

# Analytical description of shared restoration capacity for mesh networks

Manish Bhardwaj and Leon McCaughan

*Electrical and Computer Engineering Department, University of Wisconsin-Madison,  
1415 Engineering Drive, Madison, Wisconsin 53706  
mbhardwaj@wisc.edu*

Steven K. Korotky and Iraj Saniee

*Bell Laboratories, Lucent Technologies,  
101 Crawfords Corner Road, Holmdel, New Jersey 07733-3030  
skk@lucent.com*

RECEIVED 11 JUNE 2004; REVISED 20 JANUARY 2005;  
ACCEPTED 13 FEBRUARY 2005; PUBLISHED 3 MARCH 2005

We derive analytical expressions for the extra capacity requirements of mesh networks to ensure survivability against single-link failures. We study both shared link- and path-based restoration schemes and formulate the network global expectation value of extra capacity. We analyze a wide range of planar and toroid mesh networks using a linear programming optimization tool and find our analytic model in good agreement with the numerical simulation data.

© 2005 Optical Society of America

OCIS codes: 060.2330, 060.4250.

## 1. Introduction

The recent trend in optical transport networks toward mesh topologies—driven primarily by their capacity efficiency and flexibility—has led to the development of new survivability schemes as well as numerical and mathematical tools to analyze and optimize the routing and assignment of working and restoration capacity. To provide the desired availability, communication networks must promptly restore traffic when a link or node fails. There is, however, a trade-off between the time required for reestablishing the connectivity and the amount of restoration capacity deployed in the network. Shared path-based restoration schemes, which sometimes reroute traffic along a path that is disjoint from the working path, have been shown in general to be more capacity efficient than link-based restoration schemes [1–6]. However, the trade-offs often cannot be easily and readily evaluated because of the lack of a general description of the extra capacity requirements that is applicable over wide ranges of network size and topology. It is the objective of this study to provide a useful description of the requirements of shared path-based restoration capacity for general mesh networks by formulating and deriving bounds and estimates of the average extra capacity requirement analytically.

The detailed analysis and design of mesh networks with shared restoration requires complex, and sometimes heuristic, algorithms and often requires significant computer resources and long computation time. Optimum and near-optimum solutions can be accurately determined with integer linear programming (ILP) and linear programming (LP) techniques, which provide detailed routing information and required capacities; however, these can become time consuming for large networks employing path-based restoration, especially if the demand matrix is dense [2–6]. Analytical descriptions of the optimized

solutions can significantly reduce the computation time when the user does not require specific knowledge of the details of the demand routing and may also provide valuable insights. The lower bound for the extra capacity requirement for link-based restoration and also estimations of the requirement for path-based restoration have previously been described [5, 6]. Other researchers have also derived the average restoration capacity specifically for mesh toroid and chordal ring networks [7–9]. Previously we have shown that the extra capacity requirements for path-based schemes in regular planar mesh networks can be approximated by compact expressions [10]. Here we extend the formulation and derivation of the extra capacity requirements for nominally planar mesh networks to nonregular nodal degree and a wider range of demand profiles and introduce a new estimator for path-based restoration. We test this more general formulation by comparing it with simulations for instances of nominally planar networks having both regular and nonregular nodal degree and for uniform and nonuniform demand. We also compare our approximate model with results for uniform mesh toroid networks.

In Section 2 we define the network variables and state the problem explicitly. In Section 3 we derive the global expectation value of the extra capacity for both link restoration (LR) and path-disjoint restoration (PRd). In Section 4 we compare our expressions with simulation data and conclude the paper in Section 5.

## 2. Network Model

In this section we establish the relationships between the working and restoration capacity and the network variables using the Network Global Expectation model formalism [10–12]. The average value of a set  $\{q\}$  of the variable  $q$  will be represented by  $\langle q \rangle$ , and its variance will be represented by  $\sigma^2(q)$ . The covariance of two sets  $\{p\}$  and  $\{q\}$  having the same number of elements is represented by  $\sigma^2(p, q)$ . The network is represented as a graph  $G(N, L)$ , where  $\{n\}$  is the set of nodes and  $\{l\}$  is the set of links and  $N$  and  $L$  are the numbers of nodes and links in the network, respectively. The degree of a node,  $\delta$ , is the number of links attached to the node. The global average of degrees of nodes is then

$$\langle \delta \rangle = \frac{2L}{N}. \quad (1)$$

Throughout this work we consider all demands between nodes to be two-way demands, and we refer to the set of links that are used to route a demand as a path. A primary path routes a demand in the absence of link failures, and when a failure occurs, the affected demands (demands whose primary path included the failed link) are routed over the backup paths. Once the primary and backup paths are allocated, the total capacity of a link is defined as the sum of the working capacity, allocated to primary paths, and the restoration capacity, allocated to backup paths. For specificity, here we consider that each demand occupies one unit of capacity, e.g., a channel or a wavelength. In this case the average working capacity,  $\langle W_0 \rangle$ , on a link in the network is given by [11]

$$\langle W_0 \rangle = \frac{\langle d \rangle \langle h \rangle}{\langle \delta \rangle}, \quad (2)$$

where  $\langle d \rangle$  is the average number of demands terminating at a node and  $\langle h \rangle$  is the average numbers of hops of the demands in the network. We define the average extra capacity,  $\langle \kappa \rangle$ , as the fractional increase in total network capacity to ensure survivability against any single link failure over the *minimum* network capacity necessary to support the working demands alone. The average total capacity on a link  $\langle W \rangle$  can then be expressed as

$$\langle W \rangle = \langle W_0 \rangle (1 + \langle \kappa \rangle). \quad (3)$$

The value of  $\langle W_0 \rangle$  is determined by minimum hop routing, and we note that for nominally planar networks and uniform demand  $\langle h \rangle$  can be approximated by [11]

$$\langle h \rangle \cong \left( \frac{N-2}{\langle \delta \rangle - 1} \right)^{1/2}. \quad (4)$$

For degree = 4 mesh toroid networks,  $\langle h \rangle$  is given by Eqs. (5), (6) when the value of  $N^{1/2}$  is even and odd, respectively [7]:

$$\langle h \rangle = \frac{N^{3/2}}{2(N-1)}, \quad (5)$$

$$\langle h \rangle = \frac{N^{1/2}}{2}. \quad (6)$$

### 3. Analytical Analysis

To facilitate our analysis we identify and distinguish between two distinct sets of demands on a link. We consider the  $W_0$  demands on a link to be composed of the terminating demands,  $W^t$ , i.e., demands that terminate at one of the nodes attached to the failed link, and through demands,  $W^{\text{th}}$ . (Note, if a node attached to a failed link is the source or destination of a set of demands, which we refer to as terminating demands, then  $W^t$  represents the subset of these terminating demands whose primary paths include the failed link.) In our analysis we calculate the average restoration capacity at a node and then calculate the global average over all nodes in the network. Therefore the demands on a link are counted relative to one of the nodes attached to the link. If  $d_i$  are the terminating demands at a node  $n_i$  of degree  $\delta_i$ , then on average there are  $d_i/\delta_i$  terminating demands on each link connected to  $n_i$  and thus with Eq. (2),  $\langle W^t \rangle$  and  $\langle W^{\text{th}} \rangle$  can be approximated [10] by Eqs. (7), (8), respectively:

$$\langle W^t \rangle \cong \frac{\langle W_0 \rangle}{\langle h \rangle}, \quad (7)$$

$$\langle W^{\text{th}} \rangle = \langle W_0 \rangle - \langle W^t \rangle \cong \langle W_0 \rangle \left( 1 - \frac{1}{\langle h \rangle} \right). \quad (8)$$

Our strategy is to consider the extra capacity requirements for  $\langle W^t \rangle$  and denote it as  $\langle \kappa_t \rangle$ . Later we consider the incremental extra capacity required on a link to reroute demands not terminating at the adjacent node, referred to as through demands, and denote this incremental extra capacity as  $\langle \kappa_{\text{th}} \rangle$ . The total average extra capacity on a link,  $\langle \kappa \rangle$ , is then the sum of  $\langle \kappa_t \rangle$  and  $\langle \kappa_{\text{th}} \rangle$ .

#### 3.A. Restoration Capacity Requirements of Terminating Demands

We consider the links  $l_{ij}$  terminating at node  $n_i$ , which are  $\delta_i$  in number, as shown in Fig. 1, and denote the terminating working capacity and the restoration capacity on link  $l_{ij}$  as  $W_{ij}^t$  and  $R_{ij}$ , respectively. If a particular link  $l_{ik}$  fails, then the sum of the restoration capacities of the surviving links connected to node  $n_i$  must be greater than or equal to the failed terminating demands on link  $l_{ik}$  to be able to restore the traffic. This condition specifying the lower bound on the required extra capacity on the links differs from the condition described by Iraschko *et al.*, who investigated the bound on the links at the end nodes of the failed demands [5]. As the number of demands terminating at the node adjacent to a link is greater than or equal to the number of demands between any given node pair present

on a terminating link, the condition introduced here will impose a larger requirement for the extra capacity. The constraint we propose may be stated mathematically by

$$\sum_{j \neq k}^{\delta_i} R_{ij} \geq W_{ik}^t. \quad (9)$$

Because Eq. (9) is true for all of the links connected to node  $n_i$ , we may sum Eq. (9) over  $k$  to obtain

$$\sum_k \sum_{j \neq k}^{\delta_i} R_{ij} \geq \sum_k^{\delta_i} W_{ik}^t. \quad (10)$$

The sum on the right-hand side is just the total terminating demands at node  $n_i$  and is equal to  $d_i$ . The double summation on the left-hand side of Eq. (10) may be rewritten as an unrestricted sum with the result that Eq. (10) becomes

$$(\delta_i - 1) \sum_j^{\delta_i} R_{ij} \geq d_i. \quad (11)$$

We introduce the symbol  $R_i$  to represent the average value of the restoration capacity of the links connected to node  $n_i$  and note that

$$\sum_j^{\delta_i} R_{ij} = \delta_i \left( \frac{1}{\delta_i} \right) \sum_j^{\delta_i} R_{ij} = \delta_i R_i. \quad (12)$$

Substituting Eq. (12) in Eq. (11) and then summing over all nodes, we obtain

$$\sum_i^N (\delta_i - 1) \delta_i R_i \geq \sum_i^N d_i \quad (13)$$

or

$$\sum_i^N (\delta_i)^2 R_i - \sum_i^N \delta_i R_i \geq \sum_i^N d_i. \quad (14)$$

Dividing Eq. (14) by  $N$  we see that this equation may be expressed in terms of the average values over all nodes, i.e.,

$$\langle \delta^2 R \rangle - \langle \delta R \rangle \geq \langle d \rangle. \quad (15)$$

The covariance of two variables may be expressed in terms of expectation values as

$$\sigma^2(p, q) = \langle pq \rangle - \langle p \rangle \langle q \rangle, \quad (16)$$

and using this we may rewrite Eq. (15) to obtain the average restoration capacity, viz.

$$\langle R \rangle \geq \frac{\langle d \rangle}{\langle \delta \rangle (\langle \delta \rangle - 1) + \sigma^2(\delta)} + \frac{\sigma^2(\delta, R) - \sigma^2(\delta^2, R)}{\langle \delta \rangle (\langle \delta \rangle - 1) + \sigma^2(\delta)}. \quad (17)$$

For typical backbone networks the variance of the degrees of nodes,  $\sigma^2(\delta)$ , which appears in the denominator of Eq. (17), is of order  $\sim 1$  and therefore smaller than  $\langle \delta \rangle (\langle \delta \rangle - 1)$  by an order of magnitude or more for the networks we studied. With Eq. (2) the average terminating extra capacity can be expressed as

$$\langle \kappa_t \rangle \equiv \frac{\langle R \rangle}{\langle W_0 \rangle} \geq \left[ \frac{1}{\langle h \rangle (\langle \delta \rangle - 1)} + \frac{\sigma^2(\delta, R) - \sigma^2(\delta^2, R)}{\langle d \rangle \langle h \rangle (\langle \delta \rangle - 1)} \right] \left[ \frac{1}{1 + \sigma^2(\delta) / [\langle \delta \rangle (\langle \delta \rangle - 1)]} \right]. \quad (18)$$

The dominant term on the right-hand side of Eq. (18) is the first term from the left within the first parentheses. It has a contribution from the ratio of  $\langle W^t \rangle$  and  $\langle W_0 \rangle$  in the form of  $1/\langle h \rangle$  [see Eq. (7)] and a contribution from the ratio of the failed to surviving links connected to the adjacent node in the form of  $1/(\langle \delta \rangle - 1)$ . The term in the second parentheses from the left is a multiplication factor that incorporates the effects of topological variations and is nominally 1 when  $\sigma^2(\delta)$  is much smaller than  $\langle \delta \rangle(\langle \delta \rangle - 1)$ .

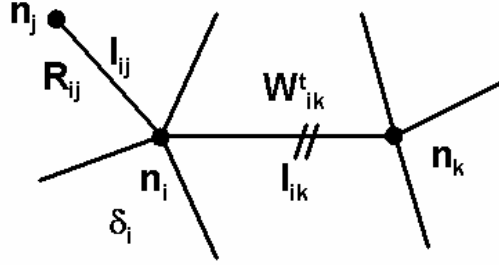


Fig. 1. Example link failure scenario.

The derivation of Eq. (18) assumed that all surviving links connected to the node participate in restoring the terminating demands on the failed link; i.e., there are  $\delta - 1$  disjoint backup paths available. This therefore represents the maximal sharing of restoration capacity and consequently the minimum extra capacity requirement. At the other extreme only one backup path might be found, and in this scenario two links attached to a node must be assigned restoration capacity equal to  $\langle W^t \rangle$ . (If only one link is assigned backup capacity, then the demand is not restorable in case that particular link fails.) Thus, in this case the total restoration capacity on all links attached to the node is on average  $2\langle W^t \rangle$ , and the total working capacity is  $\langle \delta \rangle \langle W_0 \rangle$ . The dominant term contributing to  $\langle \kappa_t \rangle$  is then  $2\langle W^t \rangle / (\langle \delta \rangle \langle W_0 \rangle) = 2 / (\langle h \rangle \langle \delta \rangle)$ . In this scenario the constraint expressed by Eq. (9) is modified so that the summation on the left-hand side of the inequality reduces to one term; i.e., the link chosen for restoration must have restoration capacity greater than the failed terminating demand. Because both links attached to node  $n_i$  that are assigned restoration capacity are equally likely to be chosen to restore the failed terminating demand (except when one of these links has failed, in which case the other must necessarily be chosen), they contribute equally to the total restoration capacity at  $n_i$ . This sum is equal to  $\delta_i R_i$  from the definition Eq. (12), as was previously outlined above. The remainder of the detailed analysis for the mean restoration capacity is similar to the one presented above leading to Eq. (18), and it can be shown that  $\langle R \rangle$  and  $\langle \kappa_t \rangle$  for the limiting condition where there is only a single backup path can be expressed as

$$\langle R \rangle \geq \frac{2\langle d \rangle}{\langle \delta \rangle^2 + \sigma^2(\delta)} - \frac{\sigma^2(\delta^2, R)}{\langle \delta \rangle^2 + \sigma^2(\delta)}, \quad (19)$$

$$\langle \kappa_t \rangle \equiv \frac{\langle R \rangle}{\langle W_0 \rangle} \geq \left[ \frac{2}{\langle h \rangle \langle \delta \rangle} - \frac{\sigma^2(\delta^2, R)}{\langle d \rangle \langle h \rangle \langle \delta \rangle} \right] \left[ \frac{1}{1 + \sigma^2(\delta) / \langle \delta \rangle^2} \right]. \quad (20)$$

Equations (18), (20) represent the approximate lower bounds of the average optimal extra capacity to restore the terminating demands and shall be referred to as the divisible (maximum sharing of restoration capacity) and indivisible (minimum sharing of restoration capacity) bounds, respectively. Since the average of the number of restoration paths lies between 1 and  $\langle \delta \rangle - 1$ , the optimum value of  $\langle \kappa_t \rangle$  lies between the divisible and indivisible

bounds. The covariance terms may be computed using an ansatz for the local dependence of the extra capacity of a link,  $\kappa_{ij}$ , on the local degrees of nodes,  $\delta_i$  and  $\delta_j$ , and local terminating demands,  $d_i$  and  $d_j$ , such as introduced in Eq. 15t of our earlier work [11].

### 3.B. Restoration Capacity Requirements of through Demands

When a link fails in the network, the failed demands—on average  $\langle W_0 \rangle$  in number—are routed over restoration paths of average length denoted as  $\langle h_r \rangle$ . The restoration path of a failed demand may be selected from among  $L - \langle h \rangle$  links, as the restoration path is disjoint from the working path. Of the  $\langle W_0 \rangle$  failed demands, the terminating  $\langle W^t \rangle$  demands can be restored by the assignment of  $\langle \kappa_t \rangle$  extra capacity to all links in the network as derived above. Next we denote  $\langle R_{th} \rangle_{\max}$  as the maximum additional average restoration capacity required on all links in the network to restore the  $\langle W^{th} \rangle$  through demands. As the total additional restoration capacity required is  $\langle W^{th} \rangle \langle h_r \rangle$ ,  $\langle R_{th} \rangle_{\max}$  can be expressed as

$$\langle R_{th} \rangle_{\max} = \frac{\langle W^{th} \rangle \langle h_r \rangle}{L - \langle h \rangle} = \langle W_0 \rangle \left( 1 - \frac{1}{\langle h \rangle} \right) \left( \frac{\langle h_r \rangle}{L - \langle h \rangle} \right), \quad (21)$$

and the maximum contribution to  $\langle \kappa \rangle$  by the through demands,  $\langle \kappa_{th} \rangle_{\max} = \langle R_{th} \rangle_{\max} / \langle W_0 \rangle$ , can be expressed as

$$\langle \kappa_{th} \rangle_{\max} = \left( 1 - \frac{1}{\langle h \rangle} \right) \left( \frac{\langle h_r \rangle}{L - \langle h \rangle} \right). \quad (22)$$

From Eqs. (7), (8) we see for networks where  $\langle h \rangle$  is greater than 2, that  $\langle W^{th} \rangle$  is larger than  $\langle W^t \rangle$ . While at first it would seem that the through demands therefore place the more stringent requirement on the extra capacity, we realize that the various demands that make up  $\langle W^{th} \rangle$  will in general be rerouted over paths spread across many different links. Therefore a given link in the network will be used in the backup paths of only a few of the through demands of a failed link. Consequently, we anticipate that the average extra capacity  $\langle \kappa_t \rangle$  derived in Subsection 3.A in consideration of the terminating demands should be nearly sufficient also to protect the through demands. It should be noted that expressing the total restoration capacity on a link of the network as the sum of the restoration capacity required for restoring the terminating demands ( $\langle R \rangle$  calculated in Subsection 3.A) and the restoration capacity required for restoring the through demands ( $\langle R_{th} \rangle_{\max}$ ) assumes that the backup paths of all through demands include links whose terminating restoration capacity has been depleted in servicing backup routes of the  $\langle W^t \rangle$  terminating demands. However, in general only a fraction of through demands will require extra capacity in excess of the terminating restoration capacity already assigned to the network. Denoting the average *incremental* restoration capacity required on a link to restore through demands as  $\langle R_{th} \rangle$ , with  $0 \leq \langle R_{th} \rangle \leq \langle R_{th} \rangle_{\max}$ , the contribution to  $\langle \kappa \rangle$  by the through demands is  $\langle \kappa_{th} \rangle = \langle R_{th} \rangle / \langle W_0 \rangle$ , where  $0 \leq \langle \kappa_{th} \rangle \leq \langle \kappa_{th} \rangle_{\max}$ . The average extra capacity required on a link  $\langle \kappa \rangle$  can thus be represented as the sum of the extra capacity required for protecting the terminating demands,  $\langle \kappa_t \rangle$ , and the incremental extra capacity required for protecting the through demands  $\langle \kappa_{th} \rangle$ , i.e.,

$$\langle \kappa \rangle = \langle \kappa_t \rangle + \langle \kappa_{th} \rangle. \quad (23)$$

In planar networks the routing choices are constrained by the boundary of the network [Fig. 2(a)], and we find that  $\langle \kappa_{th} \rangle$  approaches its maximum value of  $\langle \kappa_{th} \rangle_{\max}$  for nominally planar networks.

As an aside, note that for ring topologies, which have a linear dimensional characteristic, the average length of the restoration path is related to  $\langle h \rangle$  by  $\langle h_r \rangle = L - \langle h \rangle$ . Consequently, for rings  $\langle \kappa_t \rangle$  and  $\langle \kappa_{th} \rangle$ , Eqs. (18), (22), reduce to  $1/\langle h \rangle$  and  $1 - 1/\langle h \rangle$ , respectively, as the variance and covariance terms are 0 because of the regular nature of the ring topology. In this case the total average extra capacity, Eq. (23), is  $\langle \kappa \rangle = 1$ , as expected.

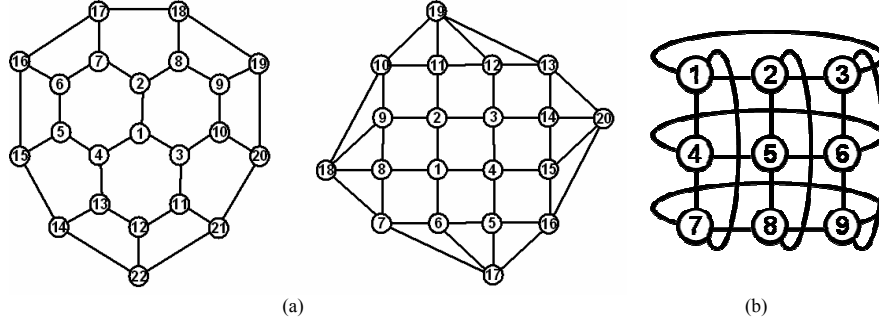


Fig. 2. (a) Example uniform planar networks of constant degree 3 and 4 and (b) mesh toroid network with  $N = 9$ .

Equations (18), (20), (22) represent the average extra capacity requirements for terminating and through demands with the only assumption being that the backup path is disjoint from the primary path, a necessary condition of PRd. This description can be modified to calculate the average extra capacity for LR as described below.

### 3.C. Link Restoration

Although LR is distinct from PRd in many aspects of the restoration process (e.g., signaling scheme, backup route calculation), for capacity estimation purposes we can consider LR to be a special case of PRd where the working capacity on a link, represented by  $\langle W_0 \rangle$ , is a set of one hop demands terminating at the end nodes of the link and in case of link failure will be rerouted around it. Thus, the condition of disjointness of the working and backup paths is satisfied, and we can use the results derived in Subsection 3.B to calculate the average extra capacity for LR by setting the length of all primary paths to 1, i.e.,  $\langle h \rangle = 1$ . Considering the short length of the primary paths, we conjecture that the number of disjoint backup paths available for restoring terminating demands may be fewer than the maximum, and in that situation the extra capacity should be described more accurately by the larger (indivisible) approximation [Eq. (20)] rather than the smaller (divisible) bound [Eq. (18)] [11]. Substituting  $\langle h \rangle = 1$  in Eq. (22), we find that  $\langle \kappa_{th} \rangle$  is 0 and the lower bound of the average extra capacity for LR can be expressed by

$$\langle \kappa \rangle \geq \left[ \frac{2}{\langle \delta \rangle} - \frac{\sigma^2(\delta^2, R)}{\langle d \rangle \langle \delta \rangle} \right] \left[ \frac{1}{1 + \sigma^2(\delta) / \langle \delta \rangle^2} \right]. \quad (24)$$

The lead term in Eq. (24),  $2/\langle \delta \rangle$ , which is slightly larger than the well-known estimate for LR of  $1/(\langle \delta \rangle - 1)$ , was previously recognized as a possible bound by Doucette and Grover [6]. Recently, we have proven that  $\langle \kappa \rangle = 2/\langle \delta \rangle$  is exact for LR and PRd for the case of uniform, unit demand on the smallest regular network of a given degree [10].

### 3.D. Path Restoration in Mesh Toroid Networks

The three-dimensional topology of mesh toroid networks presents routing choices not present in planar networks. Acampora *et al.* [7] have derived the extra capacity requirement for uniform degree = 4 mesh toroid networks for PRd as

$$\langle \kappa \rangle = \frac{(3 - 1/\sqrt{N})}{2(1 + \sqrt{N})}. \quad (25)$$



Although the routing constraints imposed by a boundary do not exist in mesh toroid networks, the analysis in Ref. [7] imposes new constraints for primary and backup routes with load balancing of the working and total capacity. This limits the routing choices for backup paths and suggests that the indivisible bound is a more accurate description of the extra capacity requirements for terminating demands and  $\langle \kappa_{th} \rangle$  approaches its maximum value. We also observe from Eq. (22) that  $\langle \kappa_{th} \rangle_{max}$  is 0 for the smallest size planar networks of a given degree,  $\delta$ , ( $N = \delta + 1$ ), which have  $\langle h \rangle = 1$ . Imposing a similar condition for the smallest size mesh toroid network,  $N = 9$  shown in Fig. 2(b), we see from Eq. (6) that  $\langle \kappa_{th} \rangle_{max} = 0$  for  $\langle h \rangle = 1.5$ . Additionally, the variance and covariance terms in Eqs. (18), (20) are identically 0 because of the regular nature of the topology of the regular toroid. Thus the average extra capacity requirement for mesh toroid networks for PRd with load balancing can be expressed as

$$\langle \kappa \rangle = \frac{2}{\langle h \rangle \langle \delta \rangle} + \left( 1 - \frac{1.5}{\langle h \rangle} \right) \left( \frac{\langle h_r \rangle}{L - \langle h \rangle} \right). \quad (26)$$

Finally if all constraints due to topology or load balancing are removed then  $\langle \kappa_{th} \rangle$  approaches its minimum value and the extra capacity can be approximated in the divisible bound as

$$\langle \kappa \rangle = \frac{1}{\langle h \rangle (\langle \delta \rangle - 1)}. \quad (27)$$

#### 4. Simulations and Comparisons

To test the formulations presented above we simulated planar mesh networks of varying average degrees and sizes using a LP tool, SPIDER [13]. We computed the average total capacity of the links for a network designed to be survivable under all single-link failure scenarios. The tool sought to minimize total capacity (working + spare) with the cost function of all links equal to 1; i.e., the cost of a primary route is the number of hops in the route. Therefore the cost of a link after optimization is the number of demands assigned to primary and backup paths. The capacity occupied by a primary route that has failed and subsequently been rerouted was considered not to be available to route other failed demands. We also separately computed the average minimum working capacity using minimum hop routing of all demands. Capacity was counted in units of channels on a link. To investigate the dependence of  $\langle \kappa \rangle$  on the size of the network,  $N$ , we carried out simulations for constant degree (regular) networks of degree 3 and 4 of varying sizes, and also nonregular networks of mixed degrees. Two of the networks from among the 12 regular networks we simulated are shown in Fig. 2(a). For regular networks all variance and covariance terms in Eqs. (18), (20) are 0. The demand profile consists of uniform, fully connected demands.

The results for LR are shown in Fig. 3 on a log-log plot of extra capacity versus number of nodes. The rms difference between the curves for the extra capacity corresponding to Eq. (24) and the simulation data set is 12.5%. The data indicates that the extra capacity for link restoration is roughly independent of the number of nodes,  $N$ , which is not surprising, as LR is a process relatively localized near the failure.

To test our analysis for mesh networks for PRd, we simulated both regular and mixed-degree planar mesh networks with no constraints on the length of the primary and backup paths. The results of our analysis are shown in Figs. 4(a) and 4(b) for regular networks of degree 3 and 4 and in Table 1 for networks of mixed degree. The average extra capacity  $\langle \kappa \rangle$  was calculated for both the divisible and indivisible bounds, and the corresponding curves are plotted in Fig. 4. Also plotted for reference is the contribution of  $\langle \kappa_{th} \rangle$  to the sum. The average extra capacity for through demands,  $\langle \kappa_{th} \rangle$ , was computed with Eq. (22), wherein the average backup path length  $\langle h_r \rangle$  was approximated as being equal to twice  $\langle h \rangle$ , the average number of hops over all demands. This relationship, which we have observed



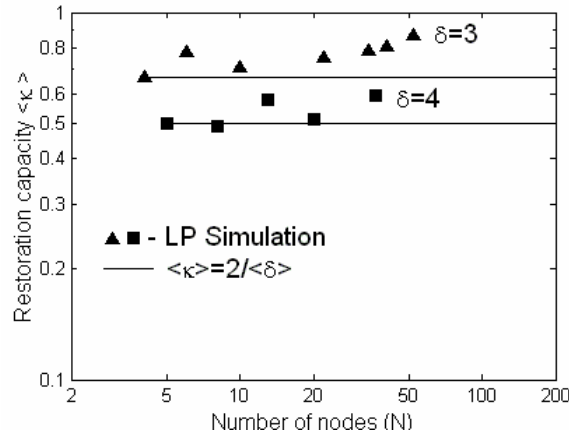


Fig. 3. Extra capacity for uniform mesh networks using LR.

for regular networks, was previously also observed for nonuniform networks employing dynamic restoration heuristics to achieve optimal capacity assignment [14]. Another indication of the validity of this approximation for  $\langle h_r \rangle$  is noted by analyzing the smallest size regular network of a given degree, e.g.,  $N = 4$  for  $\delta = 3$  and  $N = 5$  for  $\delta = 4$ , where  $h = 1$  and  $h_r = 2$  for every demand. For the smallest size network the contribution of  $\langle \kappa_{th} \rangle$  is 0 and the value of  $\langle \kappa \rangle$  in the indivisible bound is  $2/\langle \delta \rangle$ , the same as the LR requirement. Having constrained the constant of proportionality between  $\langle h \rangle$  and  $\langle h_r \rangle$ , we note that no free parameters remain within the present model for the extra capacity for PRd.

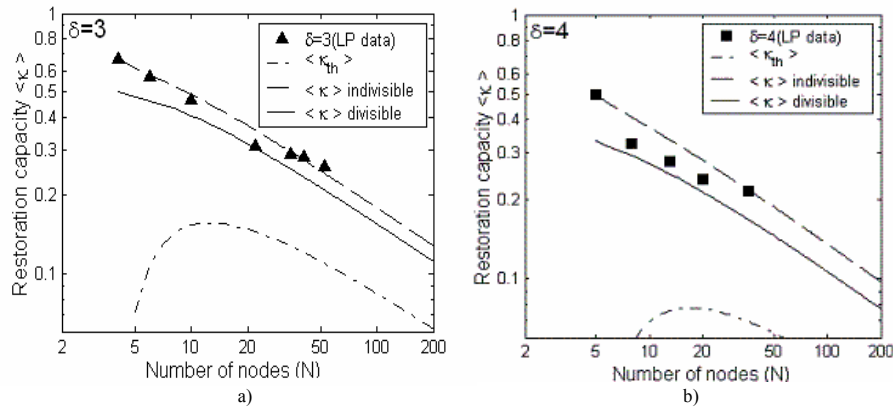


Fig. 4. Extra capacity for PRd in uniform degree planar networks of degree (a) 3 and (b) 4.

We observe from the figures for the cases we have investigated that the requirements imposed by consideration of the terminating demands are the dominant contribution to the extra capacity for shared restoration. Also, the reader will observe that the contribution of  $\langle \kappa_{th} \rangle$  increases initially with the network size and then decreases monotonically. This is because  $\langle \kappa_{th} \rangle$  is a product of two terms, the ratio of through demands to the average working capacity on a link, represented by  $(1 - 1/\langle h \rangle)$ , and the ratio of the average length of the backup path  $\langle h_r \rangle$  of a failed demand to the number of links that are candidates to be part of the backup path, represented by  $\langle h_r \rangle / (L - \langle h \rangle)$ . These two terms increase and decrease monotonically with network size, respectively, and result in  $\langle \kappa_{th} \rangle$  having a

maximum, as shown in Fig. 4.

In Table 1 we list the variables for PRd for planar mixed-degree, i.e., nonregular, networks. In the two columns on the right-hand side, we tabulate the average extra capacity calculated by the LP tool and the analytic prediction [Eq. (20)] for  $\langle \kappa \rangle$  in the indivisible bound using an ansatz (Eq. 15t in Ref. [11]) to specify the local dependence of the average terminating restoration capacity,  $R_i$ , at a node, i.e.,  $R_i = 2d_i/\delta_i$ . The first four networks in the table are mixtures of degree-2 and degree-3 nodes and have  $\langle \delta \rangle$  between 2 and 3. The modeled indivisible bound predicts the required extra capacity with an rms error of 7.5% in these cases. The agreement is similar (7.8% underprediction) for Network 5 of average degree 3.5, which is a mixture of degree-3 and degree-4 nodes. The network with the largest mean degree, Network 6, is a mixture of degree 4, 5, and 6 nodes, and the difference between the analytic result and the simulation datum for this network is 6.4%.

**Table 1. Network Parameters of Mixed Degree Networks**

Network	$N$	$L$	$\langle \delta \rangle$	$\sigma(\delta)$	$\langle \kappa \rangle$ (LP)	$\langle \kappa \rangle$ Analytic
1	4	5	2.5	0.50	0.85	0.86
2	36	47	2.6	0.49	0.39	0.36
3	34	46	2.7	0.46	0.38	0.35
4	34	48	2.8	0.38	0.36	0.33
5	34	59	3.5	0.50	0.29	0.27
6	27	69	5.1	0.88	0.20	0.22
7	32	51	3.2	0.46	0.34	0.29
8	15	28	3.7	1.24	0.58	0.43

In addition to testing our analytic approximations for  $\langle \kappa \rangle$  with simulation data for uniform demand, we also considered a nonuniform demand profile imposed on the nonregular topology of 32 nodes and 51 links described in Ref. [6], which we denote here as Network 7. The demand profile we used was based on a gravity attraction model where the measure of attraction is the degree of a node [6]. The ratio of the maximum to minimum number of demands per node pair was 2.6 for the demands between the 496 node pairs in this network scenario. We observe that the analytic approximation of  $\langle \kappa \rangle$  [Eq. (20)] underpredicts the simulation result by approximately 16% for this case. Network 8 is a metropolitan area model with 15 nodes and 28 links and a sparse demand profile (over 1/3 of the entries are 0) between the 105 node pairs [15]. For this network the minimum number of demands between node pairs is 1, while the maximum is 22 over the entire range of node pairs that generate demands. We note that recently DeMaesschalck *et al.* have modeled the anticipated traffic demand for intelligent optical networks, and their predicted distribution of the traffic demand among points-of-presence (PoPs) for 2006 exhibits a similarly large variation with approximately 99.8% of the interterminal demands falling within a ratio of 21:1 [16]. For Network 8 the analytic approximation of  $\langle \kappa \rangle$  underpredicts the simulation result by approximately 26% ( $\langle \kappa \rangle = 0.43$  versus 0.58). In all, the rms difference between the LP data and our analytical approximations is 12.2% for the indivisible bound with  $\langle h_r \rangle = 2\langle h \rangle$  and  $\langle \kappa_{th} \rangle = \langle \kappa_{th} \rangle_{\max}$  over the entire data set of 20 networks (both regular and mixed-degree). The rms difference in comparison to the divisible bound is 19.9%.

In interpreting the quality of the agreement between the present analytic model and the simulations, at this point we remind the reader that the variable optimized by the linear programming simulations is the total capacity of the network, as it is the total capacity that reflects the total cost of the network. Because the extra capacity to implement survivability by shared restoration schemes is less than the capacity required for provisioning alone, the difference in extra capacity between the model and simulation is a more sensitive metric of

comparison than is the difference in total capacities. The reader may confirm, in fact, that the difference in predicted and simulated total capacities is smaller than the difference in extra capacities by approximately the ratio of the extra capacity to the total capacity, i.e.,  $\langle \kappa \rangle / (1 + \langle \kappa \rangle)$ . Thus, for example, for a value of  $\langle \kappa \rangle$  less than 0.5 the difference in total capacities is less than 1/3 the difference in extra capacity. When we compare the predictions of the model for the total capacity with that of the simulations, the rms differences for the divisible and indivisible bounds for the data set of 20 networks investigated here are found to be 6% and 3.3%. The difference between the model and simulation for any individual case from among the 20 network scenarios we have considered is less than 10%. Such agreement may serve useful for the purpose of the quick estimation of trends in the network requirements and costs. A word of caution is also in order, however, regarding the potential range of applicability of the present formulation, because the richness of network graphs and demand possibilities may give rise to cases for which the difference between the model and detailed simulations is larger. For example, as mentioned above, we have introduced some formulas assuming the graph to be nominally planar, i.e., two dimensional in character, and we expect deviation from these semi-empirical approximations as the network graph becomes more linear, such as when the fraction of the number of nodes having degree 2 is increased. Another regime that warrants caution is when the demand matrix is sparse or the network is small, as in such situations there is little opportunity for statistical averaging of otherwise rare routing events.

Finally, to test our analysis of mesh toroid networks, we simulated five uniform degree = 4 networks with no constraints. The results of the LP simulation are shown in Fig. 5 along with our expression for  $\langle \kappa \rangle$  for the no constraint mesh toroid case [Eq. (27)]. Also shown for comparison are the extra capacity requirements for mesh toroid networks with load balancing as derived in Ref. [7] [Eq. (25)] and our expression as shown in Eq. (26). The comparison of the present formulations with the previously reported results for degree = 4 mesh toroid networks provides further indication of the capability of the approach described here.

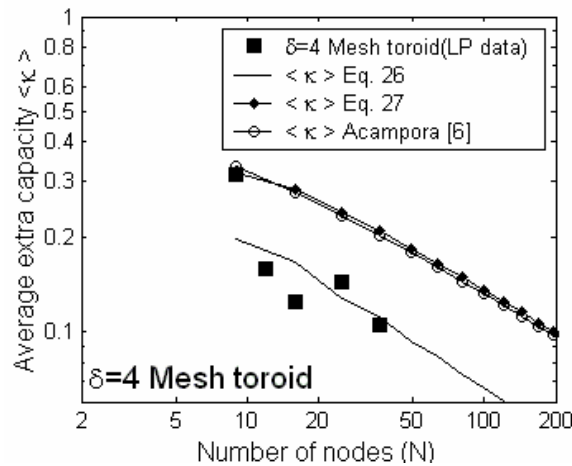


Fig. 5. Extra capacity for PRd in mesh toroid networks.

## 5. Conclusion

In this paper we have derived the average restoration capacity requirements for mesh networks survivable under single-link failure scenarios. We have shown that our description

of the extra capacity for survivability and total network capacity requirements agrees with numerical simulations for a wide range of nominally planar networks with an rms difference of less than 13% and 4%, respectively, and also agrees with previously published results for toroidal networks. Our results indicate that the restoration capacity requirement for link-based shared mesh restoration is roughly independent of the network size, which is consistent with the view that link-based restoration is a process localized near the link failure. Our analysis also shows that for path-based schemes, the restoration of terminating demands introduces the dominant contribution to the global average extra capacity. These analytic results suggest the extra capacity for both link- and path-based shared mesh restoration may be estimated quickly for a wide range of network sizes and topologies for the purpose of gauging trends in the network requirements and costs.

## References and Links

- [1] S. Ramamurthy, L. Sahasrabudde, and B. Mukherjee, "Survivable WDM mesh networks," *J. Lightwave Technol.* **21**, 870–883 (2003).
- [2] S. Baroni and P. Bayvel, "Wavelength requirements in arbitrarily connected wavelength-routed optical networks," *J. Lightwave Technol.* **15**, 242–251 (1997).
- [3] B. V. Caenegem, N. Wauters, and P. Demeester, "Spare capacity assignment for different restoration strategies in mesh survivable networks," in *Proceedings of the International Conference on Communication* (IEEE, 1997), Vol. 1, pp. 288–292.
- [4] S. Baroni, P. Bayvel, R. J. Gibbens, and S. K. Korotky, "Analysis and design of resilient multi-fiber wavelength-routed optical transport networks," *J. Lightwave Technol.* **17**, 743–754 (1999).
- [5] R. R. Iraschko, M. H. MacGregor, and W. D. Grover, "Optimal capacity placement for path restoration in STM or ATM mesh-survivable networks," *Trans. Netw.* **6**, 325–336 (1998).
- [6] J. Doucette and W. D. Grover, "Comparison of mesh protection and restoration schemes and the dependency on graph connectivity," in *Proceedings of Conference on Design Reliable Communication Networks* (IEEE, 2001), pp. 121–128.
- [7] A. S. Acampora, R. A. Gholmieh, and S. Krishnamurthy, "On tolerating single link, double link, and nodal failures in symmetric grid networks," *J. High Speed Netw.* **11**, 23–44 (2002).
- [8] J. F. Labourdette, E. Bouillet, R. Ramamurthy, and A. A. Akyamac, "Fast approximate dimensioning and performance analysis of mesh optical networks," in *Proceedings of Design Reliable Communication Networks* (IEEE, 2003), pp. 428–439.
- [9] J. Sun and E. Modiano, "Capacity provisioning and failure recovery in mesh-torus networks with application to satellite constellations," in *Proceedings of the Seventh International Symposium on Computers and Communications* (IEEE, 2002), pp. 77–84.
- [10] M. Bhardwaj, L. McCaughan, S. K. Korotky, and I. Saniee, "Global expectation values of shared restoration capacity for general mesh networks," in *Optical Fiber Communication Conference*, Vol. 95 of OSA Trends in Optics and Photonics Series (Optical Society of America, Washington, D.C., 2004), paper ThO5.
- [11] S. K. Korotky, "Network Global Expectation Model: A statistical formalism for quickly quantifying network needs and costs," *J. Lightwave Technol.* **22**, 703–722, 2004.
- [12] S. K. Korotky, "An overview of the network global expectation model," in *Optical Fiber Communication Conference*, Vol. 95 of OSA Trends in Optics and Photonics Series (Optical Society of America, Washington, D.C., 2004), paper TuH7.
- [13] R. D. Davis, K. Kumaran, G. Liu, and I. Saniee, "SPIDER: A simple and flexible tool for design and provisioning of protected lightpaths in optical networks," *Bell Labs Tech. J.* **6**, 82–97 (2001).
- [14] C. Ou, J. Zhang, H. Zang, L. H. Sahasrabudde, and B. Mukherjee, "New and improved approaches for shared-path protection in WDM mesh networks," *J. Lightwave Technol.* **22**, 1223–1232 (2004).
- [15] "The role of digital crossconnect systems in transport network survivability," Bellcore, Special Rep. SR-NWT-002514, Issue 1, January, 1993.
- [16] S. DeMaesschalck, L. Nederlof, M. Vaughn, and R. E. Wagner, "Traffic studies for fast optical switching in an intelligent optical network," *Photonic Netw. Commun.* **3**, 285–307 (2004).