Virtual Private Networks: Joint Resource Allocation And Routing Design

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Abstract

We consider the resource allocation problem in the design of intranets or virtual private networks (VPNs) that is faced by a service provider, which has service level agreements with various customers to carry their multiservice traffic. The design allocates bandwidth on each link to the VPNs such that, when the traffic of a customer is optimally routed over its VPN, a weighted aggregate measure of carried bandwidth over the service provider's infrastructure is maximized, subject to constraints that each VPN carries a specified minimum. Thus, this is a joint resource allocation and traffic routing problem. Multiplexing is across services and routes within each VPN, but not across VPNs, the latter in the interest of QoS protection. The traffic modelling is at the flow or call level, with random arrivals and holding times of calls and each call requiring (effective) bandwidth, which is characteristic of the call's service type, on all links in its route. The network is modelled as a multirate loss network. Scalability of the design process, i.e., the ability to handle OC3 and higher rates, is an important contribution, and this is achieved by the systematic use of asymptotic techniques, specifically a Refined Uniform Asymptotic Approximation. Our software package VPN DESIGNER incorporates these results. We report on numerical results for problems with up to 8 VPNs on a network with 8 nodes, 24 OC3 links, 6 services and up to 640 routes.

1 INTRODUCTION

This paper addresses the resource allocation and associated routing problem faced by a service provider which has service level agreements (SLA) with various customers to carry their multiservice traffic over its infrastructure. This problem is likely to arise increasingly in various scales and forms, from the perspective of a carrier's carrier at the large end of the spectrum, to ISPs offering network management to outsourcing enterprises at the other end. The objective is to design intranets or virtual private networks (VPNs) which will have allocated bandwidth on the links of the infrastructure network such that, when the traffic of a customer is optimally routed over its VPN, a weighted aggregate measure of carried

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bandwidth over the service provider's infrastructure is maximized, subject to constraints that each VPN carries a specified minimum. The weighted measure is called network revenue here. Note that this is a joint resource allocation and traffic routing problem. We call it the general VPN DESIGN problem. Also of considerable interest is the simpler, unconstrained version of the problem in which the required minimum revenue for each VPN is set to zero, which we refer to as the special VPN DESIGN problem. The solution to this problem is of interest to the service provider as a benchmark for crafting the SLAs, for initializing the solution procedure for the general problem, and, finally, for providing guidance to the feasibility issue and an upper bound to the optimum revenue for the general problem. We are able to exploit the simpler formulation of the special problem to substantially reduce its computational burden. Note too that in the problem formulation multiplexing occurs across services and routes within each VPN, but not across VPNs, the latter in the interest of QoS protection. This important issue is revisited later.

Let $C_\ell^{(\Omega)}$ denote the bandwidth or capacity allocated to VPN Ω on link ℓ of the infrastructure network. Then VPN Ω is defined by the collection $\{C_\ell^{(\Omega)}, \forall \ell\}$; obviously $\sum_\Omega C_\ell^{(\Omega)} = C_\ell$, the bandwidth of link ℓ, for all ℓ. The quantities $\{C_\ell^{(\Omega)}\}$ are also the design variables in the problem. We use the terms VPN and subnetwork interchangeably, and reserve the term network for the infrastructure. Each VPN is wide area and may have traffic between any two nodes in the network, for each of the various services. Typically, of course, each VPN will have distinctive traffic dispositions. The multiservice traffic rates between node pairs are an important component of the descriptor for each VPN.

The network is modelled as a loss network, with the smallest traffic entities being flows and connections, for which collectively we use the term call. Calls have random arrivals and holding times, and each call for its duration requires constant bandwidth, which corresponds to the effective bandwidth that is characteristic of the service type of the call, on all links in its route from source to destination. If the requisite bandwidth is not available in any link in the route, then the call is blocked and lost. Hence the notion of effective bandwidth is being used to encapsulate all packet-level issues, including buffering and QoS (delay, jitter, loss), at network elements. We feel that this device is essential for the tractability of network-level design problems. The interested reader will find a substantial literature on various notions of effective bandwidth, for instance in [1] and [2] and references therein.

This work addresses some of the important traffic engineering issues that are currently attracting considerable attention in the IP and ATM standards bodies. Particular notions in the IETF are QoS-based routing [3], [4] and explicit routes in Multi-Protocol Label Switching (MPLS) [5], in conjunction with the Resource Reservation Protocol (RSVP) [6], to direct QoS-homogeneous aggregate flows along specific paths [7]. The topology and data dependence of such paths are issues addressed here. In the ATM framework, the supporting mechanism is PNNI (Private Network–Network Interface) [8]. Note that, regardless of protocol, the output of the VPN DESIGN problem can be percolated down to network elements to determine the parameters and weights in schedulers and queueing engines. Specifically, the weights of WFQ and link sharing algorithms [9], such as class-based queueing [10], may be set from the design.

The objective in the general VPN DESIGN problem is to maximize network revenue W, where $W = \Sigma W^{(\Omega)}$ and $W^{(\Omega)}$ is an aggregated weighted measure of carried bandwidth on VPN Ω, subject to constraints that each VPN Ω earns at least $W^{(\Omega)}_{\min}$, a given quantity. In the special VPN DESIGN problem, $W^{(\Omega)}_{\min} = 0$, for all Ω. The decision variables are, as previously noted, the capacity allocations $\{C^{(\Omega)}_\ell\}$, and also the proportion of the offered traffic of each stream and VPN that is admitted to the network and, of the admitted portion, the proportions that are offered to the various routes in the admissible set. Thus the decision variables are clearly identified with either the resource allocation or the routing/admission control components of the design problem. Likewise, our iterative algorithm is structured so that in each outer iteration the routing problem solution is followed by a reallocation of capacities to the VPNs. Specifically, for given capacity allocations, in the first step the OPTIMUM ROUTING problem is solved for each VPN independently by using the techniques and software described in [11]. The second step is to calculate for each VPN Ω the *linearized capacity costs* on link ℓ, $w^{(\Omega)}_\ell$, to correspond to the optimum routing, where

$$\frac{\partial W^{(\Omega)}}{\partial C^{(\Omega)}_\ell} \sim w^{(\Omega)}_\ell . \tag{1.1}$$

The analytic derivation of the expression for $w^{(\Omega)}_\ell$ is a central original contribution of this paper. These quantities are used next in the resource reallocation problem to extrapolate revenue from capacities $\{C^{(\Omega)}_\ell\}$ to $\{C^{(\Omega)}_\ell - n^{(\Omega)}_\ell\}$ thus,

$$W^{(\Omega)}\left(\mathbf{C}^{(\Omega)} - \mathbf{n}^{(\Omega)}\right) - W^{(\Omega)}\left(\mathbf{C}^{(\Omega)}\right) \approx -\sum_\ell w^{(\Omega)}_\ell n^{(\Omega)}_\ell . \tag{1.2}$$

The third and final step is to solve a LP formulation of the LINEARIZED OPTIMAL

CAPACITY REALLOCATION problem, which incorporates the just computed linearized capacity costs as well as the constraints on the revenue earned by each VPN, for the optimal selection of $\{n_\ell^{(\Omega)}\}$.

In the special VPN DESIGN problem we show that the final step of the above procedure may be decomposed and the reallocation can be done independently for each link. Moreover, we are able to give a closed form solution to the reduced LP for each link.

The analysis uses the framework based on fixed point equations and implied costs, which was developed for single-service loss networks in works by Kelly [12, 13]. This framework was extended to multiservice loss networks in [14], [15] and [11]. It is worth noting that the expression for the revenue for each VPN is constituted with source-destination and service class dependent parameters, which in their default setting would simply reflect carried bandwidth. However, these parameters may be set differently for purposes of tuning QoS for specific, focussed objectives.

A key driver of the work here is scalability of the design process. The analytic techniques that are effective for DS3 rates may not scale to OC3 and OC12 rates. To handle this problem, Uniform Asymptotic Approximations (UAA) were introduced in [16] at the link level, in which computational complexity is $O(1)$, rather than $O(C)$ for large C as with non-asymptotic ("exact") techniques, where C is the link capacity or bandwidth. The UAA were used systematically for the routing problem in multiservice networks in [11] to achieve various scalability goals. More recently it was found that while the UAA is quite robust and accurate for calculating link loss probabilities, it is not very accurate for calculating implied costs and sensitivities to link loss probabilities. Consequently Refined Uniform Asymptotic Approximations (RUAA) were derived in [17], and it was shown that the RUAA to the loss probabilities is very accurate, and to their sensitivities is very good. Since single-term asymptotic expansions are extended to three-term expansions, the RUAA are more complicated than the UAA, but the computational complexity for a link remains $O(1)$ as $C \to \infty$. In this work we develop all quantities based on the RUAA. In the software package VPN DESIGNER hybrid techniques are used wherein selectively exact techniques are used for some small links and asymptotic techniques for larger links. In this paper, however, in the interest of space, we describe only the asymptotic development. The mathematical derivations are contained in Appendices A–D.

Section 5 reports on our numerical work which is based on VPN DESIGNER. A noteworthy feature of the results is that variabilities in performance across VPNs and even in the utilizations of individual link capacities by the VPNs are quite small, which is consistent with the global load-balancing objectives of the design.

We conclude with a short discussion on the issue of multiplexing across VPNs, which has been alluded to earlier. One of the objectives of our numerical work (see Sec. 5) is to get estimates of the incremental cost (in lost capacity) of not multiplexing across VPNs, i.e., for the Complete Partitioning of infrastructure resources over the alternative discipline of Complete Sharing. In several examples we find that this incremental cost is less than 2%. Of course, on the other hand, there is gain in isolation and protection from misbehaving VPNs. The cost may be higher if substantially more, smaller VPNs are involved or if incoming traffic is very different from the estimates in the design. In this case the resource-sharing technique of Hierarchical Virtual Partitioning [18] may be useful. This technique allows controlled resource-sharing across VPNs and also across service classes within each VPN, with less flexibility in sharing in the former.

2 MODEL AND PROBLEM FORMULATION

We first give information on the infrastructure and types of service. We next describe the pre-design information specific to each subnetwork. This concerns the traffic, route sets and revenue. Finally, the design problem is formulated.

2.1 Model

The infrastructure is a network with N nodes and L links which connect pairs of nodes, with link ℓ having given capacity C_ℓ. The links are unidirectional, i.e., carry traffic in one direction only. There are S types of service, where service s requires bandwidth $d_{s\ell}$ on link ℓ. Note that although we have defined the services universally, there is nothing to prevent a service to be specific to a subset of the subnetworks. It is assumed that C_ℓ and $d_{s\ell}$ are positive integers.

The subnetworks, of which there are $\bar{\Omega}$, are indexed by Ω. The capacity allocated to subnetwork Ω on link ℓ is denoted by $C_\ell^{(\Omega)}$, with $\sum_{\Omega=1}^{\bar{\Omega}} C_\ell^{(\Omega)} = C_\ell$. Let σ denote a source-destination pair of nodes and let (s,σ) denote a stream of calls of service type s between source-destination σ. These quantities which are specific to subnetwork Ω are identified by the superscript (Ω). The set of admissible routes for stream $(s,\sigma)^{(\Omega)}$ is denoted by

$\mathcal{R}^{(\Omega)}(s,\sigma)$, and assumed to be given a priori and fixed. To avoid trivialities we assume that $\mathcal{R}^{(\Omega)}(s,\sigma)$ is not empty for streams with positive offered traffic. We expect that typically route sets will depend on service type. For instance, delay sensitive traffic, like voice, will typically be carried over short routes over fewer hops in contrast to, say, data for which the routes may be much longer. Typically different subnetworks will have disjoint route sets, but it is by no means uncommon for two subnetworks to share a common source-destination pair, and also at least some of the routes between such a source and destination.

The call arrivals for stream $(s,\sigma)^{(\Omega)}$ are Poisson, with mean rate $\Lambda_{s\sigma}^{(\Omega)}$. The outcome of an independent Bernoulli trial determines if any arriving call of stream $(s,\sigma)^{(\Omega)}$ is offered to the subnetwork Ω, which is a form of admission control, and, if it is, to which route $r \in \mathcal{R}^{(\Omega)}(s,\sigma)$ it is offered. If an arriving call in stream $(s,\sigma)^{(\Omega)}$ is offered to a route r, it may be still be blocked and lost if there is insufficient bandwidth available on any link of the route. If the call is accepted, the bandwidth on each link is simultaneously held for the duration of the call. The holding period of a call in stream $(s,\sigma)^{(\Omega)}$ is generally distributed with mean $1/\mu_{s\sigma}^{(\Omega)}$, and is independent of earlier arrival and holding times.

The call arrivals of stream $(s,\sigma)^{(\Omega)}$ for route r are Poisson, with mean rate $\lambda_{sr}^{(\Omega)}$, where

$$\sum_{r\in\mathcal{R}^{(\Omega)}(s,\sigma)} \lambda_{sr}^{(\Omega)} \leq \Lambda_{s\sigma}^{(\Omega)} . \tag{2.1}$$

The corresponding traffic intensity is $\rho_{sr}^{(\Omega)} = \lambda_{sr}^{(\Omega)}/\mu_{s\sigma}^{(\Omega)}$, and it follows that

$$\sum_{r\in\mathcal{R}^{(\Omega)}(s,\sigma)} \rho_{sr}^{(\Omega)} \leq \bar{\rho}_{s\sigma}^{(\Omega)} , \tag{2.2}$$

where $\bar{\rho}_{s\sigma}^{(\Omega)} = \Lambda_{s\sigma}^{(\Omega)}/\mu_{s\sigma}^{(\Omega)}$. The probability

$$p_{s\sigma}^{(\Omega)} \triangleq \frac{1}{\Lambda_{s\sigma}^{(\Omega)}} \sum_{r\in\mathcal{R}^{(\Omega)}(s,\sigma)} \lambda_{sr}^{(\Omega)} \tag{2.3}$$

is a parameter of the admission control, i.e., exogenous calls of stream $(s,\sigma)^{(\Omega)}$ are offered to the network with Bernoulli probability $p_{s\sigma}^{(\Omega)}$ and dropped with probability $(1 - p_{s\sigma}^{(\Omega)})$. Similarly, the probability that a call of service type s is offered to a route r is $\lambda_{sr}^{(\Omega)}/\Lambda_{s\sigma}^{(\Omega)}$. The above admission and routing probabilities are obtained from the design.

2.2 Problem Formulation

Let $e_{sr}^{(\Omega)}$ denote the revenue earned per carried call per unit time by calls of service type s on route r in subnetwork Ω. Also let $L_{sr}^{(\Omega)}$ be the equilibrium probability of loss (after admission to the network) on the route. Since $\rho_{sr}^{(\Omega)}(1 - L_{sr}^{(\Omega)})$ is the carried load of service type s on route r, the long-run average revenue for the subnetwork is

$$W^{(\Omega)} = \sum_{(s,\sigma)^{(\Omega)}} \sum_{r \in \mathcal{R}^{(\Omega)}(s,\sigma)} e_{sr}^{(\Omega)} \rho_{sr}^{(\Omega)} (1 - L_{sr}^{(\Omega)}) . \tag{2.4}$$

The total network revenue is

$$W = \sum_{\Omega=1}^{\bar{\Omega}} W^{(\Omega)} . \tag{2.5}$$

Typically the earnings parameters $e_{sr}^{(\Omega)}$ are proportional to the bandwidth required by a representative call. However other criteria are not uncommon, for instance, making the parameters depend on the distance from source to destination. Also, importantly, $e_{sr}^{(\Omega)}$ can be used to tune call-level quality of service, with higher earning rate giving better service.

We formulate the general VPN DESIGN problem as follows:

$$\operatorname*{Max}_{\{C_\ell^{(\Omega)}\}, \{\rho_{sr}^{(\Omega)}\}} \quad W \tag{2.6}$$

such that

$$W^{(\Omega)} \geq W_{\min}^{(\Omega)} \qquad \forall \Omega \tag{2.7}$$

$$\left.\begin{array}{rcl} \displaystyle\sum_{r \in \mathcal{R}^{(\Omega)}(s,\sigma)} \rho_{sr}^{(\Omega)} & \leq & \bar{\rho}_{s\sigma}^{(\Omega)} \\ \rho_{sr}^{(\Omega)} & \geq & 0 \end{array}\right\} \ \forall (s,\sigma)^{(\Omega)}, \ \forall \Omega \tag{2.8}$$

$$\sum_{\Omega=1}^{\bar{\Omega}} C_\ell^{(\Omega)} = C_\ell \qquad \forall \ell \tag{2.9}$$

$$C_\ell^{(\Omega)} \geq 0 \qquad \forall \Omega, \ \forall \ell \tag{2.10}$$

As previously mentioned, the special VPN DESIGN problem is obtained when $W_{\min}^{(\Omega)} = 0$ for all Ω. Note that the problems are joint resource (bandwidth) allocation and route optimization problems. The latter in isolation, i.e., for a fixed network, has been treated earlier in [11]. In the algorithmic development here it is simply assumed that the procedure in the earlier work is available. For the benefit of the reader we state the OPTIMUM ROUTING problem for each subnetwork Ω. The inputs to this problem are $\{C_\ell^{(\Omega)}\}$, $\{\bar{\rho}_{s\sigma}^{(\Omega)}\}$, $\{e_{sr}^{(\Omega)}\}$ and $\{\mathcal{R}^{(\Omega)}(s,\sigma)\}$.

$$\begin{array}{l} \max \quad W^{(\Omega)} \\ \{\rho_{sr}^{(\Omega)}\} : \left.\begin{array}{rcl} \displaystyle\sum_{r \in \mathcal{R}^{(\Omega)}(s,\sigma)} \rho_{sr}^{(\Omega)} & \leq & \bar{\rho}_{s\sigma}^{(\Omega)} \\ \rho_{sr}^{(\Omega)} & \geq & 0 \end{array}\right\} \ \forall (s,\sigma)^{(\Omega)} \end{array} \tag{2.11}$$

We conclude the section by commenting on notation for double-indexed quantities, such as $C_\ell^{(\Omega)}$. We have need for $\mathbf{C}^{(\Omega)} = (C_1^{(\Omega)}, \ldots, C_L^{(\Omega)})$, as well as $\mathbf{C}_\ell = (C_\ell^{(1)}, \ldots, C_\ell^{(\bar{\Omega})})$. We use $\mathbf{C}$ to refer to the complete set $\{C_\ell^{(\Omega)}\}$.

3 NETWORK ANALYSIS AND ASYMPTOTIC THEORY

In this section we consider a single subnetwork, and hence suppress the index Ω. We first give the fixed point equations (FPE) for determining the link loss probabilities, and then the equations for the subnetwork implied costs. We next consider a single link with large capacity C, and traffic intensities which are also large, $O(C)$, and give Refined Uniform Asymptotic Approximations (RUAA) to the loss probabilities, and to the sensitivity of the loss probabilities to the capacity. The implications of the RUAA for the subnetwork implied costs, and for the sensitivity of the subnetwork revenue to link capacities, are then considered.

The end result of this section, which is central to the subsequent development of our algorithm, is the "linearized capacity costs" that are the gradient of the subnetwork revenue with respect to the capacities of all the links in the subnetwork. This result is new.

3.1 Fixed Point Equations

The FPE for the subnetwork are derived on the basis of the well known assumption of link independence. Each route r carrying traffic of service type s and using link ℓ adds load $\nu_{s\ell;r}$ on link ℓ which, by assumption, is Poisson, with a rate which is reduced by independent thinning by all other links in the route. By summing $\nu_{s\ell;r}$ over all routes r which use link ℓ we obtain $\nu_{s\ell}$, the total thinned load of service s offered to link ℓ. Now let $B_{s\ell}$ denote the loss probability of service s on link ℓ, and let $\mathbf{d}_\ell = \{d_{s\ell}\}_s$ and $\boldsymbol{\nu}_\ell = \{v_{s\ell}\}_s$. Then $B_{s\ell} = \mathbb{L}_s(\mathbf{d}_\ell, \boldsymbol{\nu}_\ell, C_\ell)$, which may be computed by the Kaufman–Roberts recursion [19], [20], with complexity $O(C_\ell)$. Alternatively, if $C_\ell \gg 1$ and $\boldsymbol{\nu}_\ell = O(C_\ell)$, an approximation to $B_{s\ell}$ may be computed by RUAA [17], with complexity $O(1)$. In any case, we let $\phi_{s\ell}$ denote this mapping. The complementary part of the FPE gives the thinned offered load at each link as a function of the link loss probabilities. This mapping is denoted by $\boldsymbol{\psi}$. The complete subnetwork FPE are:

$$B_{s\ell} \quad = \quad \phi_{s\ell}(\boldsymbol{\nu}_\ell), \ (s = 1, 2, \ldots, S; \ \ell = 1, 2, \ldots, L), \tag{3.1}$$

$$\boldsymbol{\nu} \;=\; \psi(\mathbf{B}),$$

where

$$\boldsymbol{\nu} = \{\boldsymbol{\nu}_\ell\} \quad \text{and} \quad \mathbf{B} = \{B_{s\ell}\}\,.$$

The numerical technique for solving the FPE is successive approximations, possibly damped. Finally, the loss probability L_{sr} of service s on route r is obtained from the solution of the FPE thus

$$L_{sr} = 1 - \prod_{\ell \in r}(1 - B_{s\ell}); \qquad r \in \mathcal{R}(s,\sigma)\,. \tag{3.2}$$

3.2 Network Implied Costs

One of the roles of the implied costs $\{c_{s\ell}\}$ ([11]–[15]), as exhibited in the following equations, is in determining the sensitivity of subnetwork revenue to offered loads:

$$\frac{\partial W}{\partial \rho_{sr}} = (1 - L_{sr})\left(e_{sr} - \sum_{\ell \in r} c_{s\ell}\right)\,. \tag{3.3}$$

The above highlights the importance of implied costs for traffic routing, where the decision variables are $\{\rho_{sr}\}$. Here our focus is on capacity costs, where implied costs also feature importantly but, as we shall see, in a different way altogether.

The calculation of $\{c_{s\ell}\}$ requires the solution of a system of SL linear equations,

$$c_{t\ell} = \sum_{\sigma,s}\left[\mathbb{L}_s(\mathbf{d}_\ell, \boldsymbol{\nu}_\ell, C_\ell - d_{t\ell}) - \mathbb{L}_s(\mathbf{d}_\ell, \boldsymbol{\nu}_\ell, C_\ell)\right] \sum_{r \in \mathcal{R}(s,\sigma):\ell \in r} \nu_{s\ell;r}\left(e_{sr} - \sum_{k \in r - \{\ell\}} c_{sk}\right), \tag{3.4}$$

where $\{\boldsymbol{\nu}_\ell\}$ are obtained from the solution of the FPE (3.1), and

$$\nu_{s\ell;r} = \rho_{sr} \prod_{m \in r - \{\ell\}} (1 - B_{sm})\,. \tag{3.5}$$

3.3 Single Link Asymptotics

Before proceeding to subnetworks, we focus on the use of the RUAA for a single link, and just in this subsection we suppress the link index ℓ. It is assumed that the link capacity C is large, and that the traffic intensities $\{\nu_s\}$ are also large, $O(C)$. Also, d_s ($s = 1, 2, \ldots, S$) are not large relative to C and their greatest common divisor is 1. The RUAA to the loss probability B_s for service s on the link takes the form

$$B_s \sim \sum_{i=1}^{3} \mathbb{B}^{(i)} \theta^{(i)}(d_s)\,, \tag{3.6}$$

where $\mathbb{B}^{(i)}$, $i = 1, 2, 3$, does not depend on s. The quantities $\mathbb{B}^{(i)}$ and $\theta^{(i)}(n)$, $i = 1, 2, 3$, are defined in Appendix A. In the next two subsections we also need an approximation to the change in the loss probabilities due to a change in the capacity. If $n = O(1)$ is a positive or negative integer, then the RUAA, as shown in Appendix B, has the form

$$\mathbb{L}_s(\mathbf{d}, \boldsymbol{\nu}, C - n) - \mathbb{L}_s(\mathbf{d}, \boldsymbol{\nu}, C) \sim \frac{\sum_{i=1}^{3} A_s^{(i)} \theta^{(i)}(n)}{\left[1 - \sum_{i=1}^{3} \mathbb{B}^{(i)} \theta^{(i)}(n)\right]}. \tag{3.7}$$

Now $\theta^{(i)}(n)$ is an analytic function of n, and $\theta^{(i)}(0) = 0$, $i = 1, 2, 3$. Although $\mathbb{L}_s(\mathbf{d}, \boldsymbol{\nu}, C) = \mathbb{L}_s(\mathbf{d}, \boldsymbol{\nu}, \lfloor C \rfloor)$ if C is not an integer, we use the approximation (3.7) when n is not an integer. In particular, if we divide equation (3.7) by n, and let $n \to 0$, we obtain the capacity gradient

$$\frac{\partial \mathbb{L}_s}{\partial C} \sim -\sum_{i=1}^{3} A_s^{(i)} \sigma^{(i)}, \tag{3.8}$$

where $\sigma^{(i)} = \lim_{n \to 0} \theta^{(i)}(n)/n$. These quantities may be given explicitly in terms of the unique positive z^* which minimizes the strictly convex function

$$F(z) = \sum_s \nu_s (z^{d_s} - 1) - C \log z, \quad (z > 0). \tag{3.9}$$

The function $F(z)$ as well as z^* are common to both UAA and RUAA. The minimizer z^* of (3.9) is readily calculated by bisection. We give below the expressions for $\sigma^{(i)}$ which are needed later, and are derived in Appendix C.

$$\sigma^{(1)} = \frac{\log z^*}{(z^* - 1)}, \quad \sigma^{(2)} = -\left[\frac{(z^* \log z^* + 1 - z^*)}{(z^* - 1)^2} + \sigma^{(1)}\right], \tag{3.10}$$

and

$$\sigma^{(3)} = \frac{[2(z^*)^2 \log z^* + 2z^*(1 - z^*) - (z^* - 1)^2]}{(z^* - 1)^3} - 2\sigma^{(2)}. \tag{3.11}$$

3.4 RUAA in Implied Costs

We consider the implication of the RUAA for the FPE (3.1) and the implied cost equations (3.4). The RUAA may be used, as previously noted, for the evaluation of the function $\phi_{s\ell}$ in (3.1). Also, from (3.7), reintroducing the link index ℓ, we have

$$\mathbb{L}_s(\mathbf{d}_\ell, \boldsymbol{\nu}_\ell, C_\ell - d_{t\ell}) - \mathbb{L}_s(\mathbf{d}_\ell, \boldsymbol{\nu}_\ell, C_\ell) \sim \sum_{i=1}^{3} A_{s\ell}^{(i)} a_{t\ell}^{(i)}, \tag{3.12}$$

where, from (3.6), $a_{t\ell}^{(i)} = \theta_\ell^{(i)}(d_{t\ell})/(1-B_{t\ell})$. From (3.4), if the RUAA (3.12) is applicable for all links of the subnetwork, it follows that the implied costs are given asymptotically by

$$c_{t\ell} \sim \sum_{i=1}^{3} a_{t\ell}^{(i)} \xi_\ell^{(i)} , \tag{3.13}$$

where

$$\xi_\ell^{(i)} = \sum_{\sigma,s} \sum_{r\in\mathcal{R}(s,\sigma):\ell\in r} A_{s\ell}^{(i)} \nu_{s\ell;r} \left(e_{sr} - \sum_{k\in r-\{\ell\}} \sum_{j=1}^{3} a_{sk}^{(j)} \xi_k^{(j)} \right) . \tag{3.14}$$

Equation (3.14) is a complete system of equations in $\{\xi_\ell^{(i)}\}$. The parameter $\{A_{s\ell}^{(i)}\}$, $\{\nu_{s\ell;r}\}$ and $\{a_{sk}^{(j)}\}$ are all obtained from the solution of the FPE (3.1). The complexity of solving the system of $3L$ linear equations (3.14) is $O(27\,L^3)$, whereas the complexity of solving the system of SL linear equations (3.4) directly for the implied costs is $O(S^3L^3)$. Independence from the number of services S constitutes a major gain if S is large. In any case, the approximation in (3.12), if applicable for link ℓ, may be used in (3.4).

3.5 Revenue Sensitivity to Link Capacity

We consider the implication of the RUAA for the sensitivity of the subnetwork revenue to the link capacities. With the smooth continuation of $\mathbb{L}_s(\mathbf{d}_\ell, \boldsymbol{\nu}_\ell, C_\ell)$ for non-integer values of C_ℓ, as previously discussed, in Appendix D we show by extending prior work [13] that the corresponding gradient of the subnetwork revenue with respect to the capacity of link ℓ is

$$\frac{\partial W}{\partial C_\ell} = -\sum_{\sigma,s} \sum_{r\in\mathcal{R}(s,\sigma):\ell\in r} \frac{\partial \mathbb{L}_s}{\partial C_\ell} \nu_{s\ell;r} \left(e_{sr} - \sum_{k\in r-\{\ell\}} c_{sk} \right) . \tag{3.15}$$

Hence, from (3.8), (3.13) and (3.14), we obtain

$$\frac{\partial W}{\partial C_\ell} \sim \sum_{i=1}^{3} \sigma_\ell^{(i)} \xi_\ell^{(i)} . \tag{3.16}$$

In summary we have

Proposition 3.1 *For large link capacities C_ℓ and traffic intensities of the same order,*

$$\frac{\partial W}{\partial C_\ell} \sim w_\ell , \tag{3.17}$$

where

$$w_\ell \triangleq \sum_{i=1}^{3} \sigma_\ell^{(i)} \xi_\ell^{(i)} . \tag{3.18}$$

Here $\sigma_\ell^{(i)}$, $i = 1, 2, 3$, are given by (3.10) and (3.11), with z^* replaced by $z^*{}_\ell$. Also, $\xi_\ell^{(i)}$ are obtained by solving the system of $3L$ linear equations in (3.14).

We call the quantities w_ℓ the *linearized capacity costs.* The above approach and result are new. Our derivation here of the sensitivities $\partial \mathbb{L}_s/\partial C$ in (3.8) differs from the interpolative approach in [11], which extended to multiple service classes the prior result in [13] for a single class. The interpolative approach gives class-dependent sensitivities (see [11]), an impediment to its use in (3.15) which is circumvented here.

4 ALGORITHMS FOR THE VPN DESIGN PROBLEM

In this section we begin by giving the mathematical basis of the iterative algorithms for, first, the general and then the special problems. A reallocation of each link's bandwidth to the subnetworks is carried out in each iteration. This reallocation is based on a linearization of the total network revenue and the linearized capacity costs derived in the preceding section. Note that after each such reallocation the optimal routing problem is solved for each of the newly resized subnetworks. Later in this section we give a complete description of the algorithm in pseudocode, and conclude by considering initialization and feasibility (in the general problem).

4.1 The General Problem

Taking note of (3.17) and applying it to subnetwork Ω which has bandwidth $C_\ell^{(\Omega)}$ allocated to it on each link ℓ, a linearized extrapolation of the revenue for the subnetwork is

$$W^{(\Omega)}\left(\mathbf{C}^{(\Omega)} - \mathbf{n}^{(\Omega)}\right) - W^{(\Omega)}\left(\mathbf{C}^{(\Omega)}\right) \approx -\sum_{\ell=1}^{L} w_\ell^{(\Omega)} n_\ell^{(\Omega)} . \tag{4.1}$$

Here $w_\ell^{(\Omega)}$, $\ell = 1, 2, \ldots, L$, are the linearized capacity costs given in (3.18), and integers $n_\ell^{(\Omega)}$ are candidates for modifying the current capacity allocations. The latter quantities are required to be $O(1)$ for (4.1) to apply.

To reflect the constraints on earned revenue of individual VPNs, we proceed as follows. Let

$$E^{(\Omega)} \triangleq W^{(\Omega)}\left(\mathbf{C}^{(\Omega)}\right) - W_{\min}^{(\Omega)} . \tag{4.2}$$

Using (4.1), the requirement that $W^{(\Omega)}(\mathbf{C}^{(\Omega)} - \mathbf{n}^{(\Omega)}) \geq W_{\min}^{(\Omega)}$ may be stated thus:

$$\sum_{\ell=1}^{L} \omega_\ell^{(\Omega)} n_\ell^{(\Omega)} \leq E^{(\Omega)} \qquad \Omega = 1, 2, \ldots, \bar{\Omega} . \tag{4.3}$$

From (2.5) note too that

$$W(\mathbf{C}-\mathbf{n}) - W(\mathbf{C}) \approx -\sum_{\Omega=1}^{\bar{\Omega}} \sum_{\ell=1}^{L} w_\ell^{(\Omega)} n_\ell^{(\Omega)} \,. \tag{4.4}$$

We proceed to select $\mathbf{n}$ such that $W(\mathbf{C}-\mathbf{n})$ is maximized. We formulate the LINEARIZED OPTIMAL CAPACITY REALLOCATION problem as a LP:

$$\min \sum_{\Omega=1}^{\bar{\Omega}} \sum_{\ell=1}^{L} w_\ell^{(\Omega)} n_\ell^{(\Omega)} \tag{4.5}$$

$$\{n_\ell^{(\Omega)}\} : \qquad \sum_{\ell=1}^{L} \omega_\ell^{(\Omega)} n_\ell^{(\Omega)} \le E^{(\Omega)}, \qquad \forall \Omega \tag{4.6}$$

$$\sum_{\Omega} n_\ell^{(\Omega)} = 0, \qquad \forall \ell$$

$$\bar{n}_\ell^{(\Omega)} \le n_\ell^{(\Omega)} \le \bar{\bar{n}}_\ell^{(\Omega)}, \qquad \forall \ell,\ \forall \Omega \,. \tag{4.7}$$

The first constraint is just (4.3), the second constraint reflects fixed link capacities and the third constraint is obtained by the following two-step process. First, the new capacity allocations must be nonnegative and cannot individually exceed any link capacity, i.e.,

$$C_\ell^{(\Omega)} - C_\ell \le n_\ell^{(\Omega)} \le C_\ell^{(\Omega)}, \qquad \forall \ell \,.$$

Next we tighten these constraints to ensure that the decision variables are not so large that the approximation in (4.1) becomes invalid. To this end we define $\bar{\bar{n}}_\ell^{(\Omega)}$ and $\bar{n}_\ell^{(\Omega)}$:

$$\bar{\bar{n}}_\ell^{(\Omega)} \triangleq \left\lfloor \min\left(C_\ell^{(\Omega)},\ (\text{const})\sqrt{C_\ell^{(\Omega)}} \right) \right\rfloor \tag{4.8}$$

$$\bar{n}_\ell^{(\Omega)} \triangleq \left\lceil \max\left(C_\ell^{(\Omega)} - C_\ell,\ -(\text{const})\sqrt{C_\ell^{(\Omega)}} \right) \right\rceil \,, \tag{4.9}$$

where a reasonable value of the constant (const) ≈ 0.5. It is important to ensure that the above bounds are integers so that the new capacity allocations are integral. Note that

$$\bar{n}_\ell^{(\Omega)} \le 0 < \bar{\bar{n}}_\ell^{(\Omega)} \,, \tag{4.10}$$

in cases where $C_\ell^{(\Omega)} > 0$.

4.2 The Special Problem

We next consider the version of the LINEARIZED OPTIMAL CAPACITY REALLOCATION problem for the special case in which the constraints that each VPN Ω earns at least $W_{\min}^{(\Omega)}$ are eliminated. In this case the constraints in (4.6) to the LP in (4.5) are eliminated. As a consequence we are able to give a closed form solution to the LP.

There are two steps in obtaining the solution. The first establishes that the LP (4.5) decomposes into L problems, one for each link. That is, the problem indexed by ℓ ($\ell = 1, 2, \ldots, L$) is

$$\begin{aligned} &\min \quad \sum_{\Omega=1}^{\bar{\Omega}} \omega_\ell^{(\Omega)} n_\ell^{(\Omega)} \\ &\{n_\ell^{(\Omega)}\} : \sum_{\Omega} n_\ell^{(\Omega)} = 0 \\ &\bar{n}_\ell^{(\Omega)} \le n_\ell^{(\Omega)} \le \bar{\bar{n}}_\ell^{(\Omega)} \end{aligned} \tag{4.11}$$

Next we give an explicit solution to the LP in (4.11). Just here the subscript ℓ is redundant and it is suppressed. Assume without loss of generality that

$$w^{(1)} \le w^{(2)} \le \cdots \le w^{(\bar{\Omega})}, \tag{4.12}$$

i.e., subnetwork 1 is "cheapest", subnetwork $\bar{\Omega}$ "most expensive", and so on. Hence, the objective function decreases whenever we reduce $n^{(\Omega_2)}$ and increase $n^{(\Omega_1)}$ by an identical amount, provided $\Omega_1 < \Omega_2$. Hence, the solution to (4.11) consists of maximizing $n^{(i)}$ for small i and minimizing $n^{(i)}$ for large i in such a manner that the global constraint $\Sigma n^{(\Omega)} = 0$ is satisfied.

A little thought shows that on account of (4.10) and (4.12) there exists an unique subnetwork index I such that

$$\begin{aligned} &\bar{\bar{n}}^{(1)} + \bar{\bar{n}}^{(2)} + \cdots + \bar{\bar{n}}^{(I-1)} + \bar{n}^{(I)} + \bar{n}^{(I+1)} + \cdots + \bar{n}^{(\bar{\Omega})} \\ &\le \quad 0 < \bar{\bar{n}}^{(1)} + \cdots + \bar{\bar{n}}^{(I-1)} + \bar{\bar{n}}^{(I)} + \bar{n}^{(I+1)} + \cdots + \bar{n}^{(\bar{\Omega})} . \end{aligned} \tag{4.13}$$

Hence there exists $n^{(I)}$, such that $\bar{n}^{(I)} \le n^{(I)} < \bar{\bar{n}}^{(I)}$, and

$$\bar{\bar{n}}^{(1)} + \bar{\bar{n}}^{(2)} + \cdots + \bar{\bar{n}}^{(I-1)} + n^{(I)} + \bar{n}^{(I+1)} + \cdots + \bar{n}^{(\bar{\Omega})} = 0 \tag{4.14}$$

Moreover, if all $\bar{n}^{(\Omega)}$ and $\bar{\bar{n}}^{(\Omega)}$ are integral, then so is $n^{(I)}$. Hence,

Proposition 4.1 *Let $n^{(I)}$, $\bar{n}^{(I)} \le n^{(I)} < \bar{\bar{n}}^{(I)}$, be the unique solution to (4.14). Then the solution to the linear program in (4.11) is*

$$\begin{aligned} n^{(\Omega)} &= \bar{\bar{n}}^{(\Omega)} \quad (\Omega = 1, 2, \ldots, I-1) \\ &= n^{(I)} \quad (\Omega = I) \\ &= \bar{n}^{(\Omega)} \quad (\Omega = I+1, I+2, \ldots, \Omega)\,. \end{aligned} \tag{4.15}$$

4.3 Algorithm Structure

To recapitulate, for the general VPN DESIGN problem the linearized problem of optimal selection of $\mathbf{n}$ such that $W(\mathbf{C} - \mathbf{n})$ is maximized has been shown to be the LP in (4.5), and for the special VPN DESIGN problem to have the explicit solution in Proposition 4.1 for each link ℓ (which index is suppressed in (4.15)). Then $(C_\ell^{(\Omega)} - n_\ell^{(\Omega)})$ is the new allocation of bandwidth to subnetwork Ω on link ℓ. The pseudocode for the algorithm is given next.

Pseudocode of the Algorithm for the VPN DESIGN PROBLEM

input: $\{C_\ell\}$, $\{\bar{\rho}_{s\sigma}^{(\Omega)}\}$, $\{e_{sr}^{(\Omega)}\}$, $\{\mathcal{R}_{(s,\sigma)}^{(\Omega)}\}$

output: $\{C_\ell^{(\Omega)}\}$, $\{W^{(\Omega)}\}$

1. Initialize: $k \leftarrow 0$

   initial allocations $\{C_\ell^{(\Omega)} : \sum_\Omega C_\ell^{(\Omega)} = C_\ell,\ \forall \ell\}$

   initial hypercubes $\mathcal{H}_\ell(\bar{\mathbf{n}}_\ell, \bar{\bar{n}}_\ell) = \{\bar{n}_\ell^{(\Omega)} \le n_\ell \le \bar{\bar{n}}_\ell^{(\Omega)}, \forall \ell\}$

   hypercube threshold H_T

   revenue threshold W_T

   $W_{\text{prev}} = 0$

   not_converged $\leftarrow$ true

2. <u>While</u> (not_converged) <u>do</u>

   2.1 for $\Omega = 1, 2, \ldots, \bar{\Omega}$ {

   solve OPTIMUM ROUTING problem (see (2.11))

   input: $\{C_\ell\}$, $\{\bar{\rho}_{s\sigma}^{(\Omega)}\}$, $\{e_{sr}^{(\Omega)}\}$, $\{\mathcal{R}_{(s,\sigma)}^{(\Omega)}\}$

   output: $\{\rho_{sr}^{(\Omega)}\}$, $W^{(\Omega)}$

   compute linearized capacity costs $\{w_\ell^{(\Omega)}, \forall \ell\}$

   }

   2.2 test for convergence

   $W = \sum_\Omega W^{(\Omega)}$

$\underline{\text{if}}$ $(W - W_{\text{prev}})/W \leq W_T$ and $|\mathcal{H}(\bar{n}, \bar{\bar{n}})| \leq H_T$

not_converged $\leftarrow$ false

$\underline{\text{else}}$ $W_{\text{prev}} \leftarrow W$

2.3 $\underline{\text{if}}$ not_converged

solve the LINEARIZED OPTIMAL CAPACITY REALLOCATIONS problem (see (4.5) and (4.15))

input: $\{C_\ell^{(\Omega)}\}$, $\{w_\ell^{(\Omega)}\}$

output: $\{n_\ell^{(\Omega)}\}$

for $\Omega = 1, 2, \ldots, \bar{\Omega}$ and $\ell = 1, 2, \ldots, L$

$C_\ell^{(\Omega)} \leftarrow C_\ell^{(\Omega)} - n_\ell^{(\Omega)}$

end_for

readjust $\bar{\mathbf{n}}$, $\bar{\bar{\mathbf{n}}}$

$k \leftarrow k + 1$

end_if

end_while

4.4 Initializations and Feasibility

Our recommendation, based on our numerical experience, is to solve in sequence the hierarchy of three problems. These are in increasing order of difficulty. Each of the solutions of the first two problems serves as initial condition to the next problem in the hierarchy. Since our solution procedure for each of the problems, is iterative, a good starting solution helps greatly to reduce the number of iterations to convergence.

The first in the hierarchy is the OPTIMUM ROUTING problem (see (2.11)) for an aggregate traffic model from which all subnetwork structure is removed. That is, the offered traffic for any source, destination pair σ and service type s, $\bar{\rho}_{s\sigma}$, is the sum of such traffic over all subnetworks,

$$\bar{\rho}_{s\sigma} = \sum_{\Omega} \bar{\rho}_{s\sigma}^{(\Omega)} \tag{4.16}$$

The routing solution thus obtained is applicable when the discipline for multiplexing across subnetworks in each link is Complete Sharing, in contrast to the assumption of Complete Partitioning across VPNs in the VPN DESIGN problem. This Complete Sharing solution is obviously of considerable independent interest.

After the aforementioned routing problem has been solved, the utilization of the bandwidth by streams of each subnetwork may be calculated, for each link. We make our allocation of bandwidth $C_\ell^{(\Omega)}$ proportional to the computed utilization, with the constant of proportionality such that $\sum_\Omega C_\ell^{(\Omega)} = C_\ell$. The quantities $\{C_\ell^{(\Omega)}\}$ thus computed may be used as initial condition to the special VPN DESIGN problem. Since the latter problem's structure has been exploited to simplify the computations, the computational burden for its solution is modest.

The capacity allocations $\{C_\ell^{(\Omega)}\}$ obtained from the aforementioned solution are used as initial allocations for the final problem in the hierarchy, the general VPN DESIGN problem.

The general VPN DESIGN problem may be infeasible if the quantities $\{W_{\min}^{(\Omega)}\}$ are too large. We propose using the classical big–M method [21], which combines in one procedure the resolution of the question of feasibility with optimization in the event that the problem is feasible. If the problem is not feasible, the natural recourse is to reduce $\{W_{\min}^{(\Omega)}\}$, with the revenue earned by each VPN in the special problem as a guide.

5 COMPUTATIONAL RESULTS

We describe the results of our computational experiments, which have been obtained by exercising VPN DESIGNER. The software structure of this package requires a separate treatment [22], and here we give only a brief overview. VPN DESIGNER consists of a master UNIX process, which communicates with $\bar{\Omega}$ TALISMAN processes, which do not interact with each other. The TALISMAN processes are responsible for solving the OPTIMUM ROUTING problem in procedure 2.1 of the pseudocode given in the preceding section. These processes compute the linearized capacity costs and pass these to the master process. The master process is responsible for procedures 2.2 and 2.3. It computes the new capacity allocations for each subnetwork and passes this information back to the appropriate TALISMAN process.

This section is organized in four parts. First, we give the basic facts of the network infrastructure on which the experiments are conducted, and of the services and the multiservice traffic specifications of the subnetworks. Second, we give the design results for a special VPN DESIGN problem, as well as on the convergence and comparisons with the Complete Sharing solution. Third, we report on a general problem. Finally, we report briefly on the design of source trees on the same network infrastructure.

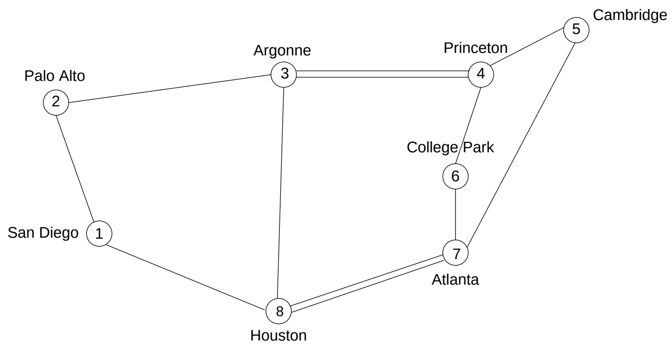

Figure 1: Infrastructure network

5.1 Infrastructure, Services and VPN Traffic

The infrastructure is a fictitious network, called Core NSF [11], with eight nodes ($N = 8$), of which 10 pairs are directly connected, as shown in Figure 1. The network has 20 unidirectional links ($L = 20$), one in each direction for each connected node pair. The typical bandwidth of a unidirectional link is OC3 = 155 Mbps, with the exception of the links in both directions connecting Argonne ③ and Princeton ④, and also Houston ⑧ and Atlanta ⑦, which have bandwidths of $2\times$ OC3 = 310 Mbps.

There are six services ($S = 6$). The effective bandwidths of individual calls of these services are 16, 48, 64, 96, 384 and 640 Kbps, respectively, uniformly over all links. Service 1 may be thought of as voice, services 2–5 as data, and service 6 as video. It is convenient to rescale and let 16 Kbps be the bandwidth unit. In this unit the bandwidth requirements are $d_{s\ell} = 1$, 3, 4, 6, 24 and 40 for $s = 1, \ldots, 6$, respectively, for all links ℓ. Also, link bandwidths of 155 Mbps and 310 Mbps translate to 9688 and 19375 units, respectively. Here C_ℓ is either 9688 or 19375 for $\ell = 1, 2, \ldots, L$.

There are four customers requiring VPNs ($\bar{\Omega} = 4$), the West, Mid West, East and South Corporations. These are all national customers with traffic from every node to every other node in the infrastructure network, for each of the services, with the traffic patterns somewhat different for the four corporations due to localization, as explained next. The four corporations have their main offices as follows: San Diego and Palo Alto for the West Corp., Argonne for the Mid West Corp., Princeton, Cambridge and College Park for the East Corp., Atlanta and Houston for the South Corp. Any node in the infrastructure network which is not a main office of a corporation is a branch office. The localization in traffic disposition states that the main–main, main–branch, branch–main, and branch–

branch traffic for any pair of nodes is roughly proportional to 4:2:2:1. This property is reflected in the traffic intensity matrices, which are described next.

To keep the description of the exogenously offered traffic intensities compact, we employ a base *traffic intensity matrix* $\mathbf{T}^{(\Omega)}$, which is $N \times N$, for each VPN, and a set of multipliers m_s ($s = 1, 2, \ldots, 6$). Then the offered traffic intensity of service s from node i to node j for VPN Ω, $\bar{\rho}_{s\sigma}^{(\Omega)} = m_s T_{ij}^{(\Omega)}$ where $\sigma = (i, j)$. The matrices $\{\mathbf{T}^{(\Omega)}\}$ are given in Table I, and the multipliers $\{m_s\}$ in Table II. The unit of measurement for traffic intensity is calls.

Table I: Base Traffic Intensity Matrices $\mathbf{T}^{(\Omega)}$ for the VPNs

	1	2	3	4	5	6	7	8
1	×	11.8	8.0	1.2	10.4	5.8	2.3	3.5
2	13.8	×	27.6	3.5	35.7	17.3	6.9	10.4
3	9.2	28.8	×	2.3	21.3	10.4	4.0	6.3
4	1.2	3.5	1.7	×	2.0	1.0	0.6	0.6
5	12.7	37.9	22.4	2.5	×	11.8	5.2	8.6
6	5.8	16.1	9.2	1.0	10.4	×	2.3	3.5
7	2.3	5.8	3.5	0.6	4.6	2.3	×	1.0
8	3.5	9.2	5.8	0.6	6.9	3.5	1.0	×

WEST CORP

	1	2	3	4	5	6	7	8
1	×	3.0	8.0	0.6	5.2	2.9	1.2	1.7
2	3.5	×	27.6	1.7	17.8	8.6	3.5	5.2
3	9.2	28.8	×	4.6	42.6	20.7	8.0	12.7
4	0.6	1.7	3.5	×	2.0	1.0	0.6	0.6
5	6.3	19.0	44.9	2.5	×	11.8	5.2	8.6
6	2.9	8.0	18.4	1.0	10.4	×	2.3	3.5
7	1.2	2.9	6.9	0.6	4.6	2.3	×	1.0
8	1.7	4.6	11.5	0.6	6.9	3.5	1.0	×

MID WEST CORP

	1	2	3	4	5	6	7	8
1	×	3.0	4.0	0.6	5.2	2.9	2.3	3.5
2	3.5	×	13.8	1.7	17.8	8.6	6.9	10.4
3	4.6	14.4	×	2.3	21.3	10.4	8.0	12.7
4	0.6	1.7	1.7	×	2.0	1.0	1.2	1.2
5	6.3	19.0	22.4	2.5	×	11.8	10.4	17.3
6	2.9	8.0	9.2	1.0	10.4	×	4.6	6.9
7	2.3	5.8	6.9	1.2	9.2	4.6	×	3.9
8	3.5	9.2	11.5	1.2	13.8	6.9	3.9	×

SOUTH CORP

	1	2	3	4	5	6	7	8
1	×	3.0	4.0	1.2	10.4	5.8	1.2	1.7
2	3.5	×	13.8	3.5	35.7	17.3	3.5	5.2
3	4.6	14.4	×	4.6	42.6	20.7	4.0	6.3
4	1.2	3.5	3.5	×	7.9	3.9	1.2	1.2
5	12.7	37.9	44.9	9.9	×	47.3	10.4	17.3
6	5.8	16.1	18.4	3.9	41.4	×	4.6	6.9
7	1.2	2.9	3.5	1.2	9.2	4.6	×	1.0
8	1.7	4.6	5.8	1.2	13.8	6.9	1.0	×

EAST CORP

Table II: Multipliers applied to VPN's base traffic intensity matrix to obtain matrices for services.

Service s	1	2	3	4	5	6
Multiplier m_s	0.5	0.5	1.0	0.5	0.5	0.5

In the expression for revenue in (2.4) and (2.5) we let the revenue earned per call per unit time of service s on route r for subnetwork Ω, $e_{sr}(\Omega) \equiv d_s \ \forall r, \ \forall \Omega$. With this selection of the earnings parameters, the revenues for the subnetworks $\{W^{(\Omega)}\}$ as well as the total network revenue W are simply equal to the carried bandwidth over the corresponding networks in units of bandwidth (not calls).

The results given below are for route sets generated by the following simple rule:

$$\mathcal{R}^{(\Omega)}(s, \sigma) = \{r \mid \#(\text{hops}) \text{ in route } r \leq 4\},$$

which is independent of Ω and s. It turns out that for each VPN the total number of routes is 160, giving a total of 640 routes.

5.2 Results for the Special Problem

We give the results obtained by exercising VPN DESIGNER for the special design problem. In Table III we give a summary of the offered and carried bandwidth over the VPNs for the optimized design. The lack of significant variability in performance as measured by blocking is noteworthy. This is consistent with the global load balancing objective in the VPN DESIGN problem.

Table III: Summary Statistics of the Optimized Design for the Special Problem

VPN	West	Mid West	East	South	Total
Offered Bandwidth	18484	17269	23167	15758	74679
Carried Bandwidth	17904	16739	22530	15230	72400
Blocking (%)	3.14	3.07	2.75	3.35	3.05

It helps to put this data in perspective by comparing it to the performance of the Complete Sharing design. As noted in Sec. 4.4 the latter design is oblivious of the VPN structure and is obtained by solving the OPTIMUM ROUTING problem for the aggregate traffic by exercising TALISMAN. The total Carried Bandwidth in the Complete Sharing design is 73,795, which is equivalent to 1.2% blocking. Hence about 1.85% incremental blocking may be attributed to the Complete Partitioning of resources among the VPNs.

In Tables IV and V we consider the Palo Alto–Argonne and the Atlanta–Houston links, respectively, and give the capacity allocations to the VPNs in the optimized design on these sample links. Note that in the Palo Alto–Argonne link the allocations to the West and South

Table IV: Palo Alto – Argonne Link (capacity = OC3)

VPN		West	Mid West	East	South
Relative Capacity Allocation to VPN (%)		32.9	25.0	24.9	17.1
Relative Utilization in VPN by service	1	1.0	0.3	2.1	1.8
"	2	2.7	0.1	6.2	5.7
"	3	7.3	0.6	16.7	14.7
"	4	4.6	0.3	12.4	10.8
"	5	21.7	23.2	14.0	38.7
"	6	62.7	75.5	48.6	28.3
Total Utilization in VPN (%)		82.6	79.2	83.4	80.5

Corporations are almost 2:1. Also given in the tables are the utilizations by each of the six services. The data in the tables also show that the utilization in the Palo Alto–Argonne link is approximately 20% higher than in the Atlanta–Houston link, which is certainly in part due to the capacity in the latter being twice as large. Quite remarkable is the lack of significant variability in the utilization of both links by the VPNs, possibly due to effective global load balancing.

Table V: Atlanta – Houston Link (capacity $= 2 \times$ OC3)

VPN		West	Mid West	East	South
Relative Capacity Allocation to VPN (%)		27.2	19.0	32.0	21.8
Relative Utilization in VPN by service	1	0.4	2.1	0.2	0.7
"	2	1.1	6.4	0.7	2.6
"	3	3.1	15.8	1.7	5.6
"	4	2.3	12.5	1.6	5.5
"	5	17.0	40.4	10.6	24.4
"	6	76.1	22.8	85.2	61.2
Total Utilization in VPN (%)		58.2	59.5	59.1	54.9

Note the hierarchy of three levels in the allocations, namely, the effective bandwidth of individual flows or calls at the lowest level, the bandwidth allocated to a service type in a VPN, and, at the highest level, the bandwidth allocated to the VPN. This hierarchical information may be used to design Hierarchical Virtual Partitioning, an algorithm for sharing resources across VPNs, and link sharing algorithms, such as class-based queueing.

We proceed to give information on the convergence process in the design calculations. Only 10 outer iterations were necessary to meet the convergence threshold of 10^{-3} and Table VI gives the total carried bandwidth after even numbers of iterations. Particularly noteworthy is the goodness of the initial design which, as explained in Sec. 4.4, is a VPN design based on the Complete Sharing solution. It is worth noting that considerable reallocations

Table VI: Convergence in the Design Process

Iterations	0 (initial)	2	4	6	8	10 (converged)
Carried Bandwidth	72042	72207	72308	72359	72384	72400

take place between the initial and final iterations. For instance, on the Houston–Argonne link the initial and final bandwidth allocations to the South Corp. were 1506 and 1842, respectively, a 22% change.

5.3 Results for the General Problem

Here we add the requirement that the blocking percentage of bandwidth is no more than 3.0, 3.5, 3.0 and 3.5 for the West, Mid West, East and South VPNs, respectively. This is equivalent to specifying $\{W_{\min}^{(\Omega)}\}$. The comparable percentages obtained by the solution of the special problem are in Table III. Table VII gives a summary of the results for the optimized design. Note that the problem is close to but not precisely feasible, with tight constraints for the West and South VPNs. Note that in this example, the redistribution of QoS over VPNs has been achieved at negligible incremental overall cost.

Table VII: Summary Statistics of the Optimized Design for the General Problem

VPN	West	Mid West	East	South	Total
Offered Bandwidth	18484	17269	23167	15758	74679
Carried Bandwidth	17920	16746	22522	15207	72395
Blocking (%)	3.05	3.03	2.78	3.50	3.06

The initial capacity allocations $\{C_\ell^{(\Omega)}\}$ for the iterative solution procedure in this example were obtained from the optimized solution for the special problem. The results reported in Table VII were obtained after just one iteration.

5.4 Results for the Optimum Source Tree Design

We briefly summarize results of another VPN DESIGN. In this case the infrastructure network, the service offerings, the earnings parameters and route sets are identical to the case considered in Sec. 5.2. The VPNs are quite different, however. Here there are 8 VPNs, one for each source node, i.e., $N = \bar{\Omega}$, and each VPN may be viewed as a source tree rooted at the source and designed to carry all traffic from the source to all destinations. This design problem arises in source routing. In MPLS-based IP networks there is a parallel notion of the sink trees, which arise in minimizing explicit routes [7], for which there is an associated joint resource allocation and routing design problem.

The traffic specification is obtained as follows: construct a base traffic intensity matrix $\mathbf{T} = \Sigma \mathbf{T}^{(\Omega)}$, where $\{\mathbf{T}^{(\Omega)}\}$ are as given in Table I; the traffic intensity matrix for each source tree-VPN is simply the restriction of $\mathbf{T}$ to the corresponding row. The service class dependent traffic is obtained as before through multipliers $\{m_s\}$.

VPN DESIGNER is used to solve the special version of this problem. Table VIII gives a summary of the results. Note that the total offered bandwidth over all VPNs is 74545, and the difference from the corresponding quantity in Table III is on account of rounding the entries in the traffic matrices. As in Sec. 5.2, the Complete Sharing, route-optimized solution carries bandwidth of 73,795, which corresponds to 1.2% blocking. Hence the incremental blocking attributable to Complete Partitioning of infrastructure resources in the optimized source tree based VPN is only 1.12%.

Table VIII: Summary Statistics of the Optimized Source Tree Design

VPN	San Diego	Palo Alto	Argonne	Princeton	Cambridge	College Park	Atlanta	Houston	Total
Offered Bandwidth	4668	13438	15560	2122	19237	9619	3960	5941	74545
Carried Bandwidth	4562	13271	15351	2045	18459	9453	3855	5820	72817
Blocking (%)	2.30	1.24	1.34	3.63	4.04	1.73	2.70	2.04	2.32

6 CONCLUDING REMARKS

We have given a framework for the design of virtual private networks in which resource allocation and routing are done jointly. We have reported numerical results on designs obtained by software which implements the algorithms described in the paper.

We are working towards developing what may be called "design-assisted real time control". This approach is SLA-centric, with real-time measurements working in tandem with the design obtained by the techniques of this paper. An important ingredient is Hierarchical Virtual Partitioning [18].

Another important direction for future work is scalability in the number of nodes in the network. While the techniques here are effective for networks with as many as about 100 nodes, other techniques are needed for networks with many more nodes. An important technique to handle this problem is aggregation [23].

Appendix A

We here give the definitions of various quantities that appear in subsection 3.3. Recall that z^* is the unique positive minimizer of the function $F(z)$ defined by (3.9). We first define the quantities $\theta^{(i)}(n)$, $i = 1, 2, 3$, for integer values of n, as follows.

$$\theta^{(1)}(n) = \begin{cases} \displaystyle\sum_{j=0}^{n-1} (z^*)^j, & n > 0, \\ \displaystyle -\sum_{j=1}^{-n} (z^*)^{-j}, & n < 0, \end{cases} \tag{A.1}$$

$$\theta^{(2)}(n) = \begin{cases} \displaystyle\sum_{j=0}^{n-1} (j-1)(z^*)^j, & n > 0, \\ \displaystyle \sum_{j=1}^{-n} (j+1)(z^*)^{-j}, & n < 0, \end{cases} \tag{A.2}$$

and

$$\theta^{(3)}(n) = \begin{cases} \displaystyle\sum_{j=0}^{n-1} (j-1)(j-2)(z^*)^j, & n > 0, \\ \displaystyle -\sum_{j=1}^{-n} (j+1)(j+2)(z^*)^{-j}, & n < 0. \end{cases} \tag{A.3}$$

Also,

$$\theta^{(i)}(0) = 0, \quad i = 1, 2, 3\,. \tag{A.4}$$

Next, we have [11]

Proposition A.1 *Let*

$$V = \sum_{s=1}^{S} \nu_s d_s^2 (z^*)^{d_s}\,, \tag{A.5}$$

and

$$M = \frac{1}{2}\ Erfc\left[sgn(1-z^*)\sqrt{-F(z^*)}\right] + \frac{K e^{F(z^*)}}{\sqrt{2\pi V}}\,, \tag{A.6}$$

where

$$K = \frac{1}{(1-z^*)} - \frac{\sqrt{V}sgn(1-z^*)}{\sqrt{-2F(z^*)}},\ z^* \neq 1\,, \tag{A.7}$$

$$K = \frac{1}{2} + \frac{1}{6V}\sum_{s=1}^{S} \nu_s d_s^3,\ z^* = 1\,, \tag{A.8}$$

and the complementary error function is given by

$$Erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-\xi^2} \, d\xi \, . \tag{A.9}$$

Also, let

$$\mathbb{B} = \frac{e^{F(z^*)}}{M\sqrt{2\pi V}} \, . \tag{A.10}$$

Then the UAA to the loss probability B_s for service s is

$$B_s \sim \mathbb{B} \; \theta^{(i)}(d_s) \, . \tag{A.11}$$

□

Further definitions are needed for the RUAA. We define

$$T = \sum_{s=1}^{S} \nu_s d_s^2 (d_s - 3)(z^*)^{d_s}, \quad Y = \sum_{s=1}^{S} \nu_s d_s^2 (d_s^2 - 6d_s + 11)(z^*)^{d_s} \, , \tag{A.12}$$

$$W = \sum_{s=1}^{S} \nu_s d_s^2 (d_s^3 - 10d_s^2 + 35d_s - 50)(z^*)^{d_s} \, , \tag{A.13}$$

and

$$\begin{aligned} E &= \frac{[1 - 3z^* + 3(z^*)^2]}{(1 - z^*)^3} + \frac{T(1 - 2z^*)}{2V(1 - z^*)^2} \\ &+ \frac{1}{8}\left(\frac{5T^2}{3V^2} - \frac{Y}{V}\right)\frac{1}{(1 - z^*)} - \frac{V\sqrt{V}\mathrm{sgn}(1 - z^*)}{[-2F(z^*)]^{3/2}}, \; z^* \neq 1 \, , \end{aligned} \tag{A.14}$$

$$E = 1 + \frac{T}{2V} + \frac{5T^2}{24V^2} + \frac{35T^3}{432V^3} - \frac{Y}{8V}\left(1 + \frac{5T}{6V}\right) + \frac{W}{40V}, \; z^* = 1 \, . \tag{A.15}$$

Also, we define

$$\mathbb{B}^{(1)} = \mathbb{B}\left\{1 + \frac{1}{V}\left[\frac{1}{8}\left(\frac{Y}{V} - \frac{5T^2}{3V^2}\right) + \mathbb{B}E\right]\right\} \, , \tag{A.16}$$

and

$$\mathbb{B}^{(2)} = \frac{\mathbb{B}T}{2V^2}, \quad \mathbb{B}^{(3)} = -\frac{\mathbb{B}}{2V} \, . \tag{A.17}$$

Then [17] the RUAA to B_s is given by (3.6).

Appendix B

The loss probability of service s is given by [19]

$$B_s = \mathbb{L}_s(C) = 1 - \frac{G(C-d_s)}{G(C)}\,, \tag{B.1}$$

where the normalization constant is

$$G(C) = \sum_{\sum_{s=1}^{S} d_s n_s \leq C} \prod_{s=1}^{S} e^{-\nu_s}\,\frac{\nu_s^{n_s}}{n_s!}\,, \tag{B.2}$$

and, for simplicity of notation, we have suppressed the dependence of $\mathbb{L}_s$ and G on $\mathbf{d}$ and $\boldsymbol{\nu}$. The UAA and RUAA to B_s were derived with the help of a contour integral representation [17] for $G(C)$. From (B.2) we obtain

$$\frac{\partial G}{\partial \nu_t} = G(C-d_t) - G(C)\,, \tag{B.3}$$

and it follows from (B.1) that

$$\frac{\partial \mathbb{L}_s}{\partial \nu_t} = \frac{G(C-d_s)G(C-d_t)}{[G(C)]^2} - \frac{G(C-d_s-d_t)}{G(C)}\,. \tag{B.4}$$

The RUAA to $\partial\mathbb{L}_s/\partial\nu_t$ was derived in [17].

Now, from (B.1), if n is an integer,

$$\frac{G(C-n)}{G(C)}\left[\mathbb{L}_s(C-n) - \mathbb{L}_s(C)\right] = \frac{G(C-d_s)G(C-n)}{[G(C)]^2} - \frac{G(C-d_s-n)}{G(C)}\,. \tag{B.5}$$

The procedure for calculating the RUAA to $1-G(C-n)/G(C)$ is analogous to that used in [17] to obtain the RUAA to B_s, but since n may be a negative integer, a modification of the formulas is required. Similarly, the procedure for calculating the RUAA to the right-hand side of (B.5) is analogous to that used in [17] for the expression on the right-hand side of (B.4). Consequently, we merely state the results here. First, we have

Proposition B.1 *For integer values of n,*

$$1 - \frac{G(C-n)}{G(C)} \sim \sum_{i=1}^{3} \mathbb{B}^{(i)}\theta^{(i)}(n)\,, \tag{B.6}$$

where $\theta^{(i)}(n)$, $i = 1,2,3$, are defined by (A.1)–(A.4), and $\mathbb{B}^{(i)}$, $i = 1,2,3$, by (A.16) and (A.17). □

We remark that (B.1) and (B.6) imply (3.6) if $n = d_s$, a positive integer.

We also have

Proposition B.2 *Let*

$$\begin{aligned} A_s^{(1)} &= [\mathbb{B}(\mathbb{B}-1+z^*) + (2\mathbb{B}-1+z^*)(\mathbb{B}^{(1)}-\mathbb{B})]\theta^{(1)}(d_s) \\ &\quad + \mathbb{B}^{(2)}[\mathbb{B}\theta^{(2)}(d_s) + d_s(z^*)^{d_s}] + \mathbb{B}^{(3)}[\mathbb{B}\theta^{(3)}(d_s) + d_s(d_s-1)(z^*)^{d_s}]\,, \end{aligned} \tag{B.7}$$

and

$$A_s^{(2)} = \mathbb{B}^{(2)}(\mathbb{B}-1+z^*)\theta^{(1)}(d_s) + 2\mathbb{B}^{(3)}d_s(z^*)^{d_s},\ A_s^{(3)} = \mathbb{B}^{(3)}(\mathbb{B}-1+z^*)\theta^{(1)}(d_s)\,, \tag{B.8}$$

where $\mathbb{B}$ *is defined by (A.10). Then,*

$$\frac{G(C-n)}{G(C)}\,[\mathbb{L}_s(C-n) - \mathbb{L}_s(C)] \sim \sum_{i=1}^{3} A_s^{(i)}\theta^{(i)}(n)\,. \tag{B.9}$$

□

We obtain (3.7) from (B.6) and (B.9). We remark that if $n = d_t$, then (B.4), (B.5) and (B.9) lead to the RUAA to $\partial\mathbb{L}_s/\partial\nu_t$ obtained in [17].

Appendix C

We here derive expressions for

$$\sigma^{(i)} = \lim_{n\to 0} \frac{\theta^{(i)}(n)}{n}, \tag{C.1}$$

where $\theta^{(i)}(n)$, $i = 1,2,3$, are given by (A.1)–(A.3) for integer values of n. From (A.1), we obtain

$$\theta^{(i)}(n) = \begin{cases} [(z^*)^n - 1]/(z^* - 1), & z^* \neq 1, \\ n, \quad z^* = 1. \end{cases} \tag{C.2}$$

Hence $\theta^{(1)}(n)$ is an analytic function of n, and it follows from (C.1) and (C.2) that $\sigma^{(1)}$ is given by (3.10) for $z^* \neq 1$, and $\sigma^{(1)} = 1$ for $z^* = 1$. Also, from (A.1)–(A.3), we find that

$$\theta^{(2)}(n) = z^* \frac{d\theta^{(1)}(n)}{dz^*} - \theta^{(1)}(n), \quad \theta^{(3)}(n) = (z^*)^2 \frac{d^2\, \theta^{(1)}(n)}{d(z^*)^2} - 2\theta^{(2)}(n)\,. \tag{C.3}$$

It follows from (C.1) and (C.3) that

$$\sigma^{(2)} = z^* \frac{d\sigma^{(1)}}{dz^*} - \sigma^{(1)}, \quad \sigma^{(3)} = (z^*)^2 \frac{d^2\sigma^{(1)}}{d(z^*)^2} - 2\sigma^{(2)}, \tag{C.4}$$

which leads to the expressions for $\sigma^{(2)}$ and $\sigma^{(3)}$ in (3.10) and (3.11).

It was shown in [17] that

$$\begin{aligned} \Gamma^{\{m\}}(z) &\triangleq z^m \frac{d^m}{dz^m}\left[\frac{\log z}{(1-z)}\right] \\ &= (-1)^{m+1}(m!)^2 \sum_{i=0}^{\infty} \frac{i!(1-z)^i}{(i+m+1)!}, \quad |1-z| < 1\,, \end{aligned} \tag{C.5}$$

for $m = 0,1,2,\cdots$. The expression for $\sigma^{(1)}$ in (3.10), together with (C.4) and (C.5), provide numerically accurate evaluations of $\sigma^{(i)}$, $i = 1,2,3$, when $|1-z|$ is small.

We remark that, since $F(1) = 0$, $F'(z^*) = 0$ and $(z^*)^2F''(z^*) = V$, the expression for K in (A.7) remains finite as $z^* \to 1$, and its limiting value is given by (A.8). However, the numerical evaluation of (A.7) loses accuracy when z^* is close to 1. Consequently, a representation for K, involving $\Gamma^{\{1\}}(z^*)$ and $\Gamma^{\{2\}}(z^*)$, which gives numerically accurate results whether or not z^* is close to 1, was derived in [17]. Moreover, the numerical evaluation of the expression in (A.14) loses accuracy significantly when z^* is close to 1, and an analogous representation for E, which also includes $\Gamma^{\{3\}}(z^*)$ and $\Gamma^{\{4\}}(z^*)$, was derived in [17].

Appendix D

We here establish (3.15), by extending the approach of Kelly [13] to the multirate case. To simplify the notation, we define $\rho_{sr} = 0$ if $r \notin \mathcal{R}(s,\sigma)\forall\sigma$. Then

$$W = \sum_s \sum_r e_{sr}\rho_{sr}(1 - L_{sr})\,. \tag{D.1}$$

For this appendix only, we define

$$\nu_{sk} = \sum_{r:k\in r} \rho_{sr} \prod_{m\in r-\{k\}} (1 - B_{sm}) + \kappa_{sk},\ \ (s = 1,\ldots,S;\ k = 1,\ldots,L)\,, \tag{D.2}$$

where eventually κ_{sk} will be set equal to zero, so that (D.2) will be consistent with (3.5) and

$$\nu_{sk} = \sum_{r:k\in r} \nu_{sk;r}\,. \tag{D.3}$$

We consider an infinitesimal increment dC_ℓ of C_ℓ, for fixed ℓ, and the corresponding increments

$$d\kappa_{sk} = \delta_{k\ell}\ \chi_{s\ell}\ dC_\ell,\ \ (s = 1,\ldots,S;\ k = 1,\ldots,L)\,, \tag{D.4}$$

where $\delta_{k\ell} = 0$ if $k \neq \ell$, and $\delta_{\ell\ell} = 1$. We will choose $\chi_{k\ell}$ so that this is consistent with

$$dB_{sk} = 0,\ \ (s = 1,\ldots,S;\ k = 1,\ldots,L)\,. \tag{D.5}$$

Then, from (D.2), (D.4) and (D.5), we have

$$d\nu_{sk} = \delta_{k\ell}\ \chi_{s\ell}\ dC_\ell,\ \ (s = 1,\ldots,S;\ k = 1,\ldots,L)\,. \tag{D.6}$$

Since $B_{sk} = \mathbb{L}_s(\mathbf{d}_k, \boldsymbol{\nu}_k, C_k)$, and $d\boldsymbol{\nu}_k = \mathbf{0}$ for $k \neq \ell$, this is consistent with $dB_{sk} = 0$, $(s = 1,\ldots,S;\ k \neq \ell)$. Also, if we consider a smooth continuation of $\mathbb{L}_s(\mathbf{d}_\ell, \boldsymbol{\nu}_\ell, C_\ell)$ for non-integer values of C_ℓ,

$$\mathbb{L}_s(\mathbf{d}_\ell, \boldsymbol{\nu}_\ell + d\boldsymbol{\nu}_\ell, C_\ell + dC_\ell) - \mathbb{L}_s(\mathbf{d}_\ell, \boldsymbol{\nu}_\ell, C_\ell) = \sum_t \frac{\partial B_{s\ell}}{\partial \nu_{t\ell}}\ \chi_{t\ell}\ dC_\ell + \frac{\partial \mathbb{L}_s}{\partial C_\ell}\ dC_\ell = 0\,, \tag{D.7}$$

since $dB_{s\ell} = 0$, $(s = 1,\ldots,S)$. It follows that

$$\sum_t \frac{\partial B_{s\ell}}{\partial \nu_{t\ell}}\ \chi_{t\ell} = -\frac{\partial \mathbb{L}_s}{\partial C_\ell},\ \ (s = 1,\ldots,S)\,, \tag{D.8}$$

which is a system of S linear equations for $\chi_{t\ell}$, $(t = 1,\ldots,S)$.

From (3.2), (D.1), (D.4) and (D.5),

$$0 = dW = \sum_t \frac{\partial W}{\partial \kappa_{t\ell}} \chi_{t\ell}\, dC_\ell + \frac{\partial W}{\partial C_\ell}\, dC_\ell\,. \tag{D.9}$$

Hence,

$$\frac{\partial W}{\partial C_\ell} = -\sum_t \frac{\partial W}{\partial \kappa_{t\ell}} \chi_{t\ell}\,. \tag{D.10}$$

But [11],

$$c_{t\ell} = -\frac{1}{(1-B_{t\ell})} \frac{\partial W}{\partial \kappa_{t\ell}}\,, \tag{D.11}$$

and

$$\frac{\partial B_{s\ell}}{\partial \nu_{t\ell}} = (1-B_{t\ell})[\mathbb{L}_s(\mathbf{d}_\ell, \boldsymbol{\nu}_\ell, C_\ell - d_{t\ell}) - \mathbb{L}_s(\mathbf{d}_\ell, \boldsymbol{\nu}_\ell, C_\ell)]\,. \tag{D.12}$$

We obtain (3.15) from (3.4), (D.8) and (D.10)–(D.12).

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