A DISTRIBUTED POWER CONTROL ALGORITHM FOR BURSTY TRANSMISSIONS ON CELLULAR, SPREAD SPECTRUM WIRELESS NETWORKS

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ABSTRACT

We propose a distributed algorithm for power control in cellular, wideband networks for bursty traffic sources, such as data and speech with silence detection. The algorithm adapts power at mobiles on the basis of only local measurements of the mean and variance of the interference. The first part of the paper introduces a probabilistic quality of service specification and gives an asymptotic framework for estimating orders of magnitudes of the quantities which affect it. The second part gives the algorithm and the condition for its convergence at a geometric rate. The condition is not burdensome.

1 INTRODUCTION

A major challenge facing designers of services on cellular wireless networks is the efficient integration of data traffic, which belongs to the general category of traffic sources which transmit in relatively short bursts and are silent at other times. Speech with silence detection belongs to this category, although its parameters are different from data. We propose a distributed algorithm for power control in cellular, wideband networks for such stochastic traffic sources, in which the adaptation of power is driven by *local* measurements of the mean and variance of the interference. The paper is in two parts. The first part introduces a probabilistic quality of service specification and gives an asymptotic framework for estimating the orders of magnitudes of various quantities which affect it. The second part gives the algorithm and an analysis of its dynamic behavior. The condition for geometric convergence is obtained and, surprisingly, it is not burdensome. The condition is only slightly more demanding than what is obtained by substituting mean activity values in the condition derived in prior work with sources which are on all the time. The algorithm and analysis are new. Indeed, we are not aware of any prior work which considers the design of power control algorithms explicitly for bursty traffic sources.

We consider a cellular network with many sources or mobiles in each cell, each transmitting for only a, possibly small, fraction of time. The transmission framework is spread spectrum, which implies that the interference to any user's transmission at any time is the superposition of the effects of all other current transmissions in the network. The statistical multiplexing that is inherent to such a wideband system would appear to make it a natural choice for the bursty traffic conditions which are considered here.

A considerable amount of prior work has been done on power control in distributed [15, 16, 1, 7, 4, 8, 6], distributed-asynchronous [10], and centralized [5] frameworks. Hanly [7, 8] considers a cellular model with interference fluctuations, rather than burstiness, i.e., he assumes that the power control algorithm can adapt to the interference fluctuations. Also, of particular interest is work on power control in spread spectrum systems [12]. Recent work in which power control is combined with network management is noteworthy [2, 9, 14]. On-off sources are considered in [13, 11].

2 MODEL

An active mobile m in cell j transmits at power p_{jm}, and the gain to its cell site is g_{jm}, so that the received power at the cell site is $g_{jm}p_{jm}$. We assume that the received power from all mobiles in any cell is common and given by P_j, for cell j, i.e.,

$$P_j \triangleq g_{jm}p_{jm} \quad \forall\ m \in \text{cell } j\ . \tag{1}$$

We prefer to work with the quantities $\{P_j\}$ since they are fewer than $\{p_{jm}\}$ and this leads to a simpler analytic treatment. However, it should be straightforward to recover information on p_{jm} from P_j via (1).

Let X_{jm} be the activity indicator for mobile m in cell j, i.e., $X_{jm} \in \{0, 1\}$ and $X_{jm} = 1$ if and only if the mobile is active at a point in time. Let the received power at cell site j due to an active mobile m in cell k $(k \neq j)$ be $g_{jm}p_{km}$. Let

I_{jk} denote the received power at cell j due to all mobiles in cell k $(k \neq j)$, i.e.,

$$I_{jk} = \sum_{m \in \text{ cell } k} g_{jm} p_{km} X_{km} = \sum_{m \in \text{ cell } k} \frac{g_{jm}}{g_{km}} X_{km}(g_{km} p_{km}) = P_k \sum_{m \in \text{ cell } k} \frac{g_{jm}}{g_{km}} X_{km} \ . \tag{2}$$

We define

$$M_{jk} \triangleq \sum_{m \in \text{ cell } k} \frac{g_{jm}}{g_{km}} X_{km} \quad (1 \leq j \leq J, \ 1 \leq k \leq J) \ , \tag{3}$$

so that $I_{jk} = P_k M_{jk}$.

Now, for any active mobile m in cell j, $X_{jm} = 1$ and the interference to its transmission is, using (1),

$$I_j = P_j \sum_{m' \in \text{ cell } j - \{m\}} X_{jm'} + \sum_{k : k \neq j} I_{jk} + \eta_j W \tag{4}$$

where $\eta_j W$ $(1 \leq j \leq J)$ is the local receiver noise power at cell j, and W is the spread bandwidth (and also the processing gain). Since we are considering a wideband system, W is large. From (2), (3) and (4) we obtain

$$I_j = \sum_{k=1}^{J} P_k M_{jk} - P_j + \eta_j W \qquad (1 \leq j \leq J) \ . \tag{5}$$

The quality of service requirement in terms of the carrier-to-inference ratio is

$$P_j / I_j \geq \alpha / W \quad (1 \leq j \leq J) \tag{6}$$

where α is a prespecified number; the right-hand quantity is typically small, since the processing gain W is large. It will be slightly more convenient to write (6) as

$$P_j \geq \frac{\tilde{\alpha}}{W} \tilde{I}_j \qquad (1 \leq j \leq J) \tag{7}$$

where $\tilde{\alpha} = \alpha / (1 + \frac{\alpha}{W})$, and

$$\tilde{I}_j \triangleq \sum_{k=1}^{J} P_k M_{jk} + \eta_j W \qquad (1 \leq j \leq J) \ . \tag{8}$$

Typically $\tilde{\alpha} \approx \alpha$.

Now consider the stochastic traffic model of a bursty mobile to be on-off, i.e., in cell j it is on or active with probability w_j:

$$Pr[X_{jm}=1]=w_j\,,\quad Pr[X_{jm}=0]=1-w_j\,,\quad \forall\ m\in \text{cell}\,j\ . \tag{9}$$

On account of the random source behavior, the quality of service requirement has to be probabilistic, with probability of compliance at best less than 1. Let the requirement be

$$\pi_j \geq 1-L_j \qquad (1\leq j\leq J) \tag{10}$$

where

$$\pi_j \triangleq Pr\left[P_j \geq \frac{\tilde{\alpha}}{W}\tilde{I}_j\right] \qquad (1\leq j\leq J) \tag{11}$$

and $\{L_j\}$ are given parameters.

From (9) we have

$$E(X_{km})=w_k,\quad \text{var}(X_{km})=w_k(1-w_k),\quad \forall m\in \text{ cell } k\ . \tag{12}$$

Hence, from (3),

$$E(M_{jk})=\sum_{m\in \text{ cell } k}\frac{g_{jm}}{g_{km}}w_k=K_kG_{jk}w_k\ , \tag{13}$$

where K_k is the number of mobiles in cell k, and

$$G_{jk}=\frac{1}{K_k}\sum_{m\in \text{ cell } k}\frac{g_{jm}}{g_{km}}\ . \tag{14}$$

Note, in particular, that $G_{jj}=1$. From (3) and (12), we obtain

$$\text{var}(M_{jk})=\sum_{m\in \text{ cell } k}\left(\frac{g_{jm}}{g_{km}}\right)^2 w_k(1-w_k)=K_kH_{jk}^2w_k(1-w_k)\ , \tag{15}$$

where

$$H_{jk}^2=\frac{1}{K_k}\sum_{m\in \text{ cell } k}\left(\frac{g_{jm}}{g_{km}}\right)^2\ . \tag{16}$$

We assume that g_{jm}/g_{km} are uniformly bounded for $m\in$ cell k. Since $X_{km}\in\{0,1\}$, it follows that the random variables $g_{jm}X_{km}/g_{km}$ are uniformly bounded for $m\in$ cell k. Hence, from the Lindeberg theorem [3], the central limit theorem holds for M_{jk}, as $K_k\to\infty$. Hence

$$M_{jk}=K_kG_{jk}w_k+H_{jk}\sqrt{K_kw_k(1-w_k)}Z_{jk}\ , \tag{17}$$

where Z_{jk} is asymptotically normally distributed, with zero mean and unit variance, as $K_k \to \infty$. We note that Z_{jk} $(1 \le k \le J)$ are independent random variables.

3 ASYMPTOTICS, ORDERS OF MAGNITUDES

We introduce a natural scaling, which allows us to make order of magnitude estimates and to develop meaningful and efficient approximations by dropping negligibly small terms. Inherent to wideband systems in which the bandwidth (W) is shared by a large number of users is the following scaling, in which the large parameter K is the average number of mobiles in a cell, i.e., $K = \frac{1}{J}\sum_{j=1}^{J} K_j$:

$$\frac{\alpha}{W} = \frac{a}{K}, \quad K_j = \gamma_j K \qquad (1 \le j \le J) \tag{18}$$

where $a = O(1)$ and $\gamma_j = O(1)$ as $K \to \infty$ and $W \to \infty$. Note, α and η_j are fixed in this scaling. Also, W and K are of the same order. Let

$$P_j = \left(\widehat{P}_j + \frac{1}{\sqrt{K}} Q_j\right) \qquad (1 \le j \le J) \tag{19}$$

where $\widehat{P}_j$ and Q_j are $O(1)$. The orders of magnitudes of the first order (dominant) and second order terms in the expansion of P_j are dictated by consistency, as will be seen.

Now let us investigate the implications of (18) and (19) on the terms P_j and $\tilde{\alpha}\tilde{I}_j$ appearing in the quality of service specifications, (10) and (11). By substituting (17), (18) and (19) in the expression for $\tilde{I}_j$ in (8), it may be verified that

$$\begin{aligned} \frac{\tilde{\alpha}}{W}\tilde{I}_j &= a\sum_k G_{jk}\gamma_k w_k \widehat{P}_k + \alpha\eta_j \\ &\quad + \frac{a}{\sqrt{K}} \sum_k \left[G_{jk}\gamma_k w_k Q_k + H_{jk}\sqrt{\gamma_k w_k(1-w_k)} Z_{jk}\widehat{P}_k\right] + O(1/K)\,. \end{aligned} \tag{20}$$

To achieve $P_j \gtrsim \frac{\tilde{\alpha}}{W}\tilde{I}_j$ $(1 \le j \le J)$, we compare coefficients of, first, order 1 and, second, $1/\sqrt{K}$ and obtain the systems of inequalities given below in (21) and (22). These constitute sufficient conditions to give $P_j \geq \frac{\tilde{\alpha}}{W}\tilde{I}_j$, to within $O(1/K)$.

$$\widehat{P}_j - \frac{\alpha}{W}\sum_k G_{jk}K_k w_k \widehat{P}_k \geq \alpha\eta_j \quad (1 \leq j \leq J) \tag{21}$$

and

$$Q_j - \frac{\alpha}{W}\sum_k G_{jk}K_k w_k Q_k \geq Y_j \quad (1 \leq j \leq J) \tag{22}$$

where

$$Y_j \triangleq a\sum_k H_{jk}\sqrt{\gamma_k w_k(1-w_k)}\widehat{P}_k Z_{jk} \quad (1 \leq j \leq J)\ . \tag{23}$$

Equation (21) is a system of relations free of random variables, which is in contrast to (22). We first treat (21) before returning to (22).

In matrix form (21) is

$$\left[\mathbf{I} - \frac{\alpha}{W}\mathbf{F}\right]\widehat{\mathbf{P}} \geqq \alpha\boldsymbol{\eta}\ , \tag{24}$$

where

$$F_{jk} = G_{jk}K_k w_k\ , \tag{25}$$

which will be recognized to be mean values of dominant terms in the interference. Also, (24) is similar in algebraic structure to relations for power levels necessary to combat co-channel interference in narrowband systems, see for instance, [4] and [10]. We assume that $\mathbf{F}$ is an irreducible matrix and since it is also nonnegative, it has an eigenvalue of maximum modulus, called the Perron-Frobenius eigenvalue, which is real, positive and simple. Denote this eigenvalue by r_F. We recall the following result quoted in [10].

The following statements are equivalent:

$$\begin{array}{ll} \text{(i)} & \frac{\alpha}{W}r_F < 1\ . \\ \text{(ii)} & \text{A solution to (24) with } \widehat{\mathbf{P}} \geqq \mathbf{0} \text{ exists.} \\ \text{(iii)} & \left[\mathbf{I} - \frac{\alpha}{W}\mathbf{F}\right]^{-1} \text{ exists and is element-wise positive.} \end{array} \tag{26}$$

If (26) holds then a particular solution to (24) is

$$\widehat{\mathbf{P}}^* = \alpha\left[\mathbf{I} - \frac{\alpha}{W}\mathbf{F}\right]^{-1}\boldsymbol{\eta}\ . \tag{27}$$

This solution is Pareto optimal since any other solution to (24) will have components which are at least as large and at least one component which is larger.

Henceforth we shall assume that (26), which is to be interpreted as a mean-value capacity constraint, holds.

Now we turn to (22) and (23). Note that Z_{jk} $(1 \le k \le J)$ are independent and asymptotically normally distributed, with zero mean and unit variance. Hence Y_j is asymptotically normally distributed, with zero mean and variance $a^2\sigma_j^2$ where

$$\sigma_j^2 \triangleq \sum_k H_{jk}^2 \gamma_k w_k (1-w_k)\widehat{P}_k^2 \qquad (1 \le j \le J) \; . \tag{28}$$

To satisfy the quality of service requirement in (10) and (11), we pick $\widehat{\mathbf{P}}$ as in (27) and $\mathbf{Q}$ such that

$$Pr\left[Q_j - \frac{\alpha}{W}\sum_k G_{jk}K_k w_k Q_k \ge Y_j\right] \ge 1 - L_j \qquad (1 \le j \le J) \; . \tag{29}$$

The latter condition is equivalent to the deterministic condition,

$$Q_j - \frac{\alpha}{W}\sum_k G_{jk}K_k w_k Q_k \ge a\nu_j\sigma_j \qquad (1 \le j \le J) \tag{30}$$

where $\{\nu_j > 0\}$ is defined as follows:

$$1 - L_j = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\nu_j} e^{-y^2/2}dy \qquad (1 \le j \le J) \; . \tag{31}$$

That is, ν_j is the multiple of the standard deviation of the asymptotic random variable Y_j, which indicates how large the left-hand quantity in (30) has to be for the probabilistic requirement in (29) to be satisfied. In matrix form the system of inequalities in (30) is

$$\left[\mathbf{I} - \frac{\alpha}{W}\mathbf{F}\right]\mathbf{Q} \geqq a\mathbf{u} \tag{32}$$

where $u_j \triangleq \nu_j\sigma_j$ and $\mathbf{u} = \{u_j\}$. The Pareto optimal solution is

$$\mathbf{Q}^* = a\left[\mathbf{I} - \frac{\alpha}{W}\mathbf{F}\right]^{-1}\mathbf{u} \; . \tag{33}$$

Note that $\mathbf{Q}^* > \mathbf{0}$.

In the final part of this section we lift the veil of asymptotics and give relations without reference to the scaling in (18) and (19). Let

$$\mathbf{A} \triangleq \widehat{\mathbf{P}}^* \quad \text{and} \quad \mathbf{B} = \frac{1}{\sqrt{K}}\mathbf{Q}^* \; , \tag{34}$$

so that, from (19),

$$\mathbf{P}^* = \mathbf{A} + \mathbf{B} \ . \tag{35}$$

From (27) and (33) we obtain relations for $\mathbf{A}$ and $\mathbf{B}$:

$$\left[\mathbf{I} - \frac{\alpha}{W}\mathbf{F}\right]\mathbf{A} \quad = \quad \alpha\boldsymbol{\eta} \ , \tag{36}$$

and

$$\left[\mathbf{I} - \frac{\alpha}{W}\mathbf{F}\right]\mathbf{B} \quad = \quad \frac{\alpha}{W}\mathbf{U}(\mathbf{A}) \ , \tag{37}$$

where

$$U_j(\mathbf{A}) \triangleq \nu_j V_j^{1/2}(\mathbf{A}) \quad \text{and} \quad V_j(\mathbf{A}) = \sum_k H_{jk}^2 K_k w_k (1 - w_k) A_k^2 \ . \tag{38}$$

We will need to keep in mind that in the decomposition (35), $\mathbf{B} << \mathbf{A}$. Finally, from (35), (36) and (37)

$$\left[\mathbf{I} - \frac{\alpha}{W}\mathbf{F}\right]\mathbf{P}^* = \frac{\alpha}{W}[\boldsymbol{\eta} W + \mathbf{U}(\mathbf{A})] \ . \tag{39}$$

In view of the above observation, $\mathbf{A} \approx \mathbf{A} + \mathbf{B} = \mathbf{P}^*$, hence

$$\boxed{\left[\mathbf{I} - \frac{\alpha}{W}\mathbf{F}\right]\mathbf{P}^* = \frac{\alpha}{W}[\boldsymbol{\eta} W + \mathbf{U}(\mathbf{P}^*)] \ .} \tag{40}$$

Note that $\mathbf{U}(\mathbf{P}^*)$ is the term which represents the combined effect of the probabilistic quality of service requirement, and the variance of the interference at the cell sites, which, importantly, depends on the power at all cell sites. Also, note that if $w_j \equiv 1$, then $\mathbf{U}(\mathbf{P}^*) = \mathbf{0}$ and we obtain equations which are similar to those in prior works, such as [4].

4 ALGORITHM AND ITS CONVERGENCE

We now propose a distributed algorithm for power control, based on (40). Write (40) in the form

$$\mathbf{P}^* = \frac{\alpha}{W}[\mathbf{F}\,\mathbf{P}^* + \boldsymbol{\eta} W + \mathbf{U}(\mathbf{P}^*)] \ . \tag{41}$$

In view of (25) and (38), we introduce the measured quantities at cell j,

$$\overline{I}_j(n) = \sum_k G_{jk} K_k w_k P_k(n) + \eta_j W, \qquad (1 \le j \le J) \ , \tag{42}$$

and

$$\overline{V}_j(n) = \sum_k H_{jk}^2 K_k w_k (1 - w_k) P_k^2(n) \ . \tag{43}$$

These are the measured values of the mean and variance of the total interference at cell j. The distributed algorithm for power control is

$$\boxed{P_j(n+1) = \frac{\alpha}{W}\left[\overline{I}_j(n) + \nu_j \sqrt{\overline{V}_j(n)}\right], \quad (n = 0, 1, \ldots) \ ,} \tag{44}$$

where $\{\nu_j\}$, defined in (31), are fixed, known quantities. The adaptation of power is thus driven by *local* measurement of the mean and variance of the interference.

In vector form, the algorithm is

$$\mathbf{P}(n+1) = \frac{\alpha}{W}[\mathbf{F}\,\mathbf{P}(n) + \boldsymbol{\eta} W + \mathbf{U}(\mathbf{P}(n))] \ . \tag{45}$$

Hence, from (41),

$$\mathbf{P}(n+1) - \mathbf{P}^* = \frac{\alpha}{W}\{\mathbf{F}[\mathbf{P}(n) - \mathbf{P}^*] + \mathbf{U}(\mathbf{P}(n)) - \mathbf{U}(\mathbf{P}^*)\} \ . \tag{46}$$

Under the assumption that (26) holds, we have

$$||\mathbf{F}[\mathbf{P}(n) - \mathbf{P}^*]|| \le r_F ||\mathbf{P}(n) - \mathbf{P}^*|| \ , \tag{47}$$

where the norm is defined in terms of the element-wise positive Perron-Frobenius eigenvector $\mathbf{w}$, i.e.,

$$\mathbf{F}\mathbf{w} = r_F \mathbf{w}, \quad \mathbf{w} > \mathbf{0} \ . \tag{48}$$

Specifically,

$$||\mathbf{x}|| = \max_{1 \le j \le J} |x_j|/w_j \ . \tag{49}$$

It is shown in the Appendix that

$$||\mathbf{U}(\mathbf{P}(n)) - \mathbf{U}(\mathbf{P}^*)|| \le c||\mathbf{P}(n) - \mathbf{P}^*|| \ , \tag{50}$$

where c is a constant, given by (62). It follows, from (46), (47) and (50), that

$$||\mathbf{P}(n+1) - \mathbf{P}^*|| \leq \frac{\alpha}{W}(r_F + c)||\mathbf{P}(n) - \mathbf{P}^*|| \; . \tag{51}$$

Proposition: Assume that $\frac{\alpha}{W}(r_F + c) < 1$. Then, by induction,

$$||\mathbf{P}(n) - \mathbf{P}^*|| \leq \left[\frac{\alpha}{W}(r_F + c)\right]^n ||\mathbf{P}(0) - \mathbf{P}^*|| \; , \tag{52}$$

i.e., the convergence of $\mathbf{P}(n)$ to $\mathbf{P}^*$ is geometric. ■

We note that in our asymptotic analysis, with the scalings in (18) and the definitions in (38), $\frac{\alpha}{W}c = O(1/\sqrt{K})$, and hence the condition for geometric convergence is only slightly more stringent than the condition $\frac{\alpha}{W}r_F < 1$, corresponding to the substitution of mean activity values.

5 CONCLUSIONS

We have given an algorithm in (44) for distributed power control in a cellular spread spectrum system. The application of the algorithm may be extended in a natural way to multiple classes of bursty traffic, and even to combinations of constant and variable rate sources. The algorithm is generic and applies in various frameworks. For instance, statistical fluctuations in the transmission medium may cause fluctuations in the total interference, in which case, taking the variance of the interference into account, as in our algorithm, will lead to a better selection of the power level and thus to an increase in system capacity.

An area for future study is the extension of the algorithm to asynchronous operations.

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APPENDIX

We here determine the constant c in (50). If x_k and y_k $(1 \le k \le J)$ are real, the Cauchy-Schwarz inequality implies that

$$\left(\sum_{k=1}^{J} x_k^2\right)^{1/2} \left(\sum_{k=1}^{J} y_k^2\right)^{1/2} \ge \sum_{k=1}^{J} x_k y_k \ . \tag{53}$$

It follows that

$$\left[\left(\sum_{k=1}^{J} x_k^2\right)^{1/2} - \left(\sum_{k=1}^{J} y_k^2\right)^{1/2}\right]^2 \le \sum_{k=1}^{J} (x_k - y_k)^2 \ . \tag{54}$$

Hence, from (38), we obtain

$$\begin{aligned} \sum_{j=1}^{J} [U_j(\mathbf{P}) - U_j(\mathbf{P}^*)]^2 &\le \sum_{j=1}^{J} \nu_j^2 \sum_{k=1}^{J} H_{jk}^2 K_k w_k (1 - w_k)(P_k - P_k^*)^2 \\ &\le b^2 \sum_{k=1}^{J} (P_k - P_k^*)^2 \ , \end{aligned} \tag{55}$$

where

$$b^2 = \max_{1 \le k \le J} \sum_{j=1}^{J} \nu_j^2 H_{jk}^2 K_k w_k (1 - w_k), \quad b \ge 0 \ . \tag{56}$$

In other words,

$$\|\mathbf{U}(\mathbf{P}) - \mathbf{U}(\mathbf{P}^*)\|_2 \le b \|\mathbf{P} - \mathbf{P}^*\|_2 \ , \tag{57}$$

where

$$\|\mathbf{x}\|_2 = \left(\sum_{j=1}^{J} x_j^2\right)^{1/2} \ . \tag{58}$$

But,

$$\max_{1 \le j \le J} |x_j| \le \|\mathbf{x}\|_2 \le \sqrt{J} \ \max_{1 \le j \le J} |x_j| \ . \tag{59}$$

Hence, from (49), we have

$$\min_{1 \le k \le J} w_k \|\mathbf{x}\| \le \max_{1 \le j \le J} |x_j| \le \|\mathbf{x}\|_2 \ . \tag{60}$$

Also, from (59), we obtain

$$\|\mathbf{x}\|_2 \leq \sqrt{J} \max_{1\leq k\leq J} w_k \max_{1\leq j\leq J} |x_j|/w_j = \sqrt{J} \max_{1\leq k\leq J} w_k \|\mathbf{x}\| \ . \tag{61}$$

The inequality (50) follows from (57), (60) and (61), where

$$c = b\sqrt{J} \max_{1\leq k\leq J} w_k / \left(\min_{1\leq j\leq J} w_j \right) \ , \tag{62}$$

and b is given by (56).

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