DESIGN AND PERFORMANCE OF UNDERLAY-OVERLAY CELLULAR NETWORKS

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Abstract: We study the design and performance of *Underlay-Overlay* (U-O) wireless networks, in which the available spectrum is partitioned into two groups - an overlay group with a conservative reuse factor, and an underlay group with a more frequent reuse. A key performance factor is the *absorption*, i.e. the proportion of traffic in a cell that can be supported on the underlay.

To address the two principal problems in U-O networks, namely, (a). Where are the boundaries of the underlay to be placed? (b). How are the channels to be allocated? we propose an optimization problem and a traffic model respectively. Our optimization problem introduces a novel set of interference constraints, and can be effectively approximated using bounds and heuristics. The traffic model, which is a simplified queueing system with a product form solution, effectively captures capacity gains realized from absorption. Numerical results on a simple linear network suggest substantial capacity gains from the U-O design compared to a single layer network.

INTRODUCTION

We introduce techniques for the design of an *underlay-overlay* (U-O) network. This well known method for capacity enhancement (also known as Reuse Partitioning) has been described in S.W.Halpern, 1983; Pattavina et al., 1995; Whitehead, 1985; Sallberg et al., 1987. As explained in S.W.Halpern, 1983, the design involves partitioning the channels into two groups - overlay (outer cell) or A-channels assigned to the overlay, but potentially accessible by all mobiles, and B-channels, accessible only to underlay (inner cell) mobiles, or B-mobiles, and reused more frequently. B-mobiles have low path loss (less than some threshold P_i in cell i) to the serving base station, while the remaining mobiles, the A-mobiles, are in the overlay, where the path loss is higher. Usually power control is applied between the two types of mobiles and the B-mobile transmitter powers on both up and down links are set lower than the A-mobiles. In this way interference throughout the network is further reduced thereby extending the scope and effectiveness of the overlay. In this paper it is assumed that constant received power control is adopted. Also we concentrate on the uplink as it is the more onerous of the two.

A key performance factor in U-O networks is the *absorption*, i.e. the proportion of a cell's traffic that can be supported within the inner cell. This traffic is made up of mobiles who have a low path loss to their serving basestation and high path loss to nearby basestations. Only these mobiles can share channels with the tighter reuse required on the underlay. Apart from the actual level of reuse, the proportion of such mobiles depends on the propagation between cells in that part of the network together with the actual geographical distribution of mobiles themselves.

Another critical performance factor is the mechanism by which the channels are allocated. The simplest possible scheme to operate is to make a fixed allocation between the overlay and underlay networks. Unfortunately this loses some trunking efficiency, thus reducing the performance gains from adapting the tighter reuse in the underlay. In this paper we therefore consider a natural sharing scheme whereby underlay calls are allowed to overflow onto overlay channels. Under idealised assumptions this sharing scheme has a product form solution as we demonstrate subsequently. This model has the further advantage that the blocking performance of a given cell depends only on its absorption. It should be noted that limiting the scope for sharing in this way constrains the degree to which capacity can be enhanced, see Borst and Whiting, 1998; Shimada et al., 1995.

The obstacles to capacity gains, mentioned earlier, motivate us to consider the impact of the inherent inhomogeneity of real networks which is the main thrust of this paper. Setting the path loss thresholds to the same value in all cells is conservative. To make more effective use of the network resources we propose an adaptive method of choosing the underlay cell boundaries so that the risk of excessive interference is constrained within some acceptable level. In our scheme this is done by measuring the path loss to co-channel base stations in conjunction with the path loss to the mobile's own base station. As we will show, such measurements can be used to estimate the moment generating functions for the interference between any pair of cells and thus yield a bound on the risk of excessive interference as a function of path loss bound-

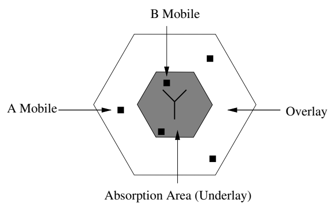

Figure 1 A Cell in an Underlay-Overlay Network.

ary. Furthermore we show that these bounds can be used as constraints to an optimisation problem which allows efficient setting of the underlay boundaries. Whilst this optimisation problem is hard to solve exactly, we compute tight upper bounds on the absorbed area and provide iterative heuristics to obtain efficient solutions.

The layout of the rest of the paper is as follows in section 1 we present a traffic model for the performance of a cell with ideal repacking of calls from the underlay to the overlay. From this model infeasible channel allocations which cannot meet a blocking target can be determined. For feasible allocations a minimum level of absorption can also be determined. In section 2 the optimisation problem itself is presented and the solution technique is outlined. Additionally a numerical example is provided.

A TRAFFIC MODEL

Figure 1 depicts the layout of a single cell within a U-O network. As explained in the introduction, there are two types of mobile, A mobiles which may only take a channel from a group of C_A channels and B mobiles which may use a channel from this group or a further group of C_B channels. The B mobiles are confined to the absorption area in the centre of the cell, whereas the A mobiles will be on the periphery, as shown in figure 1. The offered traffic of each type is assumed to be Poisson with rates ν_A and ν_B respectively. We suppose that there is *perfect repacking* so that B mobiles always use B channels as soon as they become available. Mobility is not modelled so that the mobile type remains fixed throughout the duration of its call. All of this is clearly an approximation of what would happen in an actual network. Under these assumptions the *state space* for the number of calls of each type which can be carried is

$$n_A \leq C_A, n_A + n_B \leq C_A + C_B \tag{1.1}$$

where n_A, n_B denote the numbers of A and B mobiles respectively. We may thus obtain the following product form solution for the equilibrium probability distribution, see F.P.Kelly, 1979.

$$\pi(n_A, n_B) = G^{-1}(C_A, C_A + C_B) \frac{\nu_A^{n_A}}{n_A!} \frac{\nu_B^{n_B}}{n_B!} \tag{1.2}$$

Here G is given by

$$G(C_A, C_A + C_B) = \sum_{n_A \leq C_A, n_A + n_B \leq C_A + C_B} \frac{\nu_A^{n_A}}{n_A!} \frac{\nu_B^{n_B}}{n_B!} \tag{1.3}$$

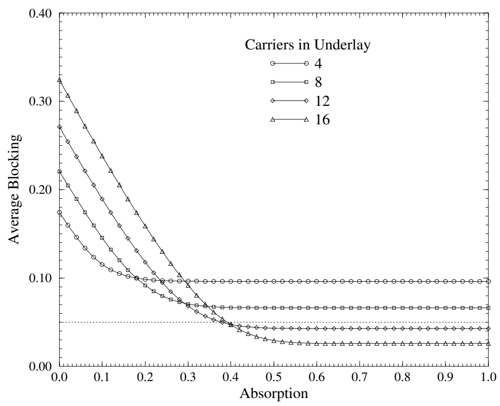

Figure 2 Blocking Performance as a Function of Absorption

In cases where the numbers of channels is relatively small, the normalisation constant G in (1.2) can be computed and used to derive the loss probabilities for each class of mobile:

$$
\begin{aligned}
B^{outer} &= 1 - \frac{G\left(C_A - 1, C_A + C_B - 1\right)}{G\left(C_A, C_A + C_B\right)} \\
B^{inner} &= 1 - \frac{G\left(C_A, C_A + C_B - 1\right)}{G\left(C_A, C_A + C_B\right)}. \\
B &= \alpha B^{inner} + (1-\alpha) B^{outer}
\end{aligned}
\tag{1.4}
$$

Here α is the *absorption*. Given the total offered traffic ν, the traffic in each layer can be computed as

$$\nu_A = (1-\alpha)\nu, \quad \nu_B = \alpha\nu. \tag{1.5}$$

In what follows it is useful to note that for a given number of channels in each layer, the above may be inverted to obtain the level of absorption for which a particular grade of service may be attained,

$$\alpha = A\left(\nu, B_T, C_A, C_B\right), \tag{1.6}$$

where B_T is the desired mean blocking. A is readily determined since A is a monotonically decreasing function of α.

Figure 2 is a graph of average blocking vs. absorption for an example network with an offered traffic of $\nu = 30.0$E, and a total of $C = 60$ channels for various allocations of channels to the inner cells. In this network the reuse is taken as $r_A = 2$ in the overlay and, $r_B = 1$ in the underlay with conservation of channels giving the constraint,

$$r_A C_A + r_B C_B = C. \tag{1.7}$$

This example will be enlarged on subsequently. As the figure shows, once a certain absorption threshold is reached little, improvement in average blocking can be obtained. As is expected, the smaller the number of channels allocated to the underlay the lower the absorption needed to reduce blocking to a given desired level. In order to obtain the target level of 5 % mean blocking marked, at least 12 channels must be allocated to the underlay.

MAXIMIZING CAPACITY IN U-O NETWORKS

As mentioned earlier, the optimal placement of the boundary between overlay and underlay is of central importance in U-O networks. Path loss serves as a natural admission criterion to the underlay, determining the underlay boundary and consequently the absorption. In this section, we formulate an optimization problem to set these path loss thresholds so as to maximize the absorption in the network, given some statistical input information regarding traffic and path loss. We assume the presence of constant-received-power control, i.e. the mobiles transmit at just enough power to be received at the base-station at a fixed target level. We also assume that we are given all possible splits (C_A, C_B) of the channels into overlay and underlay sets. Our proposed procedure for optimal design of U-O networks is hence summarized as follows:

Pruning: Determine the set of feasible channel splits (C_A, C_B).

Optimization: For each split, determine the maximum achievable absorption in the network from the solution to our optimization problem.

Channel Allocation: Choose the path loss thresholds and channel split corresponding to the largest capacity gains.

Objective Function

Given the channel split (C_A, C_B), our objective is to maximize network capacity by maximizing carried traffic, which is given by

$$\mathcal{A} = \sum_i \nu_i \mathcal{A}_i = \sum_i \nu_i (1 - B_i) = \sum_i \nu_i \{1 - \alpha_i B_i^{(inner)} - (1-\alpha_i) B_i^{(outer)}\} \quad (1.8)$$

where ν_i is the traffic offered to cell i, α_i is the absorption in cell i, and the average, underlay and overlay blocking ratios $B_i, B_i^{(inner,outer)}$, whose dependence on $(\nu_i, \alpha_i, C_A, C_B)$ has been suppressed, are given in section 1. For later convenience, we introduce the acceptance ratio $\mathcal{A}_i = (1 - B_i)$. We expect the absorption values α_i to depend on the path loss thresholds P_i in the cell. Also, we expect that the P_i, and hence the achievable α_i, for a given cell is dependent on that of its neighbors through the mechanism of interference, which is the key delimiter of capacity in wireless networks.

Important properties of the objective 1.8 are monotonicity and concavity with respect to the α_i, which can be verified by checking that

$$\frac{d\mathcal{A}_i}{d\alpha_i} \geq 0, \qquad \frac{d^2\mathcal{A}_i}{da_i^2} \leq 0$$

using the equations of section 1, or more simply by inspecting the blocking curves in figure 2. For our solution methods to be most effective however, we

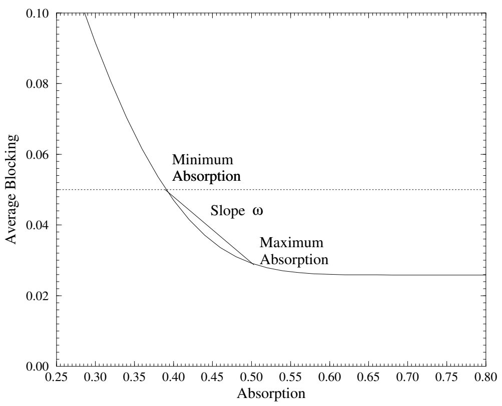

Figure 3 Linear Approximation of Blocking as a Function of Absorption.

require the concavity of the objective with respect to the P_i. Noting that the only dependence on P_i arises through the α_i, we have

$$\frac{d^2\mathcal{A}_i}{dP_i^2} = \frac{\partial^2\mathcal{A}_i}{\partial a_i^2}\left(\frac{d\alpha_i}{dP_i}\right)^2 + \frac{\partial\mathcal{A}_i}{\partial\alpha_i}\frac{d^2\alpha_i}{dP_i^2}.$$

We thus note that a sufficient condition for concavity of the objective with respect to P_i is

$$\frac{d^2\alpha_i}{dP_i^2} \leq 0$$

i.e. the concavity of the α_i.

For computational simplicity, however, we would like to approximate the objective 1.8 by a more transparent and tractable function of the absorption. To this end, note that the blocking for a single cell behaves almost in a piecewise linear manner, as seen in figure 3, with an initial drop followed by a flat portion. If we restrict ourselves to the initial part, where absorption offers perceptible gains, we may approximate the capacity gain in the cell as a linear function of the absorption, i.e.,

$$A_i = K(C_A, C_B) + \nu_i\omega_i\alpha_i$$

where $K(C_A, C_B)$ is a constant depending only on the channel split and

$$\omega_i = -\frac{dB_i(\nu_i, \alpha_i)}{d\alpha_i}$$

is the slope of the blocking curve in the range of interest. This range will be explained in greater detail in the following subsection. For a given (C_A, C_B), the constant $K(C_A, C_B)$ can be ignored in solving for the most capacity gains, but it must be taken into account when picking the final solution, which involves choosing the optimal channel split. This then suggests the following choice of objective

$$A = \sum_i \nu_i \omega_i \alpha_i.$$

This simple linear approximation of the cell blocking may be augmented by higher order refinements, or other approximations, without changing our treatment substantially. We however require the approximation to preserve the property of concavity, which is a requirement justified by the convexity of the blocking curves in figure 2. For illustrative purposes, we confine ourselves to the linear approximation in the following.

As for the functions $\alpha_i(P_i)$, one may choose them to statistically approximate the traffic distribution within the cell as accurately as possible. Again, for illustration, we restrict to the simple choice of uniform distribution of traffic within the cell. As a concrete illustration, consider a network with a path loss exponent of 4, i.e. the received signal is attenuated by a factor r^{-4} compared to the transmitted signal, where r is the distance to the transmitter. In such a situation the area of the underlay region is roughly proportional to the square root of the path loss, yielding $\alpha_i(P_i) = \sqrt{P_i}$. This suggests the following objective.

$$A(\mathbf{P}) = \sum_i \nu_i \omega_i \alpha_i(P_i) = \sum_i \nu_i \omega_i \sqrt{P_i}$$

As mentioned before, one may replace this simple choice of $\alpha_i(P_i)$ by other concave functions to reflect the true traffic distribution within the cell without changing our formulation in an essential way.

Constraints

We cannot make the underlay cells indefinitely large since the interference between cells will then become unacceptable. Again, we use statistical criteria to reflect the risk that the interference is unacceptable in a given cell. We expect the risk of excessive interference in the cell to be a monotonic function of the P_i values in neighboring co-channel cells.

To be more precise, denote the random path loss between a mobile in cell i in the underlay and base station j by l_{ij}. The actual interference caused is determined by the transmitted power of the interfering mobile. If we assume constant received power control and that mobiles are placed in the underlay provided their path loss is smaller than P_i then the contribution to interference in cell i is $l_{ij} P_i$. Denoting the set of neighbors of cell j by $c(j)$, we then propose the following probabilistic criterion for excess interference.

$$Pr\left\{ \sum_{i \in c(j)} P_i l_{ij} \geq \beta \right\} \leq \gamma \tag{1.9}$$

where β denotes the worst case interference acceptable and γ the probability with which it is exceeded. The principal difficulty in using such a constraint is that the above probability can only be evaluated by numerical convolution of the marginal probability distributions. An alternative approach is to apply large deviations (see J.Bucklew, 1990) to equation 1.9. Define the log-moment-generating-function of a random variable X, denoted $\varphi(t)$, as

$$\varphi(t) \equiv \ln \mathcal{E}[e^{tx}]$$

where $\mathcal{E}[]$ denotes expected value and t is known as the Chernoff parameter. The theory of large deviations then provides the following useful bound

$$-\ln Pr(X > C) \geq \sup_t \{Ct - \varphi(t)\}$$

Assuming independence of the l_{ij}, it follows that

$$\ln \mathcal{E}[e^{t_j \sum_{i \in c(j)} P_i l_{ij}}] = \sum_{i \in c(j)} \varphi_{ij}(t_j P_i)$$

where t_j is the Chernoff parameter for cell j and $\varphi_{ij}(t) \equiv \ln \mathcal{E}[e^{t l_{ij}}]$ is the log-moment-generating-function of the path loss from cell i to cell j. These properties can be used to restate the condition 1.9 as

$$-\ln Pr \left\{ \sum_{i \in c(j)} P_i l_{ij} \geq \beta \right\} \geq \beta t_j - \sum_{i \in c(j)} \varphi_{ij}(t_j P_i) \geq -\ln \gamma$$

which constitute the set of interference constraints for our optimization problem. Note that the first inequality is valid for arbitrary positive t_j, in particular for the value that maximizes this composite function.

These interference constraints, which are key to our formulation, perform effective trade-off between the gains in different parts of the network. The φ_{ij} can be efficiently obtained in real networks using path loss measurements reported by mobiles. In obtaining them in this way, we have a useful characterization of the interference in the network, including the effects of propagation and traffic distribution within the cells. Further, in narrow-band networks where interference for any user comes from a small number of sources, the φ_{ij} reflect only the random positions of the interferers and are hence insensitive to the traffic intensity in the interfering cells. We thus obtain a statistical representation of the actual interference in contrast to the usual worst case assumptions. However, in the present analysis, the interference constraints are conservative from the point of view that we assume all the interferers to be simultaneously active. This can be remedied by reflecting interferer activity in the log-moment-generating-functions.

Other important constraints arise as bounds on P_i for each cell based on target performance and incremental gains, which are indicated in figure 3. Given the offered traffic, each cell must meet a blocking criterion to provide adequate quality of service. This requirement imposes a lower bound on the absorption, and hence the P value, in the cell. The lower bound is determined by computing an acceptable absorption level, say $\underline{a}$, for a given blocking requirement, say 5%, from the offered traffic ν and the channel split (C_A, C_B) using the single cell behavior discussed in section 1. For a fourth power propagation law, we then obtain $\underline{P} = \sqrt{\underline{a}}$.

For the upper bound $\overline{P}$, we note that the initial linear behavior of the blocking gives way to a flatness around a certain critical absorption value $\overline{a}$ (see figure 3), indicating that the incremental improvement in cell performance drops significantly beyond this value. This determines the upper bound $\overline{P} = \sqrt{\overline{a}}$, since any further absorption in the cell would be ineffectual for the cell's performance while being detrimental to its co-channel neighbors. The upper bounds are also useful in characterizing the interference situation accurately. This is due to the fact that the range of the path loss random variables l_{ij}, used to compute the log-moment-generating-functions φ_{ij}, is restricted by the maximum allowable path loss $\overline{P}_i$ in the interfering cell. In our experiment however, we do not take this dependence into account, and we use some estimate of the largest $\overline{P}$ in the network.

Optimal Design for Given (C_A, C_B)

With these assumptions and observations, we finally arrive at the following optimization problem

$$\max \quad \sum_i \mu_i \sqrt{P_i} \tag{1.10}$$

subject to

$$\begin{aligned} t_j\beta \quad &- \sum_{i\in c(j)} \varphi_{ij}\,(t_j P_i) + \log\gamma \geq 0. \\ \underline{P}_i \leq \quad & P_i \quad \leq \overline{P}_i \end{aligned} \tag{1.11}$$

where $\mu_i = \nu_i\omega_i$ is a cell and channel-split dependent constant. A detailed mathematical discussion of bounds and heuristics for the above problem is provided in the appendix.

Pruning the Set of Channel Splits

To obtain the feasible channel splits (C_A, C_B), we first compare the best achievable blocking for each candidate. As shown in figure 2, the flattening of some of the blocking curves above the target eliminates the corresponding channel splits. For each of the remaining cases, we perform a preliminary feasibility check on the constraints, as explained in the appendix, which consists of verifying the feasibility of all interference constraints with the path loss thresholds set to their lowest values. The channel splits that clear both of the above tests are the feasible ones.

The Optimal Solution

The solution P_i^* to 1.10 for each feasible (C_A, C_B) can be used to obtain the network blocking in each case as

$$\mathcal{A} = \sum_i \nu_i(1 - B_i)$$

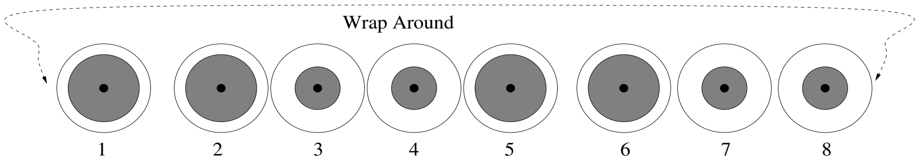

Figure 4 Schematic for the Solution of the Example Network.

and the final optimal settings $(C_A^{opt}, C_B^{opt}, P_i^{opt})$ are obtained by choosing the set yielding the maximum value of $\mathcal{A}$.

An Example Network

Consider the 8 cell network of the type depicted in figure 4. All cells are offered the same traffic which we suppose arises uniformly within each cell. Since the cells are identical, except for their relative positions, we have $\omega_i = \omega, i = 1, \cdots, 8$ and, given that $\nu_i = \nu, i = 1, \cdots, 8$, these quantities may be factored giving the objective function,

$$\mathcal{A} = \sum_{i=1}^{8} \sqrt{P_i}.$$

A 4th power law for the path loss with distance has been assumed together with lognormal shadowing, standard deviation 8 dB. Also alternate cells use the same channels on the underlay giving a reuse factor of 2. Furthermore as described above received power is held constant which we take to be 1.0 without loss of generality. The C/I requirement at the base station is 10 dB, which corresponds to $\beta = 0.1$. α is taken to be 1 % so that $-\ln \alpha \approx 4.6$.

The inhomogoneity arises from the fact that the locations of the base station within each cell are chosen with a random offset from the centre of the cell. As we will see, this simple form of inhomogoneity already captures the trade-offs in absorption that one would expect in real networks.

The results of section 1 showed that with this network the minimum allocation of channels to the underlay needed to achieve a blocking target of 5 % was $C_B \geq 12$.

Then, using the above parameters the 16 log moment generating functions, $\varphi_i^{(l)}, \varphi_i^{(r)}, i = 1, \cdots, 8$ were obtained by simulation. These functions were used to examine the feasibility of the remaining channel split. This was done by confirming that the interference constraints could be met for the minimal path loss thresholds required for each channel split. This constrained the channel allocation to the underlay to satisfy $12 \leq C_B \leq 22$. All of these splits meet the blocking target.

In order to compute the upper bound on the absorption we take each cell in turn together with its two interfering cells and compute $A(c(i))$. This gives the constraints listed in table 1.

This linear program is readily solved to show that

$$\sum_k \sqrt{P_k} \leq 4.397 \tag{1.12}$$

Cell No.	Interferers	Constraint
1	(8,2)	$\sqrt{P_8} + \sqrt{P_2} \leq 1.128$
2	(1,3)	$\sqrt{P_1} + \sqrt{P_3} \leq 1.080$
3	(2,4)	$\sqrt{P_2} + \sqrt{P_4} \leq 1.121$
4	(3,5)	$\sqrt{P_3} + \sqrt{P_5} \leq 1.190$
5	(4,6)	$\sqrt{P_4} + \sqrt{P_6} \leq 1.102$
6	(5,7)	$\sqrt{P_5} + \sqrt{P_7} \leq 1.087$
7	(6,8)	$\sqrt{P_6} + \sqrt{P_8} \leq 1.197$
8	(7,1)	$\sqrt{P_7} + \sqrt{P_1} \leq 1.173$

Table 1 Constraints in the Linear Program to compute upper bound on absorption.

with the biting constraints being 1,2,5 and 6 as numbered above. It should be noted that a tighter upper bound could be obtained by incorporating the upper and lower bounds on path loss threshold described earlier.

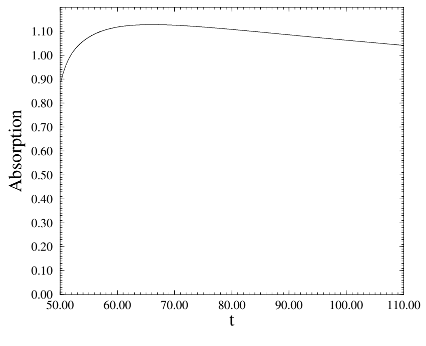

Figure 5 t_2 vs. Absorption for Cells 1 and 3

Figure 5, typical of the results we obtained for the bounds, exemplifies the insensitivity of the optimum values to the choice of t. Starting with the t parameters obtained from the bound we then applied the iterative optimization heuristic described in the appendix. We thus obtained our solution as given in table 2 below. The solution is represented in figure 4. As can be expected from the cell positions (indicated by the black dots) cells 3,4,7 and 8 are restricted, while the remaining cells have large absorption areas. Indeed it appears that cells 2 and 5 are "squeezing" cells 3 and 4 and similarly cells 6 and 1 are "squeezing" 7 and 8. The value obtained for the objective function was 4.314 which is less than 2 % smaller than the bound. As we will now see, it is not the case that the split which allocates the maximum number of channels to the

Cell No.	1	2	3	4	5	6	7	8
Threshold	0.347	0.369	0.248	0.261	0.339	0.339	0.220	0.227
Cell Area %	59	61	50	51	58	58	47	48

Table 2 Feasible solution

C_B	12	13	14	15	16	17	18	19	20	21	22
Blocking%	4.3	4.3	3.4	3.5	2.8	3.0	2.5	2.8	2.4	2.9	2.7

Table 3 Blocking Performance for Feasible Channel Allocations

underlay will attain the least blocking. Indeed by computing the mean network blocking for each feasible split using the absorption table below the following results were obtained in table 3. These show that $C_B = 20$ yields the lowest average blocking, i.e. 2.4 % which amounts to a carried traffic of 29.25 E. This should be compared with a fixed system of 30 channels per cell which has a blocking of 5 % for a carried traffic of 23.75 E traffic. For a blocking of 2.4 % using fixed, only 21.9 E can be carried. This is a capacity improvement of about 30 % in carried traffic for the same blocking criterion.

CONCLUSIONS AND RECOMMENDATIONS

In this paper, the two most critical aspects of designing U-O networks have been explored i.e. how many channels to assign to the underlay and where to place the underlay cell boundaries. The solution to these problems is complicated by the inhomogeneity of both traffic and propagation in real networks. We have presented a traffic model and an optimisation procedure to maximise the capacity of U-O networks. Our techniques rely on statistical information regarding traffic and propagation, which we believe to be readily obtainable.

Although some of our assumptions are conservative, our method can be suitably adapted to be more realistic. Examples include the assumption that all co-channel interferers of each mobile were active and that the carried traffic depended linearly on absorption. The first can be addressed by using new moment generating functions that take intermittent activity into account and the second by making a more precise representation of carried traffic.

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Appendix: Mathematical Aspects

Problem Statement and Iterative Heuristic

We proposed the following optimization problem to maximize capacity in the underlay-overlay network:

$$\max \sum_i F_i(P_i)$$

subject to

$$\mathcal{C}_j = t_j\beta - \sum_{i \in c(j)} \varphi_{ij}\,(t_i P_j) + \log\gamma \geq 0$$

$$\underline{P}_i \leq P_i \leq \overline{P}_i \tag{1.13}$$

where $\gamma, \beta, \mu_i, \underline{P}_i, \overline{P}_i$ are given constants, $F_i(P_i)$ are given concave functions of P_i, namely the carried traffic in cell i, and φ_{ij} are convex functions of their respective arguments, i.e. the logarithms of the moment generating functions of the interference from cell i to cell j. For fixed t_i, we note that (1.13) poses a Concave Program amenable to the Strong Lagrangian Principle Whittle, 1971. This implies that the objective function and the constraints have non-positive second derivatives with respect to the P_j, which are our decision variables. (The second derivative with respect to each P_j can be considered separately since the cross derivatives of the form $\partial^2/\partial P_j\,\partial P_k, j \neq k$ vanish for the objective and constraints. In general one would need to show the negative-semi-definiteness of the Hessian matrix $\mathcal{H}_{jk}(F) \equiv [\partial^2 F/\partial P_j\,\partial P_k]$.) We can hence employ, for instance, the gradient descent technique described in K.Arrow, 1958 to obtain the unique global optimum. This technique proceeds as follows:

1. Define the set of Lagrange multipliers $\Lambda_i, \underline{\lambda}_i, \overline{\lambda}_i$ for the set of constraints. We hence seek to

$$\min_{\mathbf{\Lambda}, \underline{\lambda}, \overline{\lambda}} \max_{\mathbf{P}} \sum_j \left[F_j(P_j) + \Lambda_j \mathcal{C}_j + \underline{\lambda}_j(P_j - \underline{P}_j) + \overline{\lambda}_j(\overline{P}_j - P_j)\right] \tag{1.14}$$

   where bold-face denotes vectors.

2. Starting from some positive initial values and a sufficiently small step-size ρ, repeat the following steps until the change in objective value is satisfactorily small.

$$\begin{aligned}
P_j &\leftarrow P_j + \rho\left[\frac{dF_j}{dP_j} + \sum_{i:j \in c(i)} \Lambda_i \varphi_{ij}\,(t_i P_j) + \underline{\lambda}_j - \overline{\lambda}_j\right] \\
\Lambda_j &\leftarrow \Lambda_j - \rho\left[t_j\beta - \sum_{i \in c(j)} \varphi_{ij}\,(t_i P_j)\right] \\
\underline{\lambda}_j &\leftarrow \underline{\lambda}_j - \rho(P_j - \underline{P}_j) \\
\overline{\lambda}_j &\leftarrow \overline{\lambda}_j - \rho(\overline{P}_j - P_j)
\end{aligned} \tag{1.15}$$

Unfortunately, while the objective function is concave in the P_i, the interference constraints are not concave in the variables (P_i, t_i), as can be shown by computing the determinant of the Hessian $\mathcal{H}$ of the function

$$\psi(t) = \beta t - \varphi(tP).$$

Thus, the problem ceases to be a concave program when considered as a joint optimization in the pair (t, P), while concavity holds in either one of them separately.

However, we may use the standard machinery available to solve concave programs to devise the following heuristic to obtain a local optimum. Given an initial feasible solution (P_j^0, t_j^0), we may easily re-solve for the optimal t_j for the given P_j^0 as

$$t_j = \arg\max_t \left\{ \beta t - \sum_{i \in c(j)} \varphi_{ij}\left(tP_j^0\right) \right\} \tag{1.16}$$

owing to the concavity of the constraints with respect to the t_j. Notice that the P_j retain feasibility under the new t_j, since the LHS values of the constraints are only increased by the new t_j. This implies that the optimal objective value obtained from the iterative procedure (1.15) with the new t_j is lower bounded by the current objective value. Starting from an initial feasible solution (P_j^0, t_j^0), we may hence use the following iterative procedure to obtain guaranteed convergence to a local optimum of our optimization problem (1.13), since the objective value is necessarily non-decreasing at each step of the iteration.
Heuristic **ITER**:

1. Solve for the optimal t_j using equation (1.16).
2. For this t_j, solve the concave program using the procedure described in equations (1.15).
3. Repeat the procedure till the change in objective value is satisfactorily small.

Checking for Feasibility

Before embarking on heuristic ITER to solve a specific instance of this problem, it is necessary to prove the existence of a solution. To do so, we devise the following test of feasibility. Note that the minimum risk of interference to a given cell j occurs when all its neighbors have the lowest possible threshold values, i.e. $P_i = \underline{P}_i$ for all $i \in c(j)$. It then follows that we must have

$$t_j\beta - \sum_{i \in c(j)} \varphi_{ij}\left(tP_i\right) + \log\gamma \geq 0$$

for all j and some positive set of Chernoff parameters t_j. We hence verify the conditions for the maximizing values t_j^*, which can be shown to be unique using the concavity of the φ_{ij}. Conversely, if this condition holds, then we may set $P_j = \underline{P}_j$ to obtain a feasible solution, which makes the condition necessary and sufficient. Since the tests only involve solving a set of single parameter equations for the optimizing t values and the corresponding risks, they provide us with an efficient method to check the feasibility of 1.13.

Bounds and Initial Solution

To obtain the maximum effectiveness from the above described iterative heuristic **ITER**, it is essential to have a good starting feasible solution (P_j^0, t_j^0). We obtain this by using a simple bounding procedure that examines small subnetworks of the full network. Consider the reduced problem defined as

$$A_j = \max \sum_{i \in c(j)} F_i(P_i)$$

subject to

$$t\beta - \sum_{i \in c(j)} \varphi_{ij}\,(tP_i) + \log\gamma \geq 0$$

$$\underline{P}_i \leq P_i \leq \overline{P}_i \qquad (1.17)$$

The optimal solution to this problem, which contains only a single Chernoff bound parameter t, can be obtained in a simple manner by scanning over the range of possible values using standard techniques for 1-dimensional optimization. We observed that both *Bisection* and *Golden Section* methods performed well in searching the t-space. For our 1-dimensional example networks, in fact, the objective turned out to be unimodular in t. In the full network, it then follows that

$$\sum_{i \in c(j)} F_i(P_i) \leq A_j \qquad (1.18)$$

for all j, since the solution (t^*, P_i^*) to (1.17) saturates the interference constraint on cell j while ignoring the constraints imposed by the rest of the network.

Based on the subnetwork solutions, we can obtain an upper bound on the objective, which serves as a test-bed to the solution produced by **ITER**. Consider the Linear Program (LP)

$$A^* = \max \sum_i x_i$$

subject to

$$\sum_{i \in c(j)} x_i \leq A_j, \qquad \underline{x}_j \leq x_j \leq \overline{x}_j \quad \forall j \qquad (1.19)$$

where the $x_j \equiv F_j(P_j)$, $\underline{x}_j = \underline{P}_j/\mu_j$ and $\overline{x}_j = \overline{P}_j/\mu_j$. Observe that the solution to (1.19) gives us an upper bound on the objective, i.e. $A_{max} \leq A^*$, and hence A/A^* is an indicator of the relative quality of the solutions A obtained using **ITER**. The LP (1.19) can be mapped into a flow problem, which is easily solved using the Ford-Fulkerson theorem. For the 1-dimensional networks we considered, the solution could be obtained by inspection.

The Chernoff Parameters t_j

The optimal solution is somewhat insensitive to the choice of the Chernoff parameters t_j. This insensitivity is also reflected in the auxiliary problems solved to obtain bounds. Thus the precise choice of t_j is not critical and, as the results in our example network show, using the values obtained from the bound lead to very efficient results.

To understand this robustness, we consider the solution to the bounding problem. The insensitivity to t seen in Figure 5 is a reflection of the smallness of the higher moments of the twisted interference distributions. Define the Chernoff exponent for the ith cell to be

$$\begin{aligned} \psi_i(t_i) &\equiv \beta t_i - \sum_j \varphi_{ji}(P_j t_i) \\ \frac{d\psi_i}{dt_i} &= \beta - \sum_j P_j \varphi'_{ji}(P_j t_i) \\ \frac{d^2\psi_i}{dt_i^2} &= -\sum_j P_j^2 \varphi''_{ji}(P_j t_i). \end{aligned} \tag{1.20}$$

Also observe that changes in the objective function obtained by varying t_j are obtained as

$$\delta A \approx \lambda_j \frac{d\psi_j}{dt_j} \delta t_j \tag{1.21}$$

so that unless t_j varies significantly from the choice corresponding to the optimal solution, the result will be efficient.