. \end{aligned} \tag{LP}$$

The objective function and all the constraints of (LP) are linear functions of the unknowns σ_1, σ_2, and d, and thus (LP) is indeed a linear program. The vector d of the solution of the linear program (LP) is the suggested pump change.

Note that the objective function of (LP) is just the right-hand side of (19). By minimizing the right-hand side of (19) over all σ_1, σ_2 that satisfy the constraints (17) and (18), we can guarantee that the original problem (15) and the linear program (LP) are indeed equivalent.

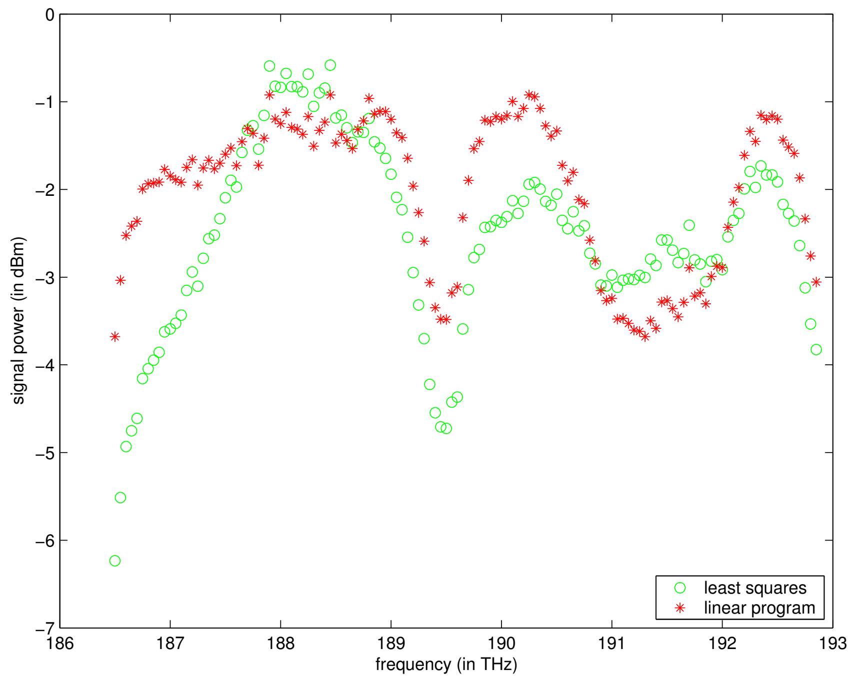

Figure 2: 128 channels, 5 spans, linear pogram vs. least squares

There are standard techniques, such as the simplex method, for the solution of the linear programs (LP). For a detailed discussion of the simplex method for linear programs, we refer the reader to [3], [4, Chapter 8], [14, Chapter 13], or [19, Chapter 4].

For Raman control of an actual system, additional features are needed to deal with OMON noise, power changes due to upstream pump adjustments, and other issues. Our linear program formulation allows to incorporate such additional features. Even though these additional features are based on known concepts, tailoring them to the Raman control problem is far from trivial. We will not be releasing any of the details of these features and their implementation.

5 Examples

In this section, we present some examples to illustrate the typical behavior of the Raman control algorithm proposed in this paper.

In all these examples, we use spans consisting of 100 km TWRS outside plant (OSP) fiber, followed by a dispersion-compensating module (DCM). There are backward Raman pumps at both the end of the OSP and the end of the DCM. Separate runs of the proposed algorithm are used to adjust the OSP and the DCM pumps. All channel powers were obtained from a Raman simulator.

The first example is a system with 128 channels from 186.50 THz to 192.85 THz, with 50 GHz spacing. There are 6 OSP backward pumps and 5 DCM backward pumps. Flat targets for the channel powers are used. In Figure 2, we display the signal powers obtained after 5 spans for the proposed linear-programming approach (red *'s) and for an algorithm that uses least-squares approximation (green +'s). Recall that our proposed algorithm minimizes peak-to-peak (p2p) ripple. For the example shown in Figure 2, the p2p minimization of

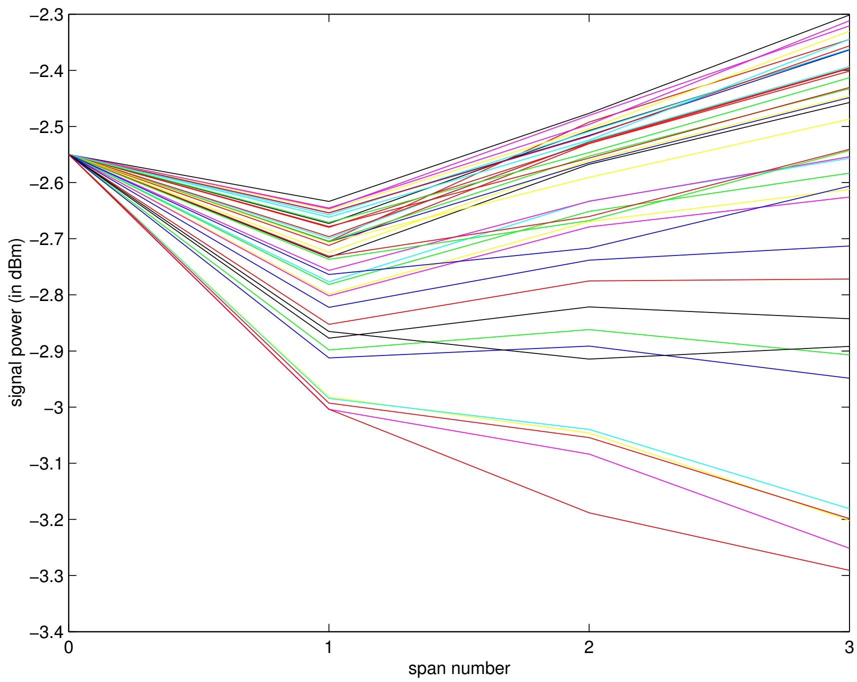

Figure 3: 40 channels, 3 spans, no noise

our algorithm results in a total p2p ripple of 2.7551 dB after 5 spans, while the least-squares minimization gives a total p2p ripple of 5.6520 dB.

Our next example is a 40-channel system consisting of 3 spans. All 3 spans are allowed to adjust their pumps independently. We employ the proposed linear-programming approach with some added features to adjust the Raman backward pumps. In Figure 3, we first show the channel power evolution for the case that the OMON readings are exact. The total p2p ripple after 3 spans is 0.9893 dB. In Figure 4, we show the results when the OMON reading are only accurate to within 0.2dB. The total ripple is now 1.0077 dB, which is only slightly higher than in the case without noise. Clearly, the linear-programming approach allows to efficiently handle OMON noise and multispan adjustments.

The next example is a 128-channel system with a tilted power target at the end of the DCM. Figure 5 shows the signal powers at the end of one span.

The final example illustrates one of the additional features that can be built into the linear-programming approach. Raman control algorithms need to respond not only to external events such as addition or deletion of channels, but also to power changes due to adjustments upstream. They must be robust with respect to inaccuracies in the monitoring devices. Figure 6 shows two pump power traces with respect to time. The one on the left results from a control strategy that ignores noise in the monitoring device; the one on the right results from a variant of the linear-programming approach with an additional feature to suppress noise.

6 Concluding remarks

We have proposed an algorithm that employs linear programming to compute optimal settings of the backward pumps of broadband Raman amplifiers. The algorithm is based on a new

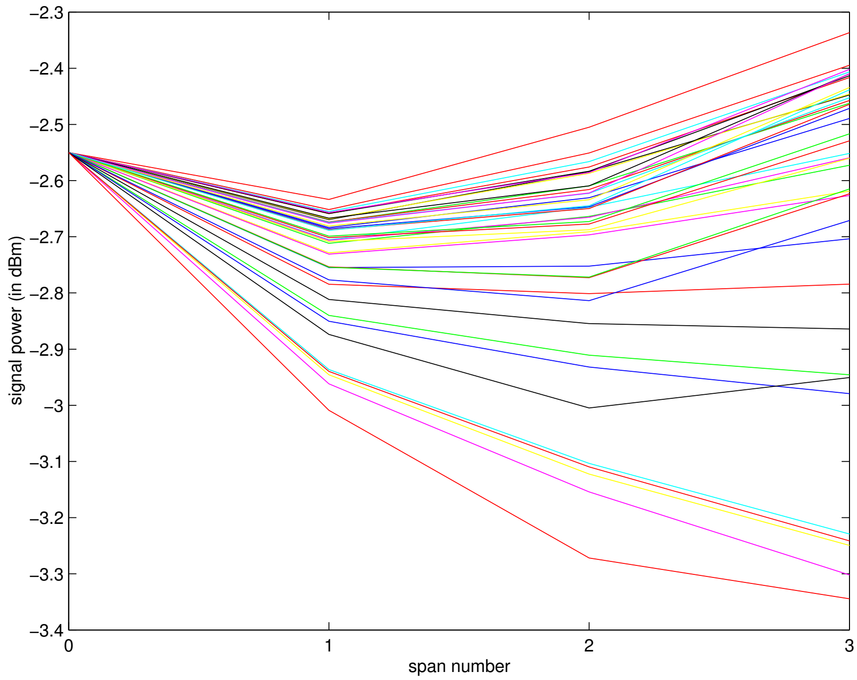

Figure 4: 40 channels, 3 spans, 0.2 dB noise

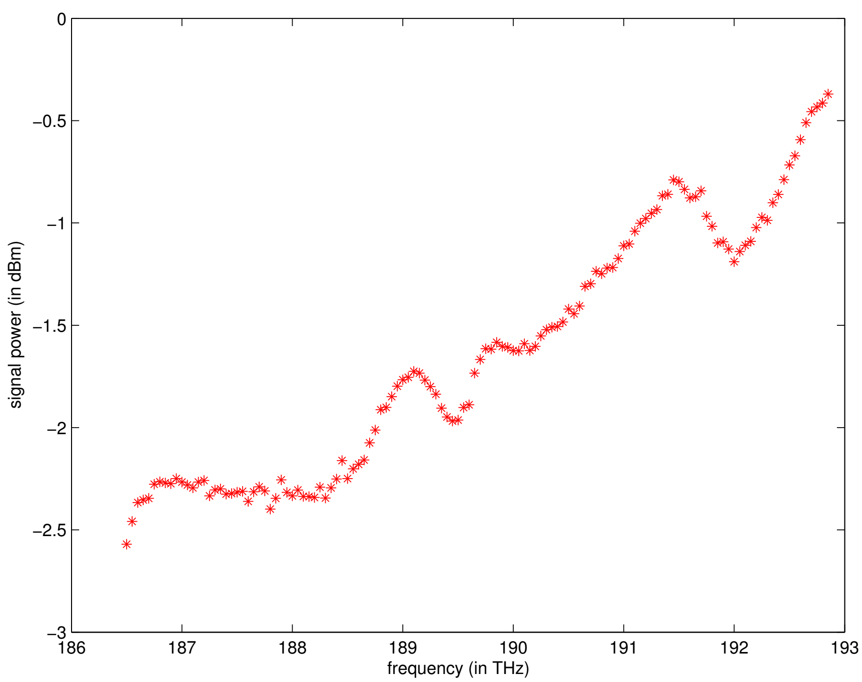

Figure 5: 128 channels, 1 span, tilted target

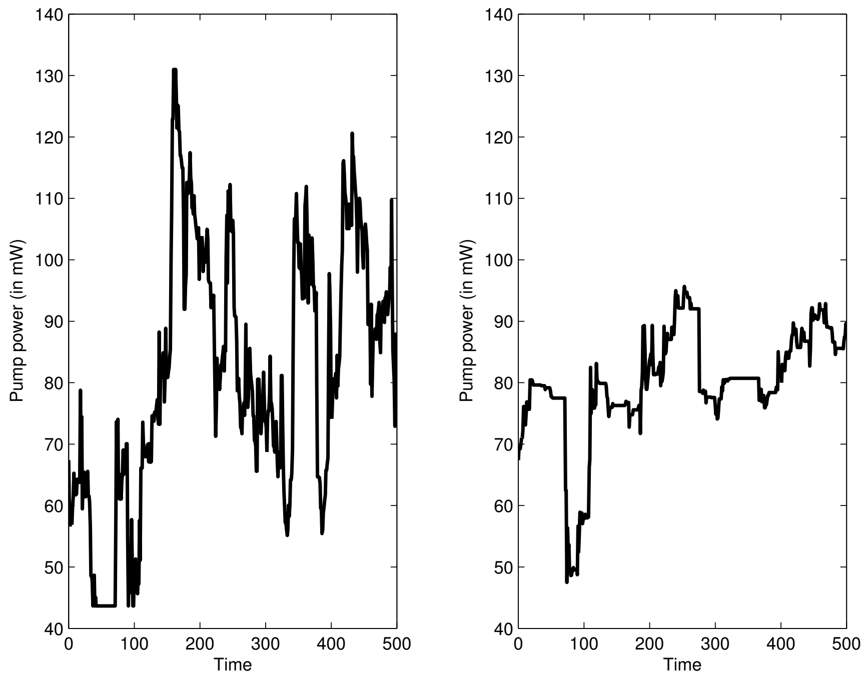

Figure 6: Power evolution of a Raman pump in a DWDM system. The left trace results from a control strategy that ignores noise in the monitoring device; the right one results from a strategy with an added feature to suppresses noise

variant of the standard linear model for Raman pump design and control. The algorithm computes pump settings that minimize peak-to-peak ripple relative to any given power target. The proposed algorithm can handle the adding and dropping of channels and pumps, and it is not restricted to constant or tilted linear power targets.

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