A Model of Soft Handoff under Dynamic Shadow Fading

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Abstract

We introduce a simple model of the effect of temporal variation in signal strength on active-set membership, for cellular phone systems that use the soft-handoff algorithm of IS-95a. This model is based on a steady-state calculation, and its applicability is confirmed by Monte Carlo studies.

1 Introduction

The CDMA cellular phone system standard uses the technique of *soft handoff*: a phone call can be carried cooperatively by multiple base-station antennas. The collection of such antennas at a given moment is called the *active set* of the call. The decisions about which antennas in a local area should be in the active set are based on continual monitoring of the radio link quality for each nearby antenna. Plainly, antennas with strong signals should be in the active set, and weak antennas should not.

The active-set-membership decision for a given antenna and a mobile (phone) at a given time is complicated by the rapid and unpredictable pseudo-random variation that the signal undergoes, largely due to the motion of the phone. Such variation has multiple sources: change in mean strength due to changing distance from the antenna, rapid variation due to constructive and destructive interference of signals arriving via different paths to the phone (fast fading), and slower variation due to obstructions (shadow fading). We will ignore fast fading here since its effect is largely averaged out in measurements.

Because of these different causes of variation, the decision algorithm cannot simply keep the strongest antennas at a given instant: it reduces performance to bring antennas constantly in and out of the active set. On the other hand, an antenna that has been left behind due to motion of the mobile should be dropped from the set, since the overall link quality of the active-set antennas should be as high as possible.

We will show that a simple steady-state model can account for a substantial fraction of the effect of shadow fading and motion on soft handoff. This holds in spite of the simplicity of the calculation, and the limited information used by the model.

2 The Decision Algorithm

The above considerations motivated the CDMA IS95a standard [TIA93]. This standard gives an active-set decision algorithm that considers multiple thresholds and observations over time: parameters t_a and t_d are *add* and *drop* thresholds for signal quality (pilot Ec/Io), and parameter τ_c specifies a time period. The parameter t_a will always be larger than t_d. The decision algorithm puts antenna a in the active-set at a decision point if either

- the pilot Ec/Io for antenna a rises above t_a, or
- antenna a was in the active set at the previous decision point, and the pilot Ec/Io for a was above t_d at some time during the previous τ_c time period.

Put another way, when the antenna is in the active set, and the Ec/Io falls below t_d, a *drop timer* is started. The timer is reset to zero whenever the Ec/Io rises above t_d. If the timer exceeds τ_c, the antenna is removed from the active set.

This model is somewhat simplified, but it captures much of the relevant effect.

3 Analysis

We can analyze the decision algorithm under simple assumptions about shadow fading.

3.1 The signal quality time series

We will model the signal quality at the mobile as a discrete time series $E(\tau)$, using a deterministic component $D(\tau)$ and a stochastic component $F(\tau)$ due to shadow fading:

$$\begin{aligned} E(\tau) &= D(\tau) + F(\tau), \quad \tau = i\tau_s, \quad i = \ldots -2, -1, 0, 1, 2 \ldots \qquad (1) \\ D(\tau) &= K_1 - K_2 \log_{10}(d(\tau)) \\ F(\tau) &\sim N(0, \sigma_F^2), \quad \text{for each } \tau. \end{aligned}$$

Here $d(\tau)$ is distance to the base station, expressed as a function of time τ, and K_1 and K_2 are constants. This is a standard model of the pilot signal strength "Ec" [ZH96, ZH98, LM02, PVar]. The following discussion can be viewed as regarding Ec, but a very similar model holds for Ec/Io, under the common and justifiable assumption that the interference Io is approximately log-normally distributed. Moreover, as discussed below, our results should be relatively insensitive to such distinctions.

The auto-correlation function $A(\hat{\tau})$ of shadow fading is well approximated as exponential in $\hat{\tau}$ [Gud91]. This suggests that $F(\tau)$ can be modeled as a first-order auto-regressive (AR) time series, since such series have an exponential auto-correlation function. However, initially we will regard the fades at distinct time steps τ, τ' as independent random variables.

3.2 A recurrence

With this model, we can now consider the combined effect of shadow fading and the decision algorithm on the likelihood of active set membership for a given antenna and a given mobile. We suppose that the mobile is traveling at constant speed, so that the number of steps of the shadow fading time series that occurs during τ_c is some integer k. Under the above assumptions, we can readily formulate a recurrence for the probability $P(\tau)$ of being in the active set at time τ. The coefficients of the recurrence will be based on the probability $P_d(\tau)$ of being above t_d at time τ, and $P_a(\tau)$, the probability of being above t_a at time τ.

The recurrence is obtained as follows. Let $Y(\tau)$ be the random variable that is 1 when the antenna is in the active set at time τ, and 0 otherwise. Let $X(\tau)$ denote the time elapsed since $E(\tau) > t_d$; that is, $E(\tau)$ was above t_d at time $\tau - X(\tau)$, and hasn't been above since that time. We have

$$\begin{aligned} P(\tau) = \mathbf{E}[Y(\tau)] &= \sum_{j\geq 0} \mathbf{E}[Y(\tau) \mid X(\tau) = j] \cdot \text{Prob}\{X(\tau) = j\} \\ &= \sum_{0\leq j\leq k} \text{Prob}\{Y(\tau) = 1 \mid X(\tau) = j\} \cdot \text{Prob}\{X(\tau) = j\}, \end{aligned} \tag{2}$$

but

$$\text{Prob}\{X(\tau) = j\} = P_d(\tau - j)Q_j(\tau), \tag{3}$$

where $Q_j(\tau)$ is the probability of being below t_d at times $\tau - j + 1 \ldots \tau$, so

$$Q_j(\tau) = \prod_{\tau-j+1\leq\tau'\leq\tau} (1 - P_d(\tau')). \tag{4}$$

Here the product is taken to be 1 if $j = 0$.

$$\begin{aligned} &\text{Prob}\{Y(\tau) = 1 \mid X(\tau) = j\} \\ &\quad= \text{Prob}\{E(\tau - j) > t_a \mid X(\tau) = j\} \\ &\quad\quad + \text{Prob}\{t_d \leq E(\tau - j) < t_a \mid X(\tau) = j\} \cdot P(\tau - j - 1) \\ &\quad= \frac{P_a(\tau - j)}{P_d(\tau - j)} + \left(1 - \frac{P_a(\tau - j)}{P_d(\tau - j)}\right) P(\tau - j - 1). \end{aligned} \tag{5}$$

Putting together (2), (3), and (5), we have

$$\begin{aligned} P(\tau) &= \sum_{0\leq j\leq k} \text{Prob}\{Y(\tau) = 1 \mid X(\tau) = j\} \cdot \text{Prob}\{X(\tau) = j\} \\ &= \sum_{0\leq j\leq k} \left[\frac{P_a(\tau - j)}{P_d(\tau - j)} + \left(1 - \frac{P_a(\tau - j)}{P_d(\tau - j)}\right) P(\tau - j - 1)\right] P_d(\tau - j)Q_j(\tau) \\ &= \sum_{0\leq j\leq k} \left[P_a(\tau - j) + (P_d(\tau - j) - P_a(\tau - j))P(\tau - j - 1)\right] Q_j(\tau), \end{aligned} \tag{6}$$

where $Q_j(\tau)$ is given in (4).

3.3 The steady-state probability

Suppose now the steady-state situation, where $P_d(\tau)$ has constant value P_d, and $P_a(\tau)$ has constant value P_a. The steady-state value P of $P(\tau)$ must satisfy

$$\begin{aligned} P &= \sum_{0 \leq j \leq k} [P_a + (P_d - P_a)P]\, Q_j \\ &= [P_a + (P_d - P_a)P] \sum_{0 \leq j \leq k} (1 - P_d)^j \\ &= [P_a + (P_d - P_a)P] \frac{1 - (1 - P_d)^{k+1}}{1 - (1 - P_d)} \\ &= [P_a + (P_d - P_a)P]\, Q/P_d, \end{aligned}$$

where

$$Q \equiv 1 - (1 - P_d)^{k+1}.$$

We have

$$PP_d/Q = P_a + (P_d - P_a)P,$$

or

$$P(P_d/Q - (P_d - P_a)) = P_a,$$

or

$$P = \frac{P_a}{P_a + P_d(1/Q - 1)} = \frac{1}{1 + (1/Q - 1)P_d/P_a}, \tag{7}$$

Observations. Note that, as they should, $P \to 1$ as $Q \to 1$, and $Q \to 1$ as $k \to \infty$ or as $P_d \to 1$. Also, when $k = 0$, we have $Q = P_d$,

$$P_a \leq P = \frac{P_a}{1 - (P_d - P_a)} = \frac{P_d}{1 + \frac{1-P_d}{P_a}(P_d - P_a)} \leq P_d.$$

4 The Local Steady-State Approximation

We propose to use the steady-state probability as our active-set probability estimate. That is, at a given location, we apply (7) to the local values of P_a and P_d, and use the resulting value as an estimate of the probability of active set membership at that location.

This local steady-state model requires only P_a, P_d, and k: whatever the variations in signal strength and speed, their effect is seen only through changes in these quantities. Moreover, when P_d and P_a remain constant, this estimate is exact. Also, for example, a straight-line trip between points equi-distant from the antenna would approximate the steady-state conditions, and hence the approximation would be close.

σ_F	5.5 dB
D_s	400 m
D_c	20 m
τ_c	4 s

Figure 1: Parameter settings holding for all experiments

Heuristically, the worst case for our approximation occurs when P_d changes as rapidly as it can: that is, the steady-state assumption is as wrong as possible. Another source of error can be values of P_d, P_a, and Q such that the active-set Markov process takes a long time to reach steady-state probabilities. Roughly, when is it very hard to get into the active set, but very easy to stay in, it can take arbitrarily long to attain steady-state probabilities.

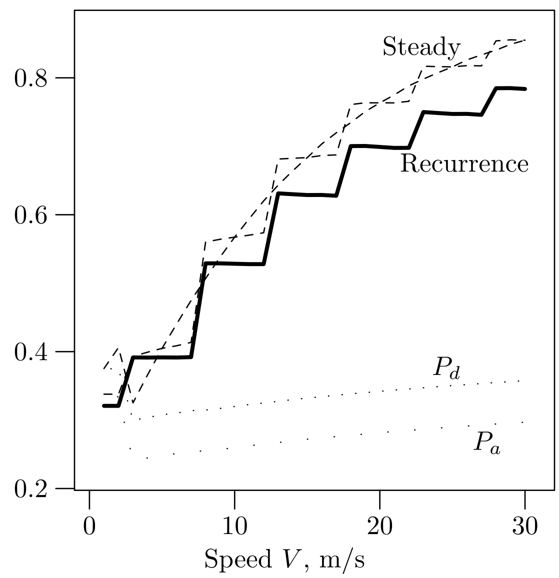
However, as our experimental studies will show, for a wide range of reasonable combinations of parameter values, the steady-state approximation works well.

4.1 Experimental Studies

To gain insight regarding the acceptability of the local steady-state approximation, we consider a mobile moving in a straight line from point s to point f, with P_d values P_d^s and P_d^f at the endpoints. Suppose the intermediate P_d values on the line are estimated by interpolating radio conditions according to a power law with respect to distance, as in (1) above. How will the active set probabilities for the mobile, determined by the active-set decision algorithm, compare with the probabilities found by our steady-state formula, but using the intermediate P_d and P_a values?

In all our studies, we have the following conditions, summarized in part in Figure 1 also:

- $\tau_c = 4$sec.
- The mobile is moving at a constant speed V.
- The distance D_c that determines the time step for the independent normal shadow fading variables is 20 meters. That is, τ_s in (1) is 20m divided by V. (Again, we are assuming the fades are independent random variables, until we stop doing so, below.)
- The number k of discrete time steps in τ_c seconds is $V\tau_c/\tau_s$, rounded to the nearest integer.
- The mobile begins at a distance D_s of 400m; this assumption affects the procedure that interpolates the radio conditions between start and finish, applying our model (1).
- The standard deviation σ_F of the shadow fading is 5.5dB.



V	1, 2, 3...30 m/s
T	14 s
$t_a - t_d$	2 dB
P_d^s	0.05
P_d^f	0.7

Figure 2: Estimated active set probability as a function of speed.

- We consider trips that last T seconds, within a range of 12 to 24. Such a span is shorter than most phone calls, but allows us a substantial averaging of predictions.

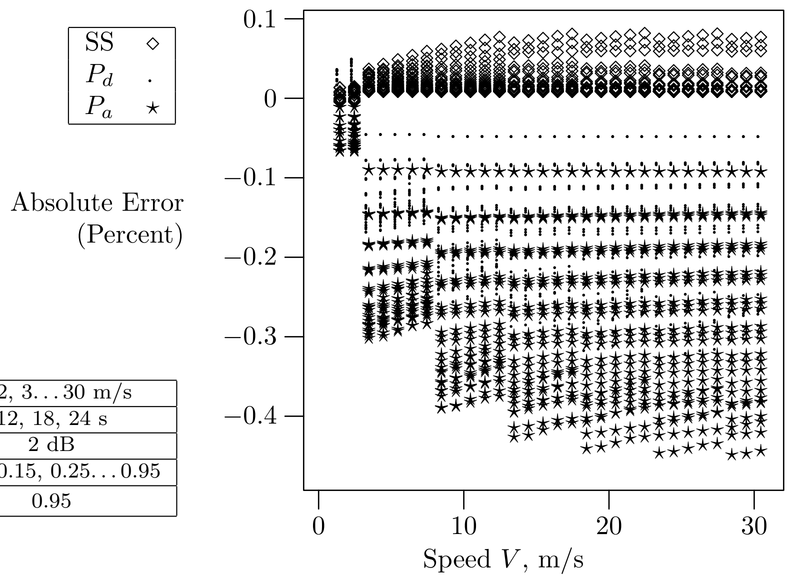
- The recurrence is solved with a "warm up" where $P_d = P_d^s$, of duration $4k$, and k is the number of discrete time steps in τ_c, as above.

- Traffic travels equally in both directions; that is, we average the recurrence results with P_d changing from P_d^s to P_d^f, and then with the same calculation, but with P_d^s and P_d^f swapped.

- The "error ratio" displayed is a ratio of the error of the steady-state estimate to an estimate based on using P_d alone. Here the "error" is the absolute value of the difference from the value from the recurrence, and we add 0.005 to the denominator to wash out large error ratios when both errors are very small.

In Figure 2, we show the dependence of our estimates of active set probability as a function of speed, for some specific parameter values, as shown. The "jagged" nature of the dependence shown by two of the curves is due to the fact that k is rounded to an integer. Of the two curves based on steady-state estimates, the smoother one is based on using the unrounded version of k.

Figure 3 shows the errors of the different estimators, as a function of speed. Note that an estimate based on P_a is quite poor at high speeds, and rarely better than one based on P_d.



V	1, 2, 3. . . 30 m/s
T	12, 18, 24 s
$t_a - t_d$	2 dB
P_d^s	0.05, 0.15, 0.25. . . 0.95
P_d^f	0.95

Figure 3: Errors of different estimates as a function of speed, e.g., ◇ denotes steady state.

Figure 4 shows the "Error ratio", discussed above, as a function of speed, under the same conditions as Figure 3.

Figures 5 and 6 show the error ratios as a function of speed, under the same conditions as before, except that $t_a - t_d$ is 4dB in Figure 5 and 6dB in Figure 6. These values of $t_a - t_d$ are rather large and unlikely in practice, but under such conditions, the steady-state estimator can become less accurate.

The study in Figure 7 has conditions the same as for Figure 4, except that we use a Monte Carlo calculation where a time series of independent Gaussians is replaced by a first-order autoregressive (AR) series. The steps of the autoregressive series are done six times faster than the steps for the independent series, and the multiplier for the AR series is chosen so that the correlation A_c between the fades at 20m is either 0.1 or 0.36; that is, the combinations of conditions for the figure include those two choices for the multiplier. With the value $0.36 \approx 1/e$ for the correlation at 20m, we have autocorrelation $A(\hat{d}) = \exp(-\frac{\hat{d}}{20m})$, so that 20m is the correlation distance.

5 Conclusions

We have seen that a simple local steady-state model of soft handoff can be an accurate predictor of the effect of motion. While a common approach to modeling of cellular systems involves the simulation of the motion of mobiles, we have shown that it is possible to avoid such simulation, at least with respect

V	1, 2, 3. . . 30 m/s
T	12, 18, 24 s
$t_a - t_d$	2 dB
P_d^s	0.05, 0.15, 0.25. . . 0.95
P_d^f	0.95

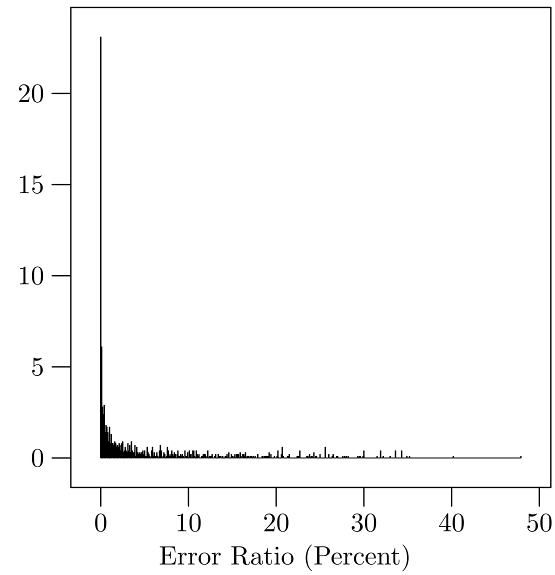


Figure 4: Ratio of error of the steady-state estimate to an estimate based on P_d.

V	1, 2, 3. . . 30 m/s
T	12, 18, 24 s
$t_a - t_d$	4 dB
P_d^s	0.05, 0.15, 0.25. . . 0.95
P_d^f	0.95

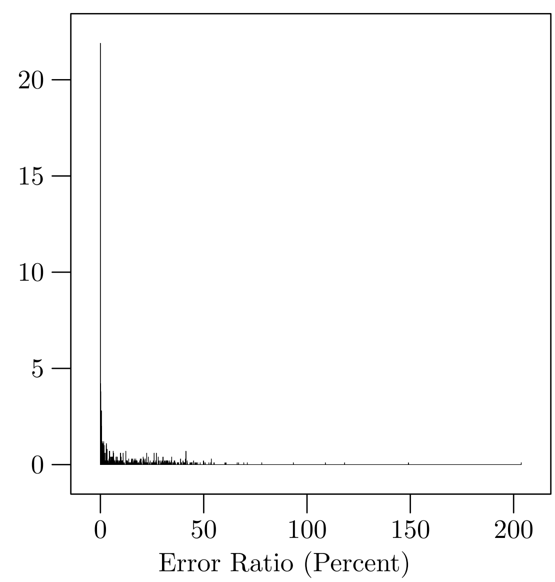
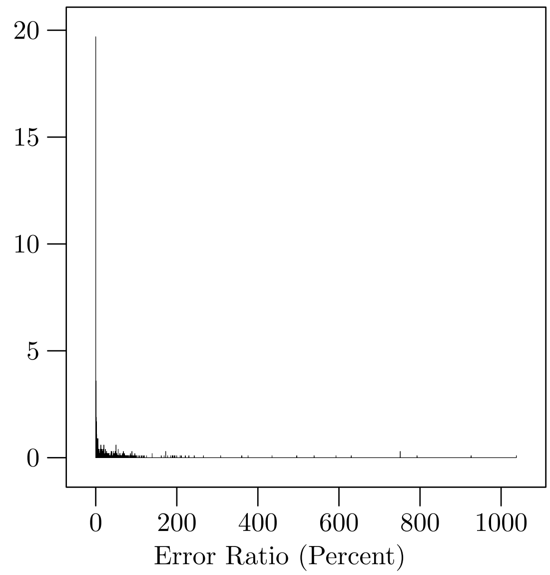
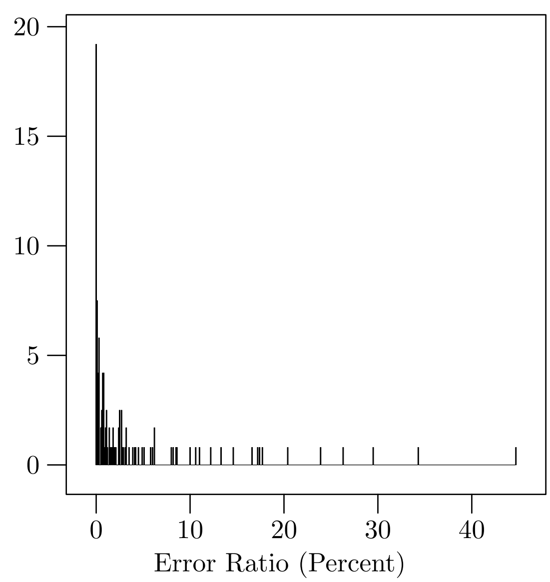


Figure 5: Ratio of error of the steady-state estimate to an estimate based on P_d with $t_a - t_d = 4$dB.



V	1, 2, 3. . . 30 m/s
T	12, 18, 24 s
$t_a - t_d$	6 dB
P_d^s	0.05, 0.15, 0.25. . . 0.95
P_d^f	0.95

Figure 6: Ratio of error of the steady-state estimate to an estimate based on P_d with $t_a - t_d = 6$dB.



V	5, 10, 15. . . 30 m/s
T	20 s
$t_a - t_d$	2 dB
P_d^s	0.05, 0.15, 0.25. . . 0.95
P_d^f	0.95
A_c	0.1, 0.36

Figure 7: Ratio of error of the steady-state estimate to an estimate based on P_d for a first-order autoregressive series.

to the phenomenon under discussion. Can other effects of motion be modeled without simulation?

It seems quite possible to obtain a computable formula for the steady-state active-set probability under AR models of shadow fading. However, a more complicated model would not seem to be needed, at the level of accuracy we are looking for, and for the ranges of parameters that seem appropriate.

An effect that is not captured here is that the conditions for entering the active set can change, depending on the active-set size; for IS-95b and CDMA2000, the conditions for entering the active set can even depend on the antennas already in it. Such conditions require us to extend these calculations, but don't change the basic mapping from P_a, P_d, and k to the steady-state probability.

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