Statistical Properties of Hadamard Matrices of Order 16

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Abstract

Hadamard matrices of order 16 due to M. Hall provide an unique opportunity for studying the statistical implications of nonregular designs because they are the smallest designs in which both full aliasing and partial aliasing occur. The full aliasing relations can be defined by a group called the incomplete defining contrast subgroup. Implications on the estimation of interactions are also considered.

Key Words: *Fractional factorial design; Incomplete defining contrasts subgroup; Interaction graph; Minimum aberration design; Nonregular design; Partial aliasing; Resolution; Word length pattern.*

1 Introduction

Hall (1961) discovered that there are exactly five non-isomorphic Hadamard matrices of order 16. (A Hadamard matrix is an orthogonal matrix with ± 1 as its entries. Two Hadamard matrices are isomorphic if one can be obtained from the other by row changes, column changes, or sign changes within any column.) Interesting as it may be from the combinatorial point of view, its statistical implications have not been explored. The main purpose of this paper is to outline some properties of these matrices as statistical designs.

Hall (1961) called his five matrices Class I, II, III, IV, and V, which we call H16.I to H16.V. The first matrix H16.I is the well known 16-run fractional factorial design, whose columns form a group under entry-wise multiplication of columns. The last four matrices do not have this property and some interesting opportunities arise as a consequence. We call the first type of designs *regular* and the second type *nonregular*. Nonregular designs have traditionally been used for studying the main effects only. Recently Hamada and Wu (1992) pointed out the potential of these designs for entertaining and studying interactions and proposed an analysis strategy. Wang and Wu (1992) gave a design-theoretical explanation for the success of the Hamada-Wu strategy. What distinguishes the two types of designs is the presence of partial aliasing of effects. For regular designs any two effects are either orthogonal or fully aliased. For nonregular designs some effects are partially aliased, i.e., the aliasing coefficient is between 1 and -1. The best known is the 12-run Plackett-Burman (1946) design, in which the aliasing coefficient between any main effect and any 2-factor interaction (2fi) is either 1/3 or $-1/3$. It is also true that only partial aliasing occurs for Plackett-Burman designs of order 20, 24, and 28. What makes the present study interesting is the observation that for each of the H16.II-V designs, both full aliasing and partial aliasing occur. The full aliasing part of the relations can be defined by a group, which we call the incomplete defining contrast subgroup. In Section 2 we spell out these groups for H16.II-V and study their design implications. In Section 3 we study the problem of factor assignment for these four designs. In Section 4, by using graph representations we demonstrate that nonregular designs like H16.II-V can be more powerful in entertaining and estimating interactions than regular designs like H16.I.

Some basic definitions for regular 2-level fractional factorial designs are given below. A 2^k factorial design has k factors, each at two levels coded by ± 1 conventionally. All possible combinations of the $\pm$ signs of k factors form 2^k runs. A $2^{k-p}(p < k)$ fractional factorial design only employs a 2^{-p} fraction of the 2^k runs and is defined by a set of p independent defining words, or generators. All 2^p defining words generated from the p generators form a group called the *defining contrast subgroup*. The number of factors (or letters) in a word is called *word-length*. The vector $W = (w_3, w_4, \ldots)$ is called the *word-length pattern*, where w_i denotes the number of words of length i in the defining contrast subgroup. *Resolution* is defined as the smallest r with $w_r \geq 1$. For two designs d_1 and d_2 with r being the smallest value such that $w_r(d_1) \neq w_r(d_2)$, we say that d_1 has less aberration than d_2 if $w_r(d_1) < w_r(d_2)$. If there is no design with less aberration than d_1, then d_1 has *minimum aberration*. The aberration criterion can further distinguish designs with the same resolution.

It is thus a useful and convenient criterion for selecting designs.

2 Hadamard matrices of order 16 due to M. Hall

Among Hall's five matrices, H16.I can be completely described by its defining contrast subgroup and the related properties such as resolution and aberration. The last four matrices have, however, received little attention in the statistical design literature. Their properties as statistical designs are rather unknown, let alone their applications in practical experiments. In this section, we present some new properties of these four designs through the notion of *incomplete defining contrast subgroup*.

We observe that for each of the four designs there are certain algebraic relations among its columns, i.e., some factors are fully aliased with the interactions of other factors. For example, in H16.II, the relation I=2345 holds, i.e., the product of columns 2 to 5 is the identity column I, which consists of all + signs. This relationship implies aliasings among the effects of factors 2, 3, 4, and 5, i.e., the main effect 2 is fully aliased with 345, the interaction 23 is aliased with 45, etc. The aliasing pattern provides information about the effects that cannot be simultaneously estimated. Here we use factor i and column i interchangeably.

We note that some defining words of length 3 were presented by Hall (1961), which he called "triples". However, Hall did not point out any further column relations which seemed irrelevant to the construction of the matrices. By close investigation of H16.II-V, we observe that there are more defining relations among the columns than what can be expressed as Hall's "triples". For example, in H16.III, the defining relation $I = 8\ t_0\ t_2\ t_4$ (we use $t_0, t_1, t_2, t_3, t_4, t_5$ to indicate column 10, 11, 12, 13, 14, 15) cannot be derived from the "triples" of H16.III. All possible defining relations among the columns of these four designs have been found through a complete search. There are totally 1023, 511, 255 and 255 defining words for H16.II–V in comparison with 2047 defining words for H16.I.

For each design, we list a set of independent defining words (generators) as follows, where the words inside the parentheses were not found in Hall (1961).

$$\begin{array}{lll}
\text{H16.I} & \text{I} & = 1\ 2\ 3 = 1\ 4\ 5 = 1\ 6\ 7 = 1\ 8\ 9 = 1\ t_0\ t_1 = 1\ t_2\ t_3 = 1\ t_4\ t_5 \\
 & & = 2\ 4\ 6 = 2\ 8\ t_0 = 2\ t_2\ t_4 = 4\ 8\ t_2 \\
\text{H16.II} & \text{I} & = 1\ 2\ 3 = 1\ 4\ 5 = 1\ 6\ 7 = 1\ 8\ 9 = 1\ t_0\ t_1 = 1\ t_2\ t_3 = 1\ t_4\ t_5 \\
 & & = 2\ 4\ 6 = 2\ 8\ t_0 = 2\ t_2\ t_4 \\
\text{H16.III} & \text{I} & = 1\ 2\ 3 = 1\ 4\ 5 = 1\ 6\ 7 = 1\ 8\ 9 = 1\ t_0\ t_1 = 1\ t_2\ t_3 = 1\ t_4\ t_5 \\
 & & = 2\ 4\ 6 \quad = (8\ t_0\ t_2\ t_4) \\
\text{H16.IV} & \text{I} & = 1\ 2\ 3 = 1\ 4\ 5 = 1\ 6\ 7 = 1\ 8\ 9 = 1\ t_0\ t_1 = 1\ t_2\ t_3 = 1\ t_4\ t_5 \\
 & & = (2\ 4\ 6\ 8\ t_0\ t_2\ t_4) \\
\text{H16.V} & \text{I} & = 1\ 2\ 3 = 1\ 4\ 5 = 1\ 6\ 7 = 2\ 4\ 6 \\
 & & = (8\ 9\ t_0\ t_1 = 8\ 9\ t_2\ t_3 = 8\ 9\ t_4\ t_5 = 8\ t_0\ t_2\ t_4)
\end{array}$$

We call the group generated from these defining relations the *incomplete defining contrast subgroup*. The word "incomplete" emphasizes the fact that this group does not completely determine all the relations among the columns. In addition to the fully aliased effects implied by the group, there are some pairs of effects that are partially aliased, which contribute to the complexity of the aliasing structure. For example, if we assign factors A, B, C, D, E, F, G to columns $1, 2, 3, 4, 5, 8, 9$ of H16.V, the contrast ℓ_1 for column 1 estimates the sum of many effects: $\ell_1 \rightarrow A + BC + DE + \frac{1}{2}FG$. So A is fully aliased with BC and DE, and partially aliased with FG with coefficient $\frac{1}{2}$.

The word-length pattern of the incomplete defining contrast group is defined in the same way as in Section 1. For H16.I–V, they are as follows:

	word-length												
	3	4	5	6	7	8	9	10	11	12	13	14	15
H16.I	35	105	168	280	435	435	280	168	105	35	0	0	1
H16.II	19	57	72	120	243	243	120	72	57	19	0	0	1
H16.III	11	33	24	40	147	147	40	24	33	11	0	0	1
H16.IV	7	21	0	0	99	99	0	0	21	7	0	0	1
H16.V	7	21	0	0	99	99	0	0	21	7	0	0	1

H16.I has the largest number of defining words of any length and the number of defining words are in descending order for H16.I–V. Interestingly, H16.IV and H16.V have the same word-length patterns.

Another piece of information obtainable from the incomplete defining contrast group is the structure of three different types of 2fi's described as:

Type-A. fully aliased with some main effects,

Type-B. fully aliased with some 2fi's but not with any main effects,

Type-C. not fully aliased with any main effects or 2fi's.

These three types of 2fi's are analogous to the notions given in Wu and Chen (1992): ineligible

2fi, eligible but not clear 2fi, and clear 2fi.

Type-A 2fi's appear in length-3 words and type-B 2fi's appear in length-4 but not length-3 words; type-C 2fi's do not appear in any length-3 or length-4 words. Type-B 2fi's put the constraint that no two aliased 2fi's can be included in the same model. Type-C 2fi's impose less constraints on the estimable models, but cause the model to have D-efficiency (see definition in Section 4) less than 1 since they are not orthogonal to other effects. From this discussion it it clear that having a large number of type-C 2fi's and a small number of type-A 2fi's is desirable. The numbers of 2fi's of different types are summarized below for each design.

design	2fi type	no. of 2fi's	comments
H16.I	A	105	
	B	0	
	C	0	
H16.II	A	57	
	B	48	each aliased with 3 other 2fi's
	C	0	
H16.III	A	33	
	B	72	48 aliased with another 2fi 24 aliased with 3 other 2fi's
	C	0	
H16.IV	A	21	
	B	84	each aliased with another 2fi
	C	0	
H16.V	A	21	
	B	28	each aliased with 3 other 2fi's
	C	56	

H16.V is the only design that has 2fi's of type-C. The reason is that any interaction ij

between columns i and j, where $i = 1, \cdots, 7$ and $j = 8, \cdots, 15$, never appears in any defining word of length 3 or 4. Altogether there are 56 (=7×8) such 2fi's. From this point of view, H16.V is the best and followed by H16.IV, H16.III, H16.II, and H16.I in descending order.

3 Experimental plans with fewer than 15 factors

So far we have assumed that all the columns are used up for factor assignment. If there are fewer than 15 factors, how should they be assigned to the columns? To use Hall's 16-run designs in practical experimentation, we are confronted with the problem of assigning factors to the columns of the matrices. This task is relatively easy in fractional factorial designs like H16.I, where the resolution and aberration criteria can be used. For nonregular designs like H16.II–V, a simple approach adopted in this paper is to apply the same resolution and aberration criteria to the incomplete defining contrast subgroup. Since partial aliasings of effects are not captured by the incomplete defining contrast subgroup, the full capacity for estimation of nonregular designs is not measured by these criteria. The authors have recently developed a more general criterion called estimation capacity, which works for fractional factorial as well as nonregular design and allows a more comprehensive comparison of designs such as H16.I–V. These results will appear soon.

For a given subset of columns, it is straightforward to obtain the corresponding incomplete defining contrast subgroup from the original incomplete defining contrast subgroup by eliminating the defining words that involve column labels not present in the subset. Minimum aberration designs of 5 to 8 factors based on the word-length pattern of the incomplete defining contrast subgroup are given here. The designs indicated by "*" are "no aberration" designs in the sense that there is no fully aliased relation among the columns.

no. of factors	design		columns	word-length pattern 3 4 5 6 7 8 9
5	H16.I		1 2 4 8 15	0 0 1
	H16.II	*	1 2 4 8 12	0 0 0
	H16.III	*	1 2 4 8 10	0 0 0
	H16.IV	*	1 2 4 6 8	0 0 0
	H16.V	*	1 2 4 8 9	0 0 0
6	H16.I		1 2 4 7 8 11	0 3 0 0
	H16.II		4 5 8 10 12 15	0 0 0 1
	H16.III	*	1 2 4 8 10 12	0 0 0 0
	H16.IV	*	1 2 4 6 8 10	0 0 0 0
	H16.V	*	1 2 4 8 9 10	0 0 0 0
7	H16.I		1 2 4 7 8 11 13	0 7 0 0 0
	H16.II		1 4 6 8 10 12 15	0 1 2 0 0
	H16.III		2 4 7 8 10 12 15	0 0 0 0 1
	H16.IV	*	1 2 4 6 8 10 12	0 0 0 0 0
	H16.V	*	1 2 4 8 9 10 12	0 0 0 0 0
8	H16.I		1 2 4 7 8 11 13 14	0 14 0 0 0 1
	H16.II		4 5 6 7 8 9 12 14	0 5 0 2 0 0
	H16.III		2 3 4 5 8 10 12 15	0 1 0 2 0 0
	H16.IV		1 2 4 6 8 10 12 15	0 0 0 0 0 1
	H16.V		1 2 4 7 8 9 10 12	0 1 0 0 0 0
9	H16.I		1 2 3 4 5 8 9 14 15	4 14 8 0 4 1 0
	H16.II		4 5 6 8 9 10 12 13 14	0 9 0 6 0 0 0
	H16.III		2 4 7 8 9 10 11 12 14	0 3 2 0 2 0 0
	H16.IV		2 3 4 5 6 8 10 12 14	0 1 0 0 2 0 0
	H16.V		1 2 4 7 8 9 10 11 12	0 2 0 0 0 1 0

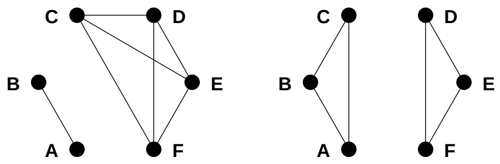
Note that there is no length-3 defining word for designs with 9 factors obtained from H16.II–V, i.e., they all have "resolution IV". For H16.I, the best design with 9 factors has only resolution III. In this sense, H16.II–V are superior to H16.I.

4 Accommodation of specific interactions

In this section we show how H16.II–V can provide better experimental plans than H16.I in case special patterns of interactions are deemed important and should be estimated clear of each other and of the main effects.

Consider an experiment consisting of six 2-level factors A, B, C, D, E, and F in two groups. A design is required to estimate all the main effects and factor interactions within each group; the inter-group factor interactions are assumed to be negligible. Also, as required by most experimenters, the run size should be as small as possible.

Two ways of grouping are considered: $\{A, B\} \cup \{C, D, E, F\}$ and $\{A, B, C\} \cup \{D, E, F\}$. The model estimation requirement in these two situations can be expressed by the interaction graphs:



where dots represent factors and lines represent interactions. There are respectively 14 and 13 numbers of parameter to be estimated (grand mean + main effects + required interactions).

Sixteen-run fractional factorial design (2^{6-2}) seems to be appropriate for this task. However, among the four possible 2^{6-2} designs, none of them provide a solution to this problem as indicated by the 11 interaction graphs in Wu and Chen (1992, pages 171-172). These 11 graphs are all subgraphs of the 4 graphs in Appendix B (*i*).

Instead, we can use the columns 1, 2, 8, 10, 11, 13 of H16.V as factors A, B, C, D, E, and F. Main effects are obviously all estimable. For the remaining 9(=15-6) degrees of freedom, we consider all *feasible interaction graphs*, which represent all the sets of 2fi's that can be simultaneously estimated assuming all other 2fi's and higher order interactions are negligible. A complete collection of graphs for this assignment is given in Appendix B (*v*). The problem in the example can be easily solved by simply picking graph no. 7 and no. 6. However, we should point out that for these designs, the 2fi's are not necessarily orthogonal to the main effects or to other 2fi's, hence resulting in some loss of estimation efficiency. We use the following measure called D-efficiency to measure design efficiency:

$$D = \frac{1}{n}|X'X|^{1/p}$$

where the columns of $X_{n\times p}$ correspond to the effects in the model including the grand mean, the main effects, and the specified interactions. The D-efficiency is given below each design in Appendix B.

For H16.I–IV, we also give their interaction graphs for 6 factors in Appendix B. In terms of the number of non-isomorphic graphs, H16.I is inferior to H16.II–V. Among H16.II–V, no design is uniformly better than the others. Why can H16.II–V allow more flexible estimation of interactions than H16.I? First, from the defining relations in the incomplete defining contrast subgroup, some interactions do not appear in length-3 or length-4 words and therefore are estimable. But this does not give a full explanation since partial aliasings are not captured by the group. Second, Wang and Wu (1992) use the notion of *hidden projection* (as opposed to *geometric projection* employed by Box and Hunter, 1961, for fractional factorial designs) to explain why, often in nonregular designs, interactions are estimable. The hidden projection properties of these designs are bestowed by the complexity of the partial aliasing relations.

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Appendix A

Hall's 16-run non-regular orthogonal designs

H16.II

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-
+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-
+	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+
+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-
+	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+
+	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+
+	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-
+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-
+	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+
+	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+
+	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-
+	-	-	+	+	-	-	+	+	-	-	+	-	+	+	-
+	-	-	+	+	-	-	+	-	+	+	-	+	-	-	+
+	-	-	+	-	+	+	-	+	-	-	+	+	-	-	+
+	-	-	+	-	+	+	-	-	+	+	-	-	+	+	-

H16.III

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-
+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-
+	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+
+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-
+	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+
+	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+
+	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-
+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-
+	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+
+	-	+	-	-	+	-	+	+	-	-	+	+	-	-	+
+	-	+	-	-	+	-	+	-	+	+	-	-	+	+	-
+	-	-	+	+	-	-	+	+	-	-	+	-	+	+	-
+	-	-	+	+	-	-	+	-	+	+	-	+	-	-	+
+	-	-	+	-	+	+	-	+	-	+	-	-	+	-	+
+	-	-	+	-	+	+	-	-	+	-	+	+	-	+	-

H16.IV

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-
+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-
+	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+
+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-
+	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+
+	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+
+	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-
+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-
+	-	+	-	+	-	-	+	+	-	-	+	-	+	-	+
+	-	+	-	-	+	+	-	-	+	-	+	+	-	-	+
+	-	+	-	-	+	-	+	-	+	+	-	-	+	+	-
+	-	-	+	+	-	+	-	-	+	-	+	-	+	+	-
+	-	-	+	+	-	-	+	-	+	+	-	+	-	-	+
+	-	-	+	-	+	+	-	+	-	+	-	-	+	-	+
+	-	-	+	-	+	-	+	+	-	-	+	+	-	+	-

H16.V

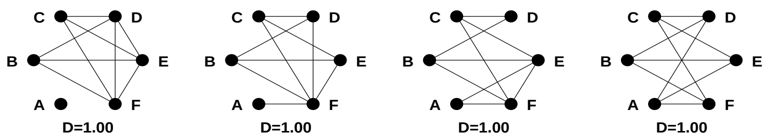
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-
+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-
+	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+
+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-
+	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+
+	+	-	-	-	-	+	+	+	-	+	-	+	-	+	-
+	+	-	-	-	-	+	+	-	+	-	+	-	+	-	+
+	-	+	-	+	-	+	-	+	+	-	-	-	-	+	+
+	-	+	-	+	-	+	-	-	-	+	+	+	+	-	-
+	-	+	-	-	+	-	+	+	-	-	+	-	+	+	-
+	-	+	-	-	+	-	+	-	+	+	-	+	-	-	+
+	-	-	+	+	-	-	+	+	-	+	-	-	+	-	+
+	-	-	+	+	-	-	+	-	+	-	+	+	-	+	-
+	-	-	+	-	+	+	-	+	-	-	+	+	-	-	+
+	-	-	+	-	+	+	-	-	+	+	-	-	+	+	-

Appendix B

Some interaction graphs for H16.I-V designs

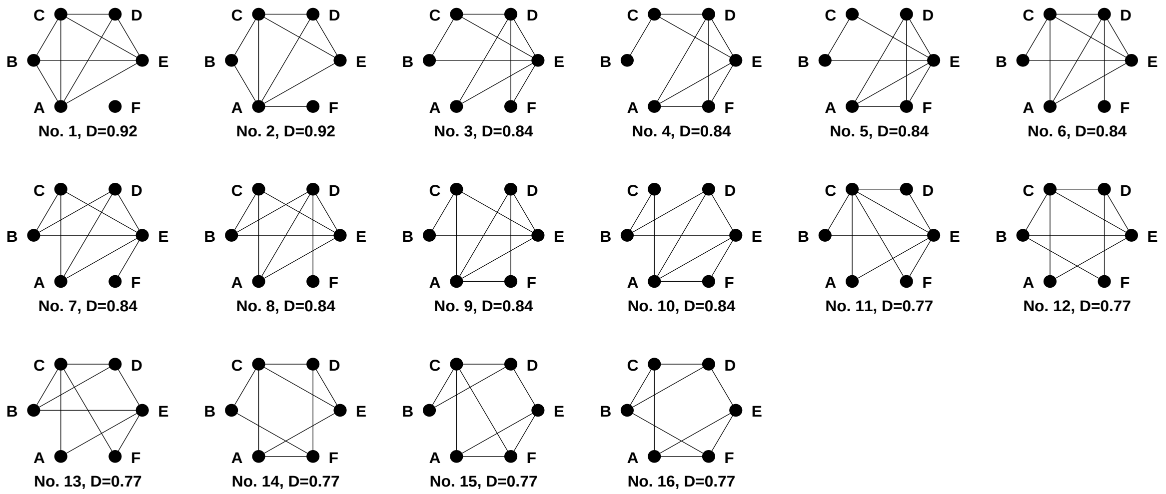
(i) **H16.I**

factors	A	B	C	D	E	F
column No.	1	2	3	4	8	13



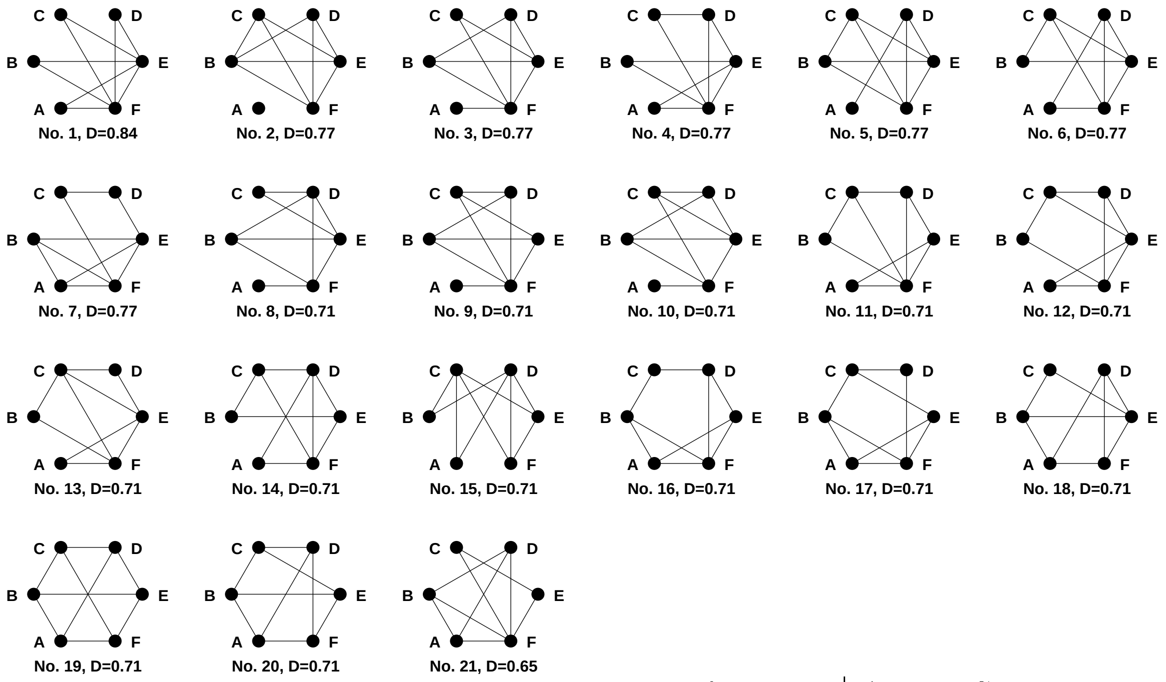
(ii) **H16.II**

factors	A	B	C	D	E	F
column No.	1	4	6	8	11	12



(iii) **H16.III**

factors	A	B	C	D	E	F
column No.	2	6	8	10	12	13



No. 1, D=0.84; No. 2, D=0.77; No. 3, D=0.77; No. 4, D=0.77; No. 5, D=0.77; No. 6, D=0.77

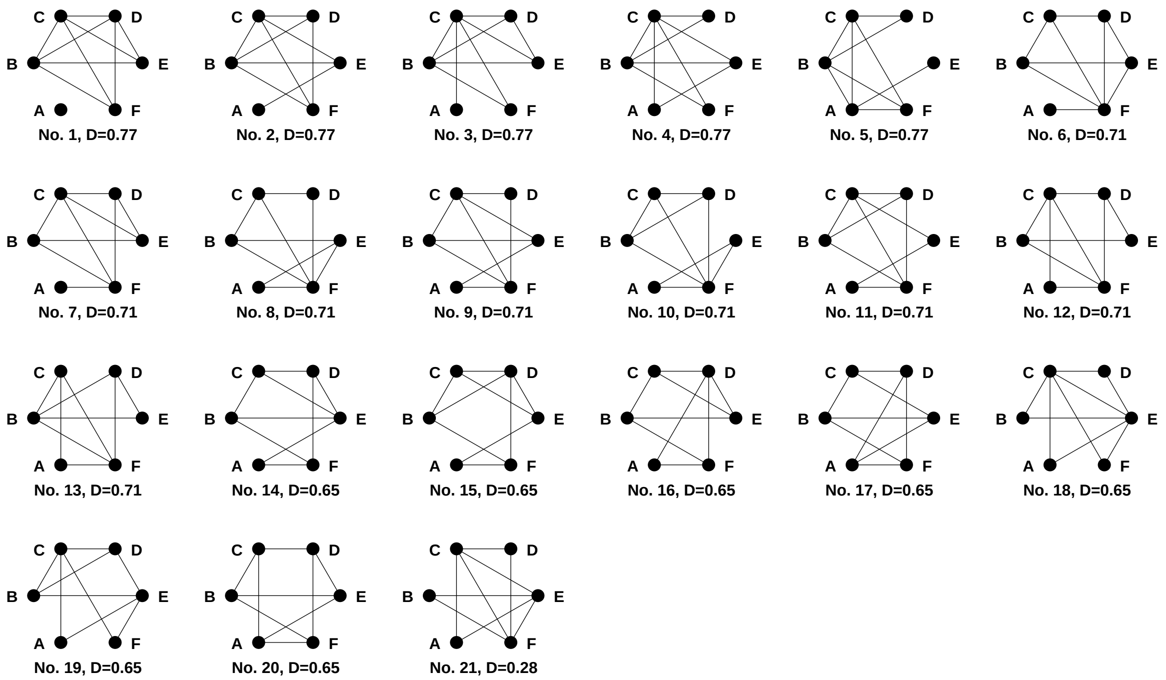
No. 7, D=0.77; No. 8, D=0.71; No. 9, D=0.71; No. 10, D=0.71; No. 11, D=0.71; No. 12, D=0.71

No. 13, D=0.71; No. 14, D=0.71; No. 15, D=0.71; No. 16, D=0.71; No. 17, D=0.71; No. 18, D=0.71

No. 19, D=0.71; No. 20, D=0.71; No. 21, D=0.65

(iv) **H16.IV**

factors	A	B	C	D	E	F
column No.	2	4	5	8	12	14



No. 1, D=0.77; No. 2, D=0.77; No. 3, D=0.77; No. 4, D=0.77; No. 5, D=0.77; No. 6, D=0.71

No. 7, D=0.71; No. 8, D=0.71; No. 9, D=0.71; No. 10, D=0.71; No. 11, D=0.71; No. 12, D=0.71

No. 13, D=0.71; No. 14, D=0.65; No. 15, D=0.65; No. 16, D=0.65; No. 17, D=0.65; No. 18, D=0.65

No. 19, D=0.65; No. 20, D=0.65; No. 21, D=0.28

(v) **H16.V**

factors	A	B	C	D	E	F
column No.	1	2	8	10	11	13