Interaction Graphs for 3-level Fractional Factorial Designs

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Abstract

Graph-aided methods for accommodating the estimation of interactions in factorial experiments have become popular among industrial users. Notable among them is the method of linear graphs due to G. Taguchi. Wu and Chen (1992) pointed out some shortcomings of Taguchi's linear graphs and proposed an alternative method. By extending their method we develop some new graphs for 3-level fractional factorial designs. The proposed graphs have two new features: (i) Each edge of the graph can have one or two lines representing the two components of interaction in a 3-level design, (ii) There are two types of vertices and lines. A collection of graphs is given for 27- and 81-run designs.

Key Words: *Interactions, Linear Graphs, Minimum Aberration, Orthogonal Arrays.*

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Introduction

In some factorial experiments background knowledge may suggest that certain interactions are potentially important and should be estimated clearly from each other and from the main effects. Wu and Chen (1992) and Kacker and Tsui (1990) gave several examples to illustrate the importance of the problem of "accommodating interactions". Any graphs for solving this problem are generically called *interaction graphs* or simply graphs. Recognizing the importance of the problem, G. Taguchi proposed a graph-aided method called linear graphs to solve this problem. A convenient reference is Taguchi (1987). Some defects of Taguchi's approach were pointed out by Wu and Chen (1992), who proposed a more comprehensive method while retaining the graphical appeal of Taguchi's method. The main purpose of this paper is to extend the results of Wu and Chen to 3-level fractional factorial designs. These designs, particularly the 27-run design, are useful when some factors have qualitative levels and/or some but not all the interactions should be estimated. A competing class of designs is the central composite designs, which can be very economical for estimating all the quadratic and interaction effects. However the central composite designs and related designs considered in response surface methodology (see Khuri and Cornell, 1987) assume that the factors are quantitative and are not flexible in the choice of interactions to be estimated. The graph approach presented here will allow different combinations of interactions to be studied in 3-level fractional factorial designs.

The extension is not straightforward because the graphs for 3-level designs have some new features which are not present in the graphs for 2-level designs. First, for each interaction, say, between factors A and B, there are two components denoted by AB and AB^2. By representing each component by a line in a graph, the resulting graph can have edges with one or two lines. Only graphs with a relatively large number of double-line edges are kept in our collection. Second, we classify the main effects into two types: those that are aliased with some 2-factor interactions (2fi's) and those that are not. In the graphs the main effects of the first type are represented by circles with dashed lines and those of the second type are represented by circles with solid lines. Similarly we classify the 2fi's into two types: those that are aliased with some main effects or 2fi's and those that are not. The 2fi's of the first type are represented by dashed lines and those of the second type are represented by solid lines. Details on these new features can be found in the next section. We then give graphs for some 27- and 81-run designs for which the proposed graphs are more likely to be useful. As in Wu and Chen (1992), we first rank the designs by the minimum aberration criterion (definition given below) and then give graphs for each design. The graphs in this paper are compared with Taguchi's graphs and the advantages of using the new graphs are demonstrated. In the last section, we give an example to show how these graphs can be used to plan a 3-level factorial experiment.

The definition of a minimum aberration design is illustrated through a simple example. Consider the 3^{5-2} design which studies 5 factors in $27 (= 3^3)$ runs and is defined by $I = ABD = AB^2CE$, where factors A, B, and C are assigned to three independent columns each with three levels (thus making up 27 runs) and factors D and E are related to A, B, and C

through the two defining relations above. To further explain these defining relations, we use $x_1, x_2, \cdots, x_5$ to denote the three levels 0, 1, 2 of the factors A, B, C, D, and E respectively. Then $I = ABD$ can be represented by the equation $x_1 + x_2 + x_4 = 0 \pmod 3$, where mod 3 denotes modulus 3 arithmetic. Since this equation is equivalent to $2x_1 + 2x_2 + 2x_4 = 0 \pmod 3$, to avoid ambiguity, we adopt the convention that the first nonzero coefficient should be 1. Notationally the second equation can be represented by $I = A^2B^2D^2$. Now, we can generate two additional defining relations by multiplying these defining relations and using modulus 3 arithmetic:

$$I = ABD \cdot AB^2CE = A^2CDE = AC^2D^2E^2,$$

$$I = A^2B^2D^2 \cdot AB^2CE = BCD^2E.$$

Algebraic details for 3-level designs can be found in Kempthorne (1973). The four defining relations and the identity element I form the defining contrast subgroup for the design:

$$I = ABD = AB^2CE = BCD^2E = AC^2D^2E^2.$$

The word-length pattern of a design is given by

$$WLP = (A_1, A_2, ..., A_k),$$

where A_i denotes the number of words with length i in the defining contrast subgroup (e.g., the word $AC^2D^2E^2$ has length 4). The word-length pattern of the design given above is WLP=(0, 0, 1, 3, 0). For designs with resolution III or higher, the first two elements in the word-length pattern are always zero, and we shall omit them in the rest of the paper. For two designs d_1 and d_2, let r be the smallest value such that $A_r(d_1) \neq A_r(d_2)$. We say that

d_1 has less aberration than d_2 if $A_r(d_1) < A_r(d_2)$. If there is no design with less aberration than d_1, then d_1 has *minimum aberration.* In general the minimum aberration criterion gives a good measure of the estimation capacity of a design. However, in some special cases as given in Chen, Sun and Wu (1993), it may be not optimal for estimating the two-factor interactions.

New features of graphs for 3-level designs

We use several simple examples to demonstrate the new features of interaction graphs for 3-level designs that are not present for 2-level designs. First, we consider the 3^{3-1} design with the defining contrast $I = ABC$. It is a 1/3-fraction design in nine runs and three factors labeled A, B and C. The aliases of each effect can be obtained by multiplying the effect to the equation $I = ABC = A^2B^2C^2$, where $I = A^2B^2C^2$ is equivalent to $I = ABC$ as explained in the previous section. For example, by multiplying A (i.e., the main effect of factor A) by each of the three entries of the equation and adopting the previous convention for uniqueness, we have

$$A = AB^2C^2 = BC;$$

similarly we have $B = AB^2C = AC$, $C = ABC^2 = AB$, $AB^2 = AC^2 = BC^2$. Effects in the same equation are called aliases, e.g., BC and AB^2C^2 are aliases of A. In factorial experiments it is common to adopt the hierarchical assumption that higher order effects are less important (or less likely to be important) than lower order effects. So the six aliases of A, B, C are not estimable. Interestingly, one of AB^2, AC^2 and BC^2 is estimable if the other two are assumed negligible. Take, for example, AB^2. It is one component (with two degrees of freedom) of the $A \times B$ interaction. In order for experimenters to designate it as an estimable interaction, we must first understand its meaning. To do so, let us represent the nine factor combinations of A and B by the nine cells in the following 3×3 square

		x_2 = 0	x_2 = 1	x_2 = 2
	0	$\alpha i\ (y_{00})$	$\beta k\ (y_{01})$	$\gamma j\ (y_{02})$
x_1	1	$\beta j\ (y_{10})$	$\gamma i\ (y_{11})$	$\alpha k\ (y_{12})$
	2	$\gamma k\ (y_{20})$	$\alpha j\ (y_{21})$	$\beta i\ (y_{22})$

where $y_{00}, y_{01}, \cdots, y_{22}$ are the nine observations obtained from a 3^2 design. The letters $\alpha, \beta, \gamma, i, j, k$ represent the groupings of the observations, e.g., α represents the group of observations y_{00}, y_{12}, y_{21}. Let $y_\alpha = y_{00} + y_{12} + y_{21}$, $y_\beta = y_{01} + y_{10} + y_{22}$, $\cdots$, $y_k = y_{01} + y_{12} + y_{20}$. The AB interaction component represents the contrasts among the three groups of cells defined by $x_1 + x_2 = 0, 1, 2 \pmod 3$ respectively, i.e., the contrasts among $y_\alpha, y_\beta, y_\gamma$. This AB interaction component contains two degrees of freedom and can be further decomposed into two orthogonal contrasts, each with one degree of freedom, e.g., $y_\alpha - y_\beta$ and $y_\alpha - 2y_\gamma + y_\beta$. Similarly the AB^2 interaction component represents the contrasts among the three groups of cells defined by $x_1 + 2x_2 = 0, 1, 2 \pmod 3$ respectively. The three groups are represented by the letters i, j, and k. For more details see John (1971, pp130-132, pp177-184) and Kempthorne (1973).

Each of the contrasts represented by AB and AB^2 may not have a natural interpretation in some practical situations. Ideally one would like to have both components to be estimable but considerations such as the choice of run size economy may lead to the choice of designs in which only one component is estimable. As shown below we can give a good interpretation and make practical use of a singly estimable interaction component. For quantitative factors a more meaningful decomposition of the $A \times B$ interaction is obtained through the linear contrast $\ell = (-1, 0, 1)$ and the quadratic contrast $q = (-1, 2, -1)$, where the coefficients in ℓ and q are applied to the observations with levels 0, 1, and 2. For example, the linear-by-linear contrast $\ell \times \ell$ represents $(y_{00} + y_{22}) - (y_{02} + y_{20})$. The $\ell \times q$, $q \times \ell$, and $q \times q$ contrasts are similarly defined. There is an important connection (suggested by the referee) between these four contrasts and the AB and AB^2 contrasts. If only one out of the four contrasts is present, say, $\ell \times \ell$, the vector representing $\ell \times \ell$ has a $45°$ angle with each of the two-dimensional spaces representing the AB interaction and the AB^2 interaction. The same result holds for each of the other three contrasts. So even when only one of the AB and AB^2 components is estimable, we can still estimate any one of the four quantitative contrasts through regression technique. (The detailed development of this method is beyond the scope of the paper.)

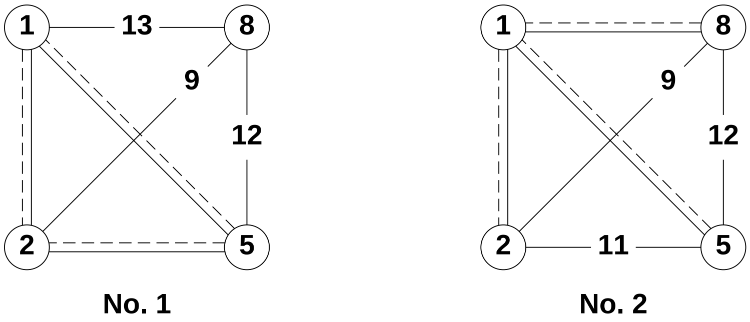
Next we consider the more interesting case of the 3^{4-1} design with $I = ABCD$. Assuming that the 3fi's and 4fi's are negligible, A, B, C, D are estimable. Among the 2fi's, $AB^2, AC^2, AD^2, BC^2, BD^2, CD^2$ are not aliased with any 2fi' and are therefore estimable. For example, by multiplying AB^2 by $I = ABCD = A^2B^2C^2D^2$, we have $AB^2 = AC^2D^2 =$

BC^2D^2. Following Wu and Chen (1992), we call a 2fi *eligible* if it is not aliased with any main effects. An eligible 2fi is called *clear* if it is not aliased with any other 2fi's. The six 2fi's given above are clear. The remaining 2fi's are aliased in three pairs: $AB = CD, AC = BD, AD = BC$. These six 2fi's are eligible but not clear. Only one in each aliased pair is estimable if the other is assumed negligible. The six clear 2fi's form a complete graph among the vertices (or factors) A, B, C, D, i.e., a graph consisting of lines between any pair of vertices. The three non-clear 2fi's can form two nonisomorphic graphs: one represented by AB, BC, AC; another by AB, AC, AD. (Two graphs are isomorphic if one can be obtained from the other by relabeling the vertices). Hence we have two graphs in Figure 1 to represent all the possible estimable 2fi's. The numbers in the circles refer to the column numbers of $L_{27}(3^{13})$ as given in Table 1 of the next section, where $L_{27}(3^{13})$ denotes the orthogonal array of size 27 having 13 columns with three levels. From the correspondence between column numbers and factorial effects in Table 1, if columns 1, 2, and 5 represent A, B, and C, then column 8 represents $D = ABC$. Explanation of column numbers for single-line edges is deferred to the next section.

Between any two vertices, say A and B, there can be two lines representing the two components AB and AB^2 of the interaction $A \times B$. Graphically using two different types of line to distinguish clear and non-clear 2fi's was first proposed by Wu and Chen (1992) and Robinson (1993). In this paper we adopt Robinson's convention of using solid lines for clear 2fi's and dashed lines for eligible but non-clear 2fi's.

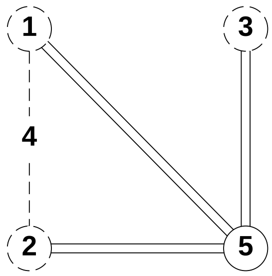
The notion of "clearness" can also be applied to the main effects. For illustration, consider

Figure 1. Graphs for the 3^{4-1} design with $I = ABCD$



the 3^{4-1} design defined by $I = ABD$. From the aliasing relations $A = BD = AB^2D^2$, A is estimable only if BD is negligible. We call A non-clear because it is aliased with a 2fi. The main effects B and D are also non-clear. On the other hand, C is clear because it is not aliased with any 2fi's. Following Robinson (1993), we use a circle drawn with a solid line for a clear main effect and a circle drawn with a dashed line for a non-clear main effect. Among the 2fi's, $AC, AC^2, BC, BC^2, CD, CD^2$ are clear and are represented by solid lines in Figure 2; AB, AD, BD are not eligible. The remaining 2fi's are aliased: $AB^2 = AD^2 = BD^2$. So only one of them, say, AB^2, is eligible but not clear, which is represented by a dashed line. For this design, there is only one graph as given in Figure 2. If, instead, AD^2 is chosen for the dashed line, the resulting graph will be isomorphic to the one given in Figure 2. The choice of column numbers in Figure 2 can be justified by the correspondence given in Table 1. If columns 1, 2, and 5 represent A, B, and C, then column 3 will represent $D = AB$.

Figure 2. A graph for the 3^{4-1} design with $I = ABD$



Non-isomorphic graphs for 27- and 81-run designs

For easy access to interaction graphs in experimental planning, we construct all the non-isomorphic graphs for 27- and 81-run designs with number of factors ranging from 4 to 8. From this collection of graphs, users can select the best graphs to suit their needs.

In the following presentation, we organize the designs in categories according to run size and number of factors. In each category, the designs are listed in ascending order of the aberration criterion. Therefore, the first design in each list is the minimum aberration design.

Although the graphical symbols were introduced in the previous section, we give a summary here for easy reference. A vertex of a graph denotes a factor and its corresponding column number. A double-line edge represents the two components of the interaction between the two corresponding factors (e.g., AB and AB^2 of the $A \times B$ interaction). A solid circle indicates a clear main effect and a dashed circle indicates an eligible but non-clear main effect. A solid line indicates a clear interaction component (e.g., AB or AB^2) and a

Table 1. Arrangement of the 13 columns in $L_{27}(3^{13})$ in Yates order

column	1	2	3	4	5	6	7	8	9	10	11	12	13
	a	b	ab	ab^2	c	ac	bc	abc	ab^2c	ac^2	bc^2	abc^2	ab^2c^2

dashed line indicates a non-clear interaction component.

The column numbers in the graphs represent the columns in the 3-level designs arranged in the standard (Yates) order. For $L_{27}(3^{13})$, the 13 columns are represented by the 13 factorial effects of a 3^3 design as in Table 1.

The column numbers for the interaction between columns $\mathcal{C}_i$ and $\mathcal{C}_j$ are obtained through $\mathcal{C}_i\mathcal{C}_j$ and $\mathcal{C}_i(\mathcal{C}_j)^2$. For example, suppose we want to know the column numbers of the double lines between vertices labeled 2 and 8 in a graph. From the symbols of columns 2 and 8 in Table 1: $\mathcal{C}_2 = b$ and $\mathcal{C}_8 = abc$, we have $\mathcal{C}_2\mathcal{C}_8 = b \cdot abc = ab^2c$ and $\mathcal{C}_2(\mathcal{C}_8)^2 = b \cdot (abc)^2 = a^2c^2 = ac$. By looking up the two symbols ab^2c and ac in Table 1, we immediately have the column numbers 9 and 6. For easy reference, we provide the interaction table for $L_{27}(3^{13})$ in Table 2, where each entry has two numbers, the first indicating the column number for $\mathcal{C}_i\mathcal{C}_j$ and the second for $\mathcal{C}_i(\mathcal{C}_j)^2$. Once the column numbers of the two vertices (i.e., factors) are known, the column numbers of their double-line edge (i.e., interaction components) can be easily read from Table 2. For clarity of graph representation we do not give column numbers to double-line edges. Column numbers for single-line edges are given in all the graphs. Tables 1 and 2 can be easily extended for $L_{81}(3^{40})$ (see Taguchi 1987, vol II, pp. 1158-1161).

Table 2. Interaction table for $L_{27}(3^{13})$

column	1	2	3	4	5	6	7	8	9	10	11	12
2	3,4											
3	4,2	4,1										
4	3,2	1,3	1,2									
5	6,10	7,11	8,12	9,13								
6	10,5	8,9	13,11	12,7	10,1							
7	8,13	11,5	9,10	6,12	11,2	12,4						
8	13,7	9,6	12,5	10,11	12,3	9,2	13,1					
9	12,11	6,8	10,7	13,5	13,4	8,2	10,3	6,2				
10	6,5	12,13	9,7	8,11	1,6	1,5	3,9	4,11	3,7			
11	12,9	7,5	13,6	10,8	2,7	3,13	2,5	4,10	1,12	8,4		
12	9,11	13,10	8,5	6,7	3,8	4,7	4,6	3,5	1,11	13,2	9,1	
13	8,7	10,12	6,11	9,5	4,9	3,11	1,8	1,7	4,5	12,2	6,3	10,2

Designs with 27 runs

Since graphs for the two 3^{4-1} designs have been given in the previous section, we present the graphs for 5–8 factors in this section.

Graphs for 3^{5-2} designs

According to Chen, Sun and Wu (1993), there are three non-isomorphic 3^{5-2} designs:

(i) $I = ABD = AB^2CE$, WLP = (1, 3, 0),

(ii) $I = ABD = ACE$, WLP = (2, 1, 1),

(iii) $I = ABD = AB^2E$, WLP = (4, 0, 0).

For design (i), there are 29 non-isomorphic graphs. Some graphs are better than the others in terms of the number of double-line edges. As argued before, the graph-aided method would be more effective if both components of the interaction are eligible so that we can decompose the four degrees of freedom of the interaction in other ways such as the $\ell \times \ell$, $\ell \times q$, $q \times \ell$, and $q \times q$ contrasts. Out of the 29 graphs only six have three or four double-line edges. Since the remaining graphs are inferior in terms of the number of double-line edges, we only present these six graphs in Figure 3.

Each of the graphs in Figure 3 has two clear main effects represented by vertices with column numbers 5 and 9 and one clear interaction component (column 13) indicated by the solid line between vertices 5 and 9. This fact suggests that users may assign the two most important factors to columns 5 and 9 so that their main effects and one component of their interaction are clearly estimable.

Among these six graphs, graph 5 is superior to the others because it has one more double-line edge than the others. The remaining five graphs can be grouped into two types: the first with all three double-line edges connected to one common factor (graphs 1, 2, and 3) and the second with the three double-line edges forming a triangle among the three factors (graphs 4 and 6). The first type of graph is useful if one factor is likely to have interactions with other factors, while the second type is useful if three factors have pairwise interactions.

For design (ii), all the graphs have fewer than three double-line edges. They are considered to be inferior to the graphs for design (i). The design is not recommended for estimating interactions.

For design (iii), there is only one graph as given in Figure 4. It has four double-line edges with all 2fi's clear. This graph is strongly recommended in the case that one factor is anticipated to be far more significant than the rest. Such a factor should be assigned to

Figure 3. Graphs for the 3^{5-2} design with $I = ABD = AB^2CE$

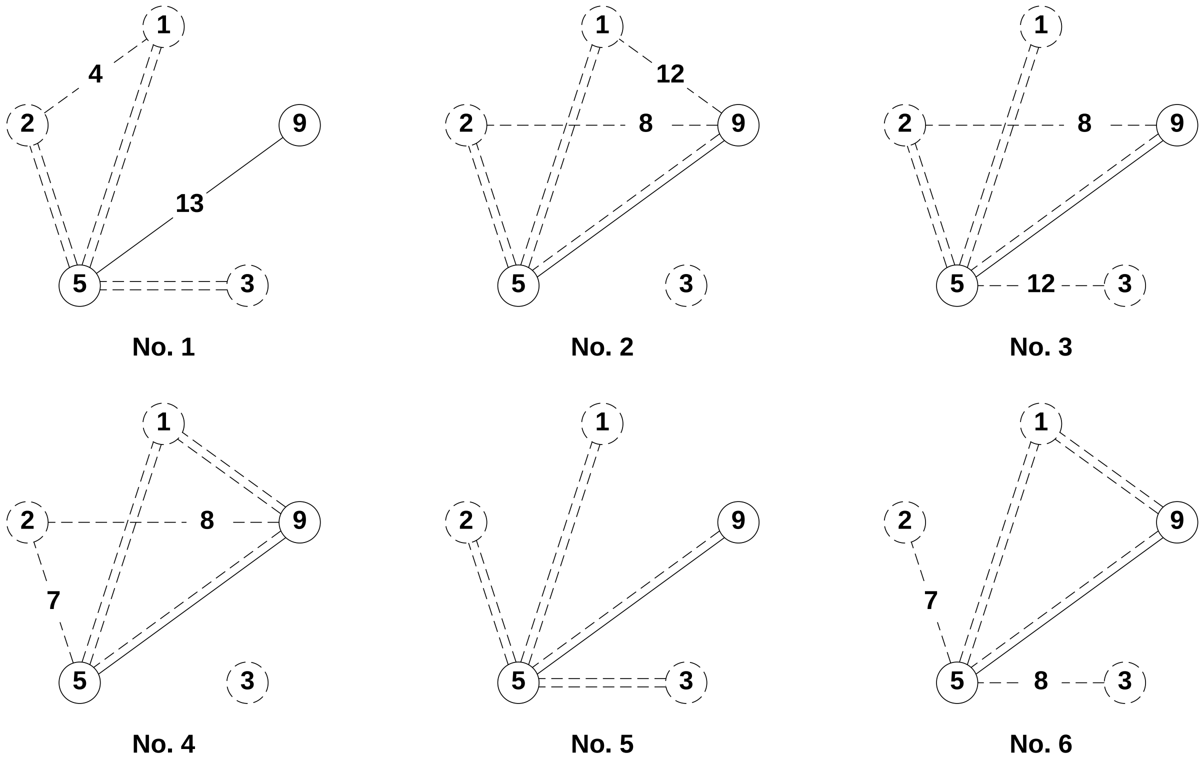
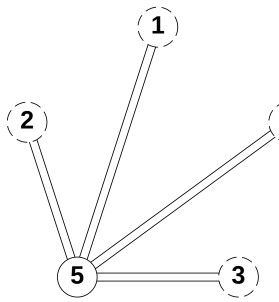


Figure 4. A graph for the 3^{5-2} design with $I = ABD = AB^2E$



column 5. This graph provides a good alternative to the graphs for design (i).

In terms of the aberration criterion and the diversity of graphs for accommodating interaction, we recommend design (i) as the best choice.

Graphs for 3^{6-3} designs

There are four non-isomorphic 3^{6-3} designs:

(i) $I = ABD = AB^2CE = AB^2C^2F$, WLP = (2, 9, 0, 2),

(ii) $I = ABD = ACE = BCF$, WLP = (3, 6, 3, 1),

(iii) $I = ABD = ACE = BC^2F$, WLP = (4, 3, 6, 0),

(iv) $I = ABD = AB^2E = ACF$, WLP = (5, 3, 3, 2).

For design (i), there are 97 graphs, out of which only three graphs (given in Figure 5) have three double-line edges. Other graphs have one or two double-line edges and are therefore omitted.

For design (ii), there are only four graphs (given in Figure 6) having three double-line

Figure 5. Graphs for the 3^{6-3} design with $I = ABD = AB^2CE = AB^2C^2F$

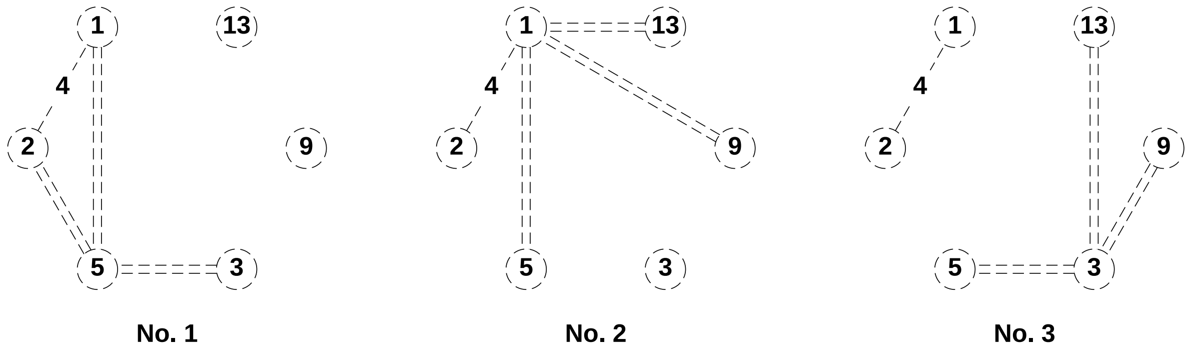
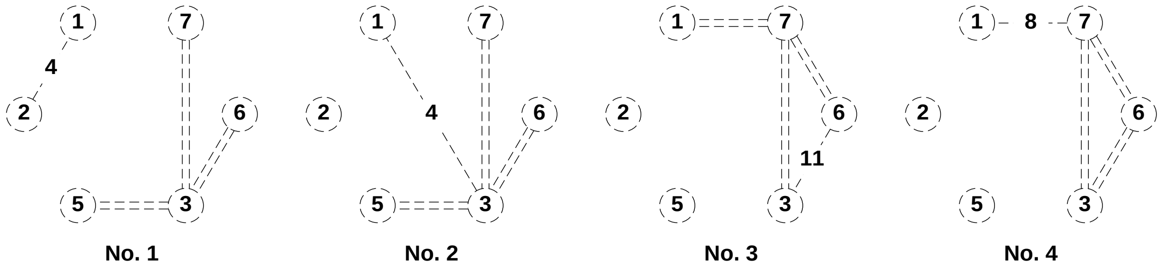


Figure 6. Graphs for the 3^{6-3} design with $I = ABD = ACE = BCF$



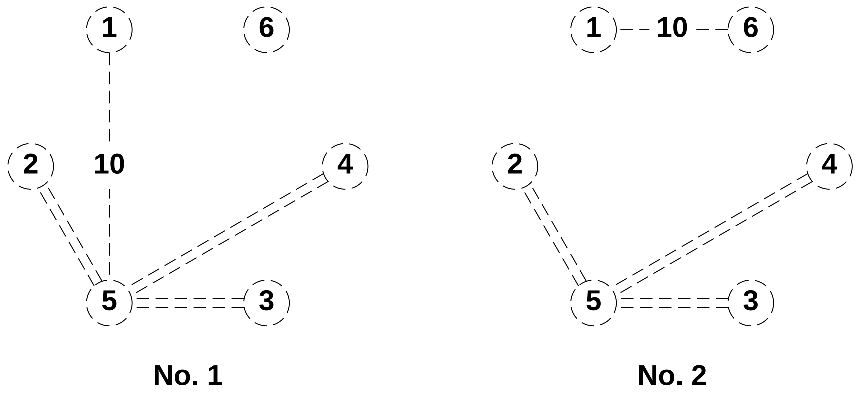
edges among the 146 graphs.

For design (iii), all the 43 graphs have at most one double-line edge. The design is not recommended for estimating interactions.

For design (iv), there are two graphs having three double-line edges (see Figure 7) out of 25 graphs.

Overall we recommend design (i) and (ii).

Figure 7. Graphs for the 3^{6-3} design with $I = ABD = AB^2E = ACF$



Graphs for 3^{7-4} designs

There are four non-isomorphic 3^{7-4} designs:

(i) $I = ABD = AC^2E = BC^2F = AB^2C^2G$, WLP = (5, 15, 9, 8, 3),

(ii) $I = AB^2D = ABCE = AB^2CF = BC^2G$, WLP = (6, 11, 15, 4, 4),

(iii) $I = AB^2D = ABCE = AC^2F = BC^2G$, WLP = (7, 10, 12, 9, 2),

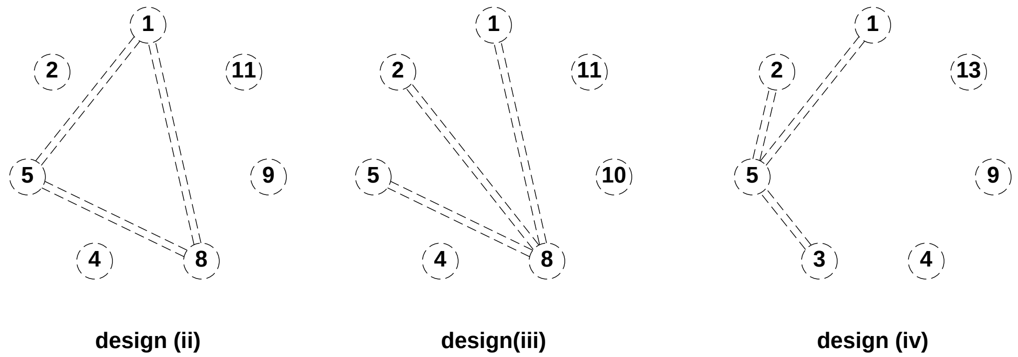
(iv) $I = ABD = AB^2E = AB^2CF = AB^2C^2G$, WLP = (8, 9, 9, 14, 0).

For design (i), all the graphs have at most two double-line edges, and are inferior to the graphs for designs (ii)-(iv).

Each of designs (ii), (iii) and (iv) has one graph with three double-line edges. The graphs are given in Figure 8.

This is an interesting case. The minimum aberration design (i) is quite poor for accommodating interactions. It may be explained by the word-length patterns. Design (i), though having the smallest number of three-letter words, has far more four-letter words than the other designs. The same explanation applies to the 3^{8-5} designs given next.

Figure 8. Graphs for 3^{7-4} designs



Graphs for 3^{8-5} designs

There are three non-isomorphic 3^{8-5} designs:

(i) $I = ABD = ABCE = AB^2CF = AC^2G = BC^2H$, WLP = (8, 30, 24, 32, 24, 3),

(ii) $I = AB^2D = ABCE = AB^2CF = AC^2G = BC^2H$, WLP = (10, 23, 32, 30, 22, 4),

(iii) $I = ABD = AB^2E = AB^2CF = BC^2G = AB^2C^2H$, WLP = (11, 21, 30, 38, 15, 6).

Design (i) has no graph with more than one double-line edge.

For design (ii), there are four graphs with two double-line edges (Figure 9), and for design (iii), there are three graphs with two double-line edges (Figure 10).

Designs with 81 runs

For five factors, the 3^{5-1} design with $I = ABCDE$ has resolution V. Since its interactions are estimable, there is no need to use a graph-aided method. Using the method in Chen et al. (1993), we can show that resolution IV designs exist for 6 to 10 factors. Note that for

Figure 9. Graphs for the 3^{8-5} design with $I = AB^2D = ABCE = AB^2CF = AC^2G = BC^2H$

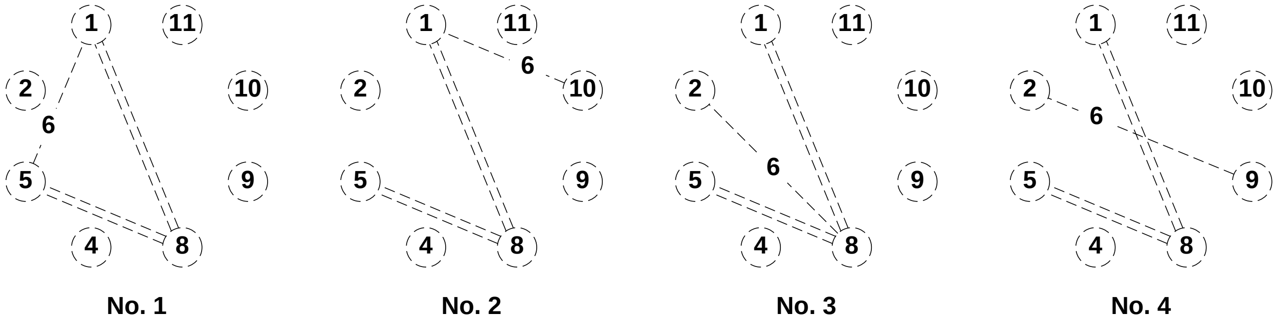
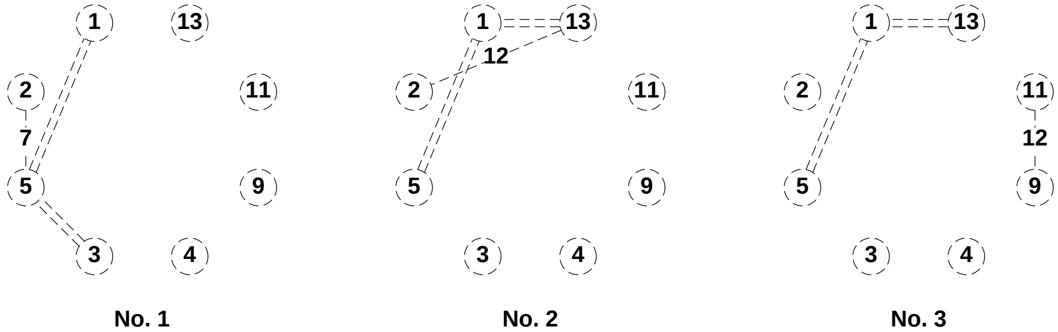


Figure 10. Graphs for the 3^{8-5} design with $I = ABD = AB^2E = AB^2CF = BC^2G = AB^2C^2H$



resolution IV designs all main effects are clear from 2fi's, i.e., all the circles in the graphs are solid.

Since the number of graphs for 8, 9, 10 factors is too large, it is impractical to give them in this paper. In the following we only consider 6 and 7 factors. All the other graphs are available from the authors.

Graphs for 3^{6-2} designs

There are two non-isomorphic 3^{6-2} designs:

(i) $I = ABCE = AB^2DF$, WLP = (0, 2, 2, 0),

(ii) $I = ABCE = ABDF$, WLP = (0, 3, 0, 1).

Design (i) has four graphs (see Figure 11) with 10 double-line edges, while design (ii) has none. Design (i) is recommended.

Graphs for 3^{7-3} designs

There are two non-isomorphic 3^{7-3} designs:

(i) $I = ABCE = ABDF = AB^2C^2DG$, WLP = (0, 5, 6, 1, 1)

(ii) $I = ABCE = ABDF = ACDG$, WLP = (0, 6, 3, 4, 0)

For both designs the maximum number of double-line edges is ten. However, even with 10 double-line edges, there are still too many non-isomorphic graphs to be enumerated. By eliminating all the single-line edges from the graphs, we have six graphs with 10 double-line edges for design (i) (see Figure 12) and another six graphs for design (ii) (see Figure 13). The two designs are comparable according to the aberration criterion and the number of

Figure 11. Graphs for the 3^{6-2} design with $I = ABCE = AB^2DF$

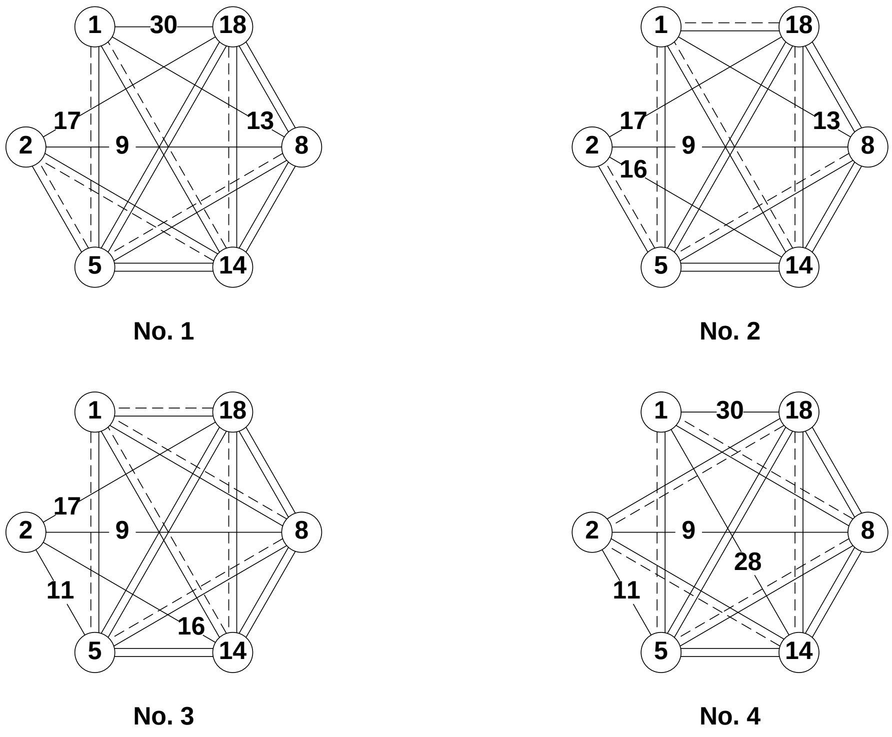
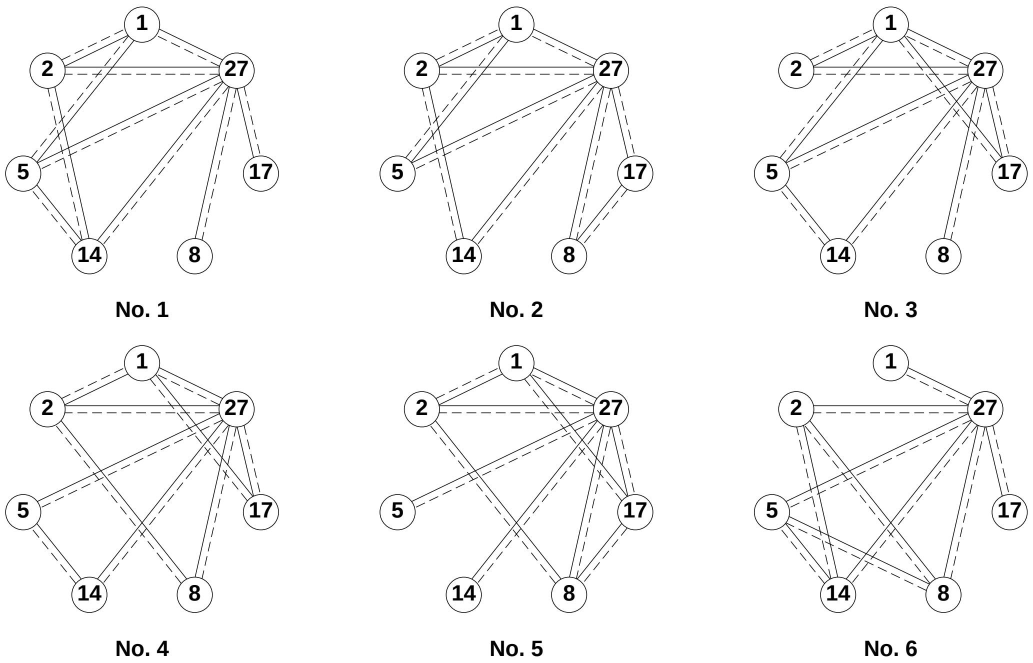


Figure 12. Graphs for the 3^{7-3} design with $I = ABCE = ABDF = AB^2C^2DG$

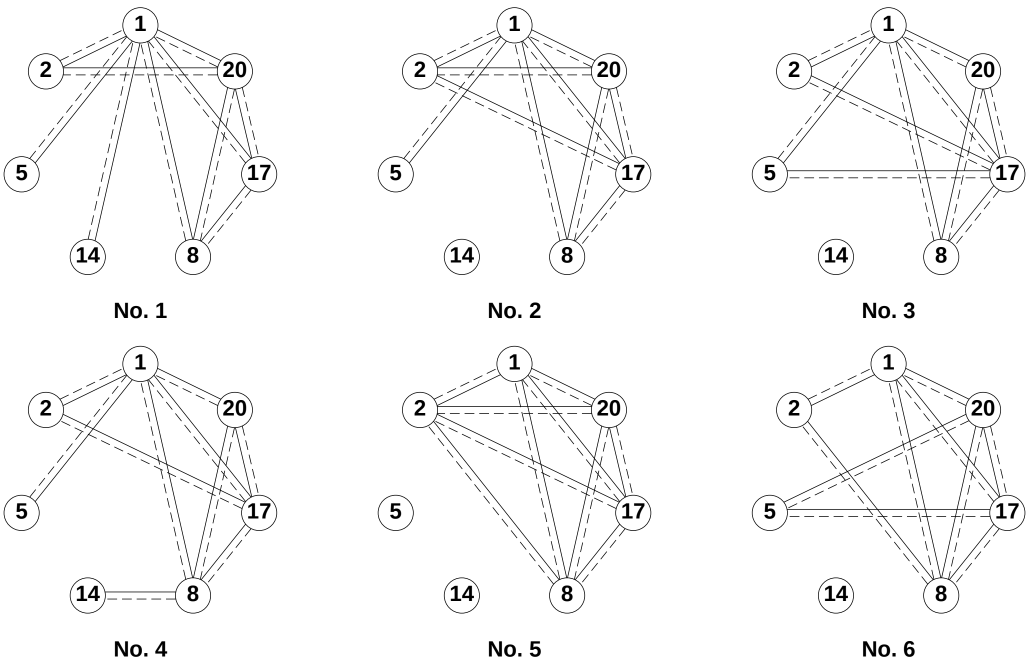


estimable interactions.

Comparison with Taguchi's linear graphs

There are three major aspects that distinguish our approach from Taguchi's. First, as argued in Wu and Chen (1992), Taguchi's collection of graphs is not exhaustive and hence incomplete, while ours is complete in that it searches through all nonisomorphic graphs for each 3-level design. Secondly, we provide some insights on the problem through the use of single-line and double-line edges which indicate how the degrees of freedom are allocated for interaction estimation. Different characteristics of the interactions are represented in

Figure 13. Graphs for the 3^{7-3} design (ii) with $I = ABCE = ABDF = ACDG$



the graphs by double-solid lines, double-dashed lines, double-mixed lines, single solid lines, and single dashed lines. Finally, we provide the graphs for all the nonisomorphic designs (including the minimum aberration designs) with given run size and number of factors. This approach gives the users more flexibility in the selection of graphs. In the following, we compare the graphs in this paper with those of Taguchi.

Graph for L_9:

For L_9, there is only one graph available (see Taguchi 1987, p.1153). To avoid redundancy, we do not present the graph in this paper.

Graphs for L_{27}:

Taguchi (1987, p.1155) gave two graphs. Our paper provides a complete collection of graphs (Figures 1-10) for 4 to 8 factors. The graph for design (ii) in Figure 8 matches Figure 22 in Robinson (1993). The graphs in Figure 3 provide a greater variety of choices than Taguchi's two graphs. For example, graphs 4 and 6 not only allow the estimation of the "triangle pattern of interactions" as in Taguchi's graph, but also provide two extra interactions with different geometrical structures.

Graphs for L_{81}:

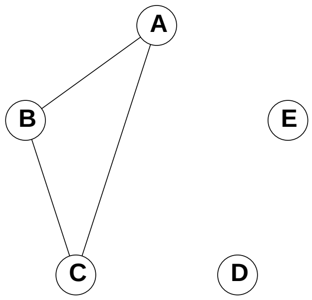
Taguchi (1987, pp1162-1168) gave a collection of 14 graphs. Neither his 14 graphs nor ours are complete. However, there is a significant difference between the two approaches. Taguchi's graphs are some special examples. It is not clear on what basis these graphs were chosen. We actually searched through all nonisomorphic graphs and stored a complete collection of graphs in computer files. Because the number of graphs is too large for publication or practical application, we select a small subset of graphs from the complete collection based on the new criterion in terms of the number of double-line edges of a graph. For users who do need graphs for L_{81} with more than 6 factors, the graphs are available upon request. It should, however, be pointed out that L_{81} is seldom used in practical experiments, while L_9 and L_{27} are much more frequently used. For example, no L_{81} was found in the five consecutive volumes of the Supplier Symposium on Taguchi Methods published by the American Supplier Institute during 1983-1987.

An example of graph-aided planning of 3-level experiment

A process of mounting components onto the surface of a printed wiring board consists of two steps: epoxy application and component placement. Three factors are considered for epoxy application: dispensing pressure (A), dispensing time (B) and dispenser speed (C). Two factors are considered for component placement: component pressure (D) and tool height (E). Each factor has three levels. It was thought that the three factors for epoxy application had pairwise interactions, i.e., the interactions $A \times B, A \times C$ and $B \times C$ should be estimated clearly from each other and from the main effects. How to find a 3-level design to accommodate the estimation of the main effects A to E and $A \times B, A \times C$ and $B \times C$?

First we draw a graph (see Figure 14) to represent the three designated interactions as a triangle among A, B and C. Each edge connecting two vertices has two lines. Next we compare this graph with the graphs given in Figure 3 for the minimum aberration 3^{5-2} design.

Figure 14. An example of interaction assignment graph



A – dispensing pressure

B – dispensing time

C – dispenser speed

D – component pressure

E – tool height

It turns out that graphs 4 and 6 in Figure 3 include the graph in Figure 14 as their subgraph regardless of the types (solid or dashed) of the edges or vertices of the triangle. By comparing graph 4 (or 6) with the graph in Figure 14, we assign factors A, B, C to columns $1, 5, 9$ and factors D, E to columns 2 and 3. The exact assignment depends on which graph is being used. Note that both graphs 4 and 6 in Figure 3 have two extra estimable interactions which are not provided in Taguchi's graph. If the experimenter has prior knowledge that factor D is more likely to have interaction with A, B, or C, we recommend using graph 4 with factor D being assigned to column 2 so that columns 7 and 8 can be used to estimate, say, AD and BD. Alternatively, if A is known to be the most important factor and the estimation of its interactions with the rest of the factors are desirable, we recommend using graph 6 with factor A being assigned to column 5 and factors D and E being assigned to columns 2 and 3.

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