Scheduling on Uplink of CDMA Packet Data Network with Successive Interference Cancellation

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***Abstract*— Uplink scheduling in wireless systems is gaining importance due to arising uplink intensive data services (ftp, image uploads etc.), which could be hampered by the currently in-built asymmetry in favor of the downlink. In prior work [1], [2], we proposed optimal algorithms for uplink scheduling in a CDMA cell that does not employ any form of interference cancellation. In this work, we modify the approach to incorporate *Successive Interference Cancellation* (SIC), which has been shown to be optimal in an information theoretic sense [3]. As in [1], [2], no statistical assumptions are made about channel or traffic behavior, but feedback to communicate current channel state and queue state are assumed. Our results demonstrate that the throughput optimal scheduling strategy takes a particularly simple form with SIC as compared to without [1], [2], apart from providing some level of performance improvement. A reasonable alternative algorithm based purely on received power can be constructed based on early work on SIC [4]. Considering decoding errors, only strongly received users can benefit from SIC. Our simulation experiments suggest that our throughput optimal scheduling improves performance over the alternative when users have similar received power. Combining the above observations, we also propose a hybrid scheduling algorithm that performs SIC for strong users and simultaneous transmission for weak users.**

I. Introduction

Data scheduling in wireless networks is a widely studied topic due to the impending explosion of high speed wireless data services in third generation (3G) systems. For natural reasons associated with the expected traffic characteristics, most of the previous research has focused on the forward-link/downlink, i.e. base to mobile communication. The traffic is expected to be dominated by web browsing and file downloads. As a result, current wireless data systems employ highly asymmetric link designs (e.g. HDR) with skinny uplinks and fat downlink pipes [5]. However, it has also often been pointed out that there could be a proportional increase in reverse-link/uplink traffic in the form of acknowledgments, feedback etc. along with the growth of other services like ftp, image/data uploads etc. which require high data rates on the uplink. These considerations have resulted in some research on the subject of uplink scheduling [6], [7], [8], [9], [10], [11], although small compared to the literature on downlink scheduling.

In this work, we consider the problem of optimal scheduling of uplink user transmissions in a single CDMA cell with Successive Interference Cancellation (SIC) employed at the base-station. SIC is a well-known technique discussed in much past work, see e.g. [4], [12], in the context of multi-user detection. It has been shown that the performance of SIC is mainly determined by user decoding order. [4], [12] also pointed out that users with high received power at the base will more likely decode correctly, hence they will benefit most from SIC. Scheduling with SIC has been analyzed in [3] and related work from an information theoretic point of view for given traffic and channel statistics and constraints on average power consumption. In this work, we analyst scheduling with SIC more from a system queuing stability/Quality of Service (QoS) point of view without any assumptions on users channel or traffic statistics, and with instantaneous transmit power constraints for each user. Alternatively, we assume *instantaneous* (short-term) feedback information in the form of of users' queue lengths/delays and channel state inferred from received power. It is our belief that these constraints better reflect todays wireless systems and devices.

In previous work [1], [2], we addressed the problem of optimal scheduling without Successive Interference Cancellation (SIC). The main observation in that work was that simultaneous transmission by "weak" users, time-shared with one-at-a-time transmission by "strong" users[1] can lead to improvements in both capacity and fairness. Further, we observed that an introduction of an orthogonality factor $f \in [0, 0.5)$, which reflects high degree of interference suppression among users, favors more simultaneous transmission. A similar conclusion seems plausible with SIC, which is a specific method of interference suppression. In fact, we will show that under our assumptions, the throughput optimal scheduling algorithms favor completely simultaneous transmission by all backlogged users, who are only differentiated by the order in which they are decoded. Assuming perfect decodability, this ordering also turns out to be particularly simple, suggesting that the algorithms are easily implemented in practical systems. However, perfect decodability is likely to be a very strong assumption in practical wireless systems, and the best user ordering would be complicated if decoding error is taken into account. The traditional ordering used in the SIC context is strongest-to-weakest [4], which is optimal if the instantaneous sum of user rates (assuming infinite backlog) were the criterion, and not total throughput or weighted QoS measure as we propose here. We use this scheme as a benchmark in our simulation study.

[1]Hereafter, we refer to users with low received power at the base even when transmitting at peak transmit power as "weak" users, and the strongly received users at the base as "strong" users.

We observe that it performs similarly to the throughput optimal algorithm when users with different channels all perform SIC assuming no decoding error. This is because both algorithms converge to the same decoding order in this scenario. However, they perform differently when users have similar received power at the base.

Combining the observation that strong users benefit the most from SIC, we propose a hybrid scheduling scheme that performs SIC for strong users while letting weak users transmit simultaneously. The former is because strong users are generally immune to decoding errors, while the latter is motivated by the observation that simultaneous transmission without SIC is beneficial for weak users [1]. We show that the system performance, especially for the weak users, improve substantially when the weak users suffer from high ($> 20\%$) decoding errors.

We begin with the derivation of the throughput optimal scheduling with SIC. We then provide the description for the optimal scheduling, benchmark, and hybrid scheme. This is followed by simulation results and observations.

II. Scheduling with Successive Interference Cancellation

We now consider the effect of Successive Interference Cancellation (SIC) on scheduling. This problem has been studied in [13] for the downlink broadcast channel from the point of view of maximizing a weighted sum of user rates subject to constraints on long-term average power consumption. They assume that the users' have pre-specified channel statistics. Here we focus on SIC for the uplink multi-access channel and on achieving stability of the user queues, or satisfying users' QoS, with just instantaneous channel conditions from feedback. No assumption is made about long-term channel statistics, and the constraints we consider are limits on the instantaneous transmit power. With SIC, the SIR equation of the individual users for a given decoding order may be written as

$$\frac{P_i}{\sum_{j<i} P_j + 1} = \gamma_i \qquad i = 1, 2, ..., N \tag{1}$$

where it is implicitly assumed that the users are ordered in some specific way. Note that the ordering is a crucial aspect of SIC, since it plays an important role in determining the users target received powers through (1). The ordering chosen above determines the *decoding order* of users at the base-station, which is the inverse order $N, ..., 2, 1$. As the name suggests, SIC involves decoding the next user after subtracting out the interference from all previously decoded users, thus making this user immune to the previously decoded ones. As before, we first derive the *feasible region* of (1), also derived in[13], to help formulate and solve our problem. Define $S_i \triangleq \sum_{j=1}^{i} P_i$ to rewrite (1) as

$$\frac{S_i - S_{i-1}}{S_{i-1} + 1} = \gamma_i \qquad i = 1, 2, ..., N \tag{2}$$

where $S_0 \triangleq 0$. (2) then leads to the identity $S_i = \gamma_i + (1 + \gamma_i)S_{i-1}$, i.e. $1+S_i = (1+\gamma_i)(1+S_{i-1})$. From these identities, it is clear that equations (1,1) can be explicitly solved as

$$1 + S_i = \prod_{j=1}^{i} (1 + \gamma_j) \qquad i = 1, 2, ..., N \tag{3}$$

which leads to

$$P_i = S_i - S_{i-1} = \gamma_i \prod_{j=1}^{i-1} (1 + \gamma_j) \qquad i = 1, 2, ..., N. \tag{4}$$

For feasibility under given instantaneous power constraints, we require $0 \leq P_i \leq \bar{P}_i$, which then leads to the final feasibility condition

$$\gamma_i \prod_{j=1}^{i-1} (1 + \gamma_j) \leq \bar{P}_i \qquad i = 1, 2, ..., N. \tag{5}$$

As before (see [1], [2]), the scheduling problem is then stated as

$$\max_{\gamma} \sum_{i=1}^{N} Q_i R_i \tag{6}$$

where $R_i = f(\gamma_i)$ is the user rate and Q_i is the user weight which is chosen here to be the queue content for user i to guarantee queue stability [14], [15], [16]. (6) must be solved subject to the feasibility constraints (5). All our analytical results below assume the Shannon formula $R_i \triangleq \beta \log(1+\gamma_i)$. With some modification of the methods, certain other choices for $f(.)$ can be used, but the nature of the optimal schedule derived below does not hold in general. In terms of the S_i, we then have the optimization problem

$$\max \sum_i Q_i \log\left(1 + \frac{S_i - S_{i-1}}{1 + S_{i-1}}\right) \tag{7}$$

subject to $S_i - S_{i-1} \leq \bar{P}_i \quad i = 1, 2, ..., N$. Using other functions $f(.)$ for the rate-SIR relationship preserves tractability of the problem for a fixed user ordering if $f\left(\frac{S_i - S_{i-1}}{1+S_{i-1}}\right)$ is jointly concave in S_i and S_{i-1}, but the solution is in general not explicit. With the Shannon formula, a surprisingly simple solution can be obtained, that we derive below. The objective function in (7) can be rewritten as

$$\begin{aligned} &\textstyle\sum_i \; Q_i \log\left(1 + \frac{S_i - S_{i-1}}{1 + S_{i-1}}\right) \\ &= \sum_i Q_i \log\left(\frac{1 + S_i}{1 + S_{i-1}}\right) \\ &= \sum_i (Q_i - Q_{i+1}) \log(1 + S_i) \end{aligned} \tag{8}$$

with the convention $S_0 = 0 = Q_{N+1}$. Recall that the optimal ordering of the users is part of the problem, and still remains to be determined. To maximize (7), we may now invoke the *polymatroid structure* of the rate region of SIC [13], but we give the simple proof below for completeness. The following theorem addresses the optimal user ordering problem.

Theorem 1: The optimal user order for the solution of (7) is given by the condition $Q_1 \geq Q_2 \geq ..... \geq Q_N$.

Proof: Consider two consecutive users in given ordering with queue lengths Q_i, Q_{i+1} and received powers P_i, P_{i+1} respectively. We assume that $Q_j, P_j, \;\; j = 1, 2, .., N$ are fixed and given, and the only unknown is the user ordering. The contribution of these two users to the objective function (7) is given by

$$Q_i \log\left(1 + \frac{P_i}{1 + S_{i-1}}\right) + Q_{i+1} \log\left(1 + \frac{P_{i+1}}{1 + S_{i-1} + P_i}\right).$$

Note that the reordering of these two users does not affect $S_j, \;\; j \neq i, i+1$ and hence the remaining terms in the objective are unaffected by the reordering. To evaluate the gain in the objective value from the reordering, consider the difference

$$\begin{aligned} Q_i \log\left(1 + \frac{P_i}{1 + S_{i-1}}\right) + Q_{i+1} \log\left(1 + \frac{P_{i+1}}{1 + S_{i-1} + P_i}\right) \\ -Q_{i+1} \log\left(1 + \frac{P_{i+1}}{1 + S_{i-1}}\right) - Q_i \log\left(1 + \frac{P_i}{1 + S_{i-1} + P_{i+1}}\right) \\ = (Q_i - Q_{i+1}) \log\left[\frac{(1 + S_{i-1} + P_i)(1 + S_{i-1} + P_{i+1})}{(1 + S_{i-1})(1 + S_{i-1} + P_i + P_{i+1})}\right] \end{aligned}$$

The latter expression is obtained by simple algebraic manipulation, and is non-negative only if $Q_i \geq Q_{i+1}$. This shows that the objective can be increased for any given fixed values of $Q_j, P_j, \;\; j = 1, 2, .., N$, by interchanging users i and $i+1$ if $Q_i < Q_{i+1}$. By induction, this argument continues to hold for multiple users, culminating in the optimality of the claimed ordering $Q_1 \geq Q_2 \geq ..... \geq Q_N$. ■

In light of Theorem 1, we hereafter assume that $Q_1 \geq Q_2 \geq ..... \geq Q_N$ in the following. We now have the optimization problem

$$\begin{aligned} &\max \sum_i (Q_i - Q_{i+1}) \log(1 + S_i) \\ \text{subject to} \quad &S_i - S_{i-1} \leq \bar{P}_i \quad\;\; i = 1, 2, ..., N. \end{aligned}$$

It is easy to see that, for the optimal ordering, the objective is monotonously increasing in each S_i, and it is hence optimal to choose $S_i = \sum_{j=1}^{i} \bar{P}_i$, i.e. all users transmit at full power! Throughput optimal uplink scheduling with SIC hence involves the following steps for each scheduling interval.

Algorithm SICQ

1) Order the users so that $Q_1 \geq Q_2 \geq ..... \geq Q_N$.
2) All users transmit at full power and rates given by $R_i = f(\gamma_i)$ where
$$\gamma_i = \frac{\bar{P}_i}{1 + \sum_{j<i} \bar{P}_j}.$$
3) The users are decoded at the base in the inverse of the Q order.

Note that the above algorithm is extremely simple in comparison to scheduling without SIC [1], [2], and extends to all $f(.)$ that satisfy the reordering property of Theorem 1, i.e.

$$\begin{aligned} Q_i f\left(\frac{P_i}{1 + \sum_{j<i} P_j}\right) + Q_{i+1} f\left(\frac{P_{i+1}}{1 + \sum_{j\leq i} P_j}\right) \\ \geq Q_{i+1} f\left(\frac{P_{i+1}}{1 + \sum_{j<i} P_j}\right) + Q_i f\left(\frac{P_i}{1 + \sum_{j<i} P_j + P_{i+1}}\right) \end{aligned}$$

if $Q_i \geq Q_{i+1}$ for any fixed $Q_j, P_j, \;\; j = 1, 2, .., N$.

As mentioned earlier, past work [4], [12] suggests scheduling the users purely based on received power. The only difference of this scheme from SICQ described above is that the user decoding order corresponds to decreasing received power. We call this scheme **Algorithm SICP**.

a) Effect of Decoding Errors: The above analysis ignore decoding errors, which is only reasonable for strong users. In practice, "weak" users with low received power do not benefit much from SIC on account of significant decoding errors. As a result, SIC is most effective for the "strong" users [12]. Thus, to get the best possible advantages from SIC, it is desirable to perform SIC only for the "strong" users, and allow simultaneous transmission without SIC for "weak" users [1], [2]. However, to maximize the throughput of the weak users, it is advantageous to perform *Multi-User Detection* (MUD) [12], which relies on "soft" decoding to minimize error probabilities. We assume that MUD performs well enough for weak users so as to provide service with negligible error rates. Based on these observations, we propose the following hybrid scheduling algorithm:

Algorithm HYSICP/HYSICQ

1) All users transmit simultaneously at each time slot.
2) All strong users are decoded according to SICP/SICQ.
3) All weak users are decoded with MUD.

The proposed HYSICP/HYSICQ combines the gain of SIC for strong users while weak users are decoded with MUD after eliminating interference from strong users who are reliably decoded with SIC. We expect it will perform better than pure SIC when decoding errors in SIC are significant.

III. Simulation Results

In order to quantify the performance gain by applying SICQ and SICP scheduling algorithms, a discrete-event simulator has been used to evaluate them in a single cell CDMA system with 20 users. The uplink is implemented as a slot based (Time Division) data transmission mechanism, for example, in 3G1xEV-DO (HDR). We will use the time-averaged queue length as the criterion to compare different uplink scheduling algorithms. Individual as well as total average queue lengths are considered for comparison.

A. Scenario I

In our first set of simulations, we assume that the users all have i.i.d. received power at the base with a narrow distribution, i.e. low variance. However, roughly half the users are "heavy" users that run high bandwidth applications and the other half are "light" and require less resources. This experiment is designed to study the advantages of queue sensitive scheduling (SICQ) over SICP without the effect of substantial channel variations. We further assume the following:

1) The scheduling decision is made by the base station for every time slot We use 1.6667 msec time slot as defined in 3G1xEV-DO (HDR).
2) It is assumed that all users share 1.25 MHz bandwidth.
3) It is assumed that the uplink traffic of each mobile user is Poisson with the same inter-arrival time 0.05 sec.
4) The mean received power values at the base are 0dB, -1dB, -2dB, -5dB respectively.
5) It is assumed that packet length is exponentially distributed with mean 2048 bits for "light" users and 5120 bits for "heavy" users.
6) Roughly equal proportions of users were chosen (at random) to be "light" and "heavy".
7) Simulation time = 10 minutes.
8) Discrete rate sets, as in 3G1xEV-DV: $9.6Kbps, 19.2Kbps, ...2.4Mbps$ roughly in powers of 2.
9) No decoding errors occur.

Figure 1 shows that, in this scenario, SICQ performs almost uniformly better than SICP, as expected, on account of the former's queue content responsiveness. We observe that the difference is about 15-20% in average queue length. While this experiment may appear a little unrealistic now, we later motivate its value by considering decoding errors.

B. Scenario II

In this set of experiments, we permit high channel variability with diverse channel conditions among the users. The following additional assumptions were used to make the simulation more "realistic".

1) The location of the mobiles are assumed to be uniformly distributed in the cell area.
2) It is assumed that the link gains have the following form

$$G_i(k) = d_i^{-4}(k)A_i(k)B_i(k) \tag{9}$$

where $d_i(k)$ is the distance from the ith mobile to the base station at time instant k, A_i is a log-normal distributed stochastic process (shadowing). B_i is a fast fading factor (Rayleigh distributed).
3) It is assumed that the cell diameter is 2 km. $d_i(k)$ is a 2-D uniformly distributed random variable.
4) It is assumed that the standard deviation of A_i is 8 dB, [17].
5) It is assumed that the Doppler frequency is 8 Hz, corresponding to pedestrian mobile users, [17].

The simulations were carried out for both uniform traffic among users, as well as the variable traffic as in Scenario I (subsection III-A) above. We do not report numbers for these experiments, as the results were almost identical for SICQ and SICP! This seemingly surprising conclusion can be explained by the observation that the user decoding order

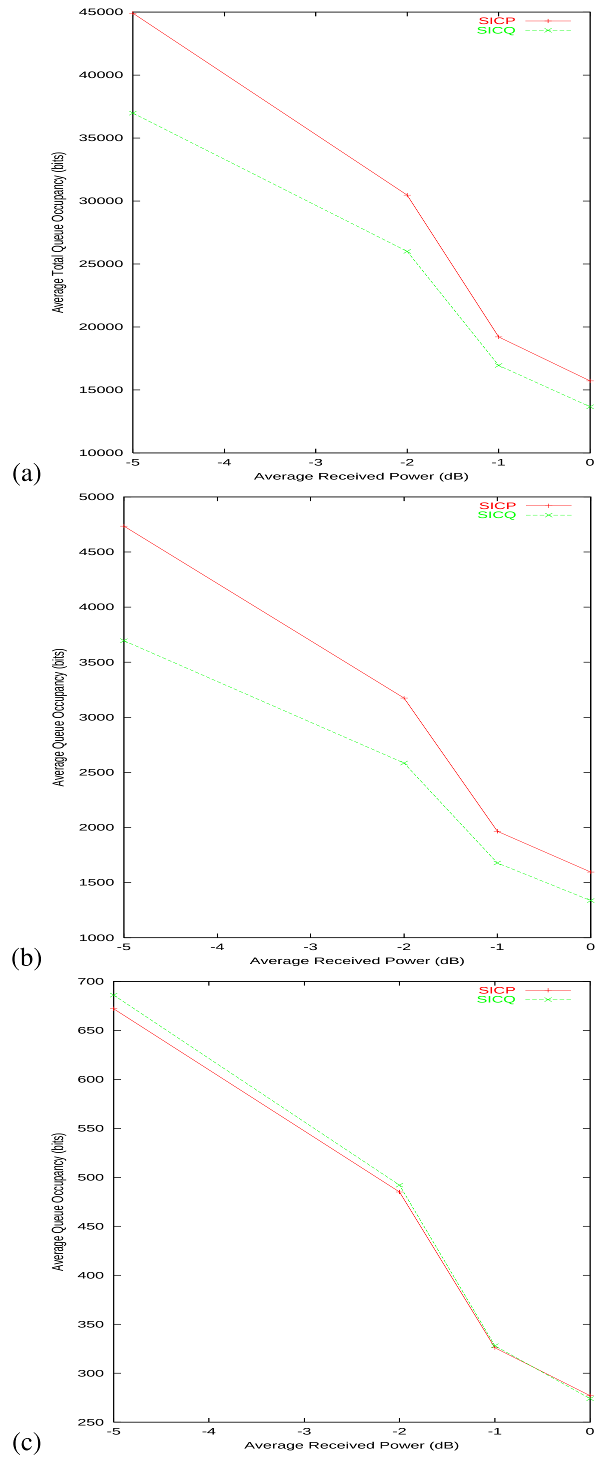

Fig. 1. Average queue length vs. Received Power. (a) Total (b) Typical "heavy" user. (c) Typical "light" user.

for the two algorithms is very likely to be identical for each scheduling interval. This is because users with good channels are very likely to have small queue content, while those with bad channels typically have long queues. Therefore, ordering based on queue lengths and received powers are likely to be identical, thus producing similar performance in both cases.

C. Scenario III

We now report on simulations for HYSICP. The assumptions of Scenarios I (except for link gain) and II (link gain model) apply to this set of experiments as well. In addition, we impose a threshold on received power from users in order to distinguish strong and weak users. We tried threshold values of 0dB and -2dB, and only report the results with 0dB, which was the worse of the two for HYSICP. Recall that the users have inhomogeneous traffic demand as before. Our benchmark is the pure SICP[2] algorithm which assumes no decoding errors, but suffers decoding errors for weak users. We uniformly assume that strong users are decoded perfectly in all cases. There are two extreme scenarios by which we may obtain bounds on the effect of the decoding errors for weak users in SICP:

SICP-BEST: When a decoding error occurs for user i in the decoding order, user i experiences an erasure while all succeeding users in the order face twice the received power of this user ($\bar{P}_i$) as interference. The succeeding user rates are lowered to the best possible rate with this lowered SIR. Clearly this is optimistic, since it is not possible to predict the correct data rates after the randomly occurring decoding errors are realized at the receiver.

SICP-WORST: A decoding error for user i results in erasure for i and all succeeding users. This is pessimistic since there is typically some probability of correctly decoding some of the succeeding users.

Figures 2, 3 summarize the simulation results for this scenario. Observe that when the frame decoding error is in excess of 20%, HYSICP uniformly gives improved performance for all users even over SICP-BEST. Of course, the major part of the gain comes from weak users with heavy traffic, as expected.

IV. Conclusion

We have proposed and studied a throughput optimal scheduling strategy (SICQ) with SIC in a single CDMA cell. When channel and queue length information are available, we show that SICQ is at least as efficient as SICP, which is closer to current implementations of SIC, and can provide some gains when applied only to a subset of reliably decodable users. As a by-product, we also show that SICP has comparable performance because it typically suggests the same decoding order as SICQ. Furthermore, we suggest a candidate hybrid

[2]In light of Scenario I (subsection III-A), we could have applied SICQ to the strong users, but in this scenario, most of the performance gains come from the weak users and the relative overall gain from SICQ over SICP was observed to be small.

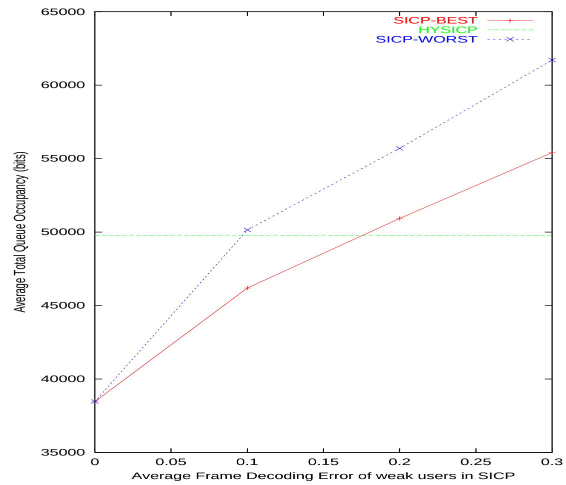

Fig. 3. Total average queue length vs. frame decoding error for weak users.

scheduling strategy to best realize the advantages of SIC accounting for decoding error. This strategy takes a very simple form, and shows significant performance gains in simulations when frame decoding error for weak users exceeds 20%.

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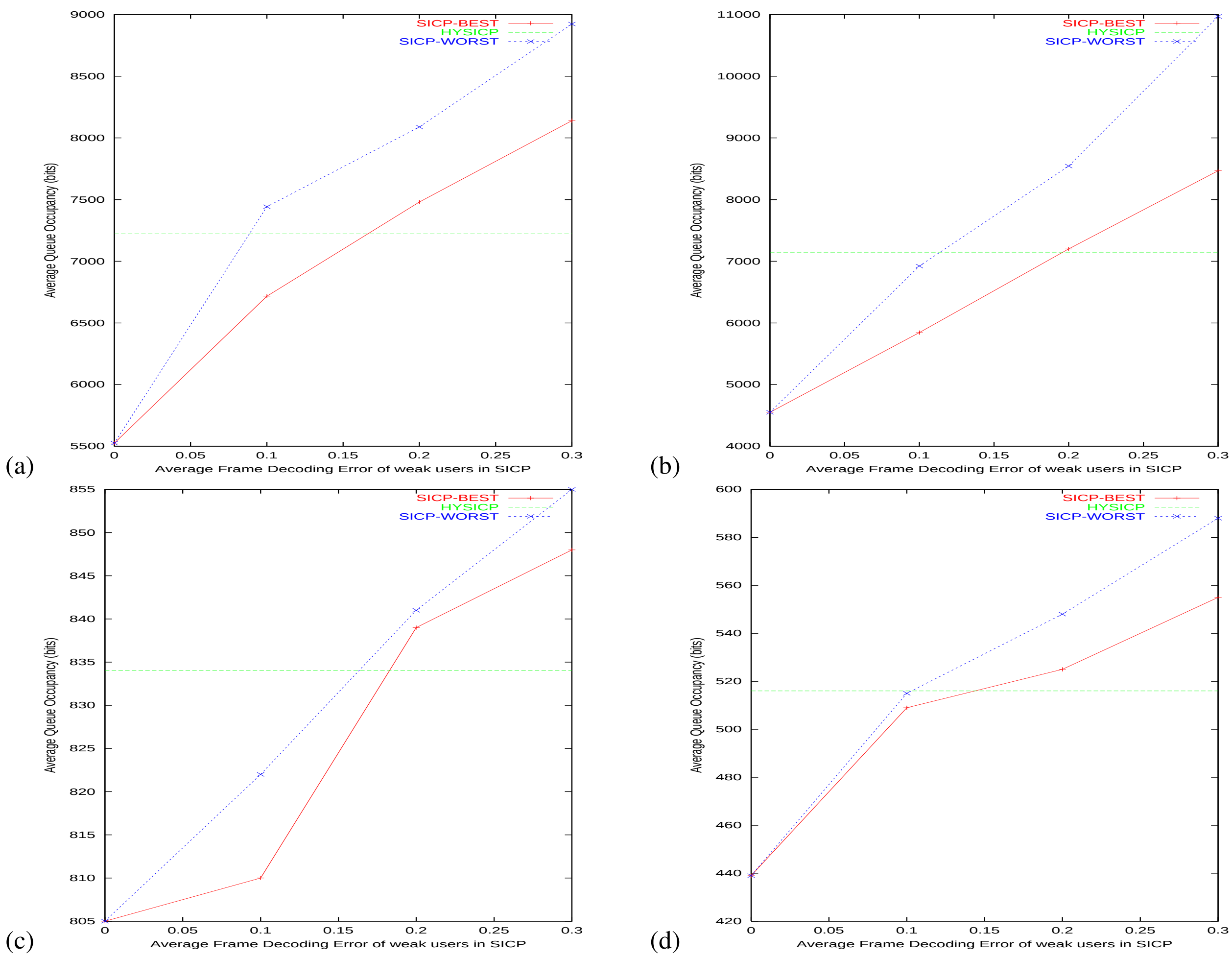

Fig. 2. Average queue length vs. frame decoding error for weak users. (a) Weak user with light traffic. (b) Weak user with heavy traffic. (c) Strong user with light traffic. (d) Strong user with heavy traffic.