A Novel Distributed Power Control Algorithm for Classes of Service in Cellular CDMA Networks

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Abstract

The paper proposes a distributed power control algorithm for integrating heterogeneous transmitting sources, which have a broad range of statistical/burstiness characteristics and quality of service requirements, in wideband cellular CDMA networks. Speech sources with silence detection and data are canonical examples of service classes. Each service class is characterized by on-off transmissions with characteristic probabilities, desired minimum carrier-to-interference ratio and minimum probability with which the latter is required to be satisfied. In the power control algorithm given here, the received power for each service class at each cell is adapted locally based on only local measurements of the mean and also, importantly, the variance of the interference. The algorithm is derived from an asymptotic analysis in which the bandwidth as well as the number of mobiles are large. The analysis leads to a Gaussian approximation to the interference at each cell, which depends on the power levels. The algorithm is remarkable for its simplicity in the decoupling between classes. This is due in part to an attractive product-form in the expression for the dominant term in the asymptotic expansion of the desired power for each service class and cell. A condition for geometric convergence of the adapted power to the ideal power is obtained. The condition is remarkably unburdensome and only slightly more demanding than the condition based on the mean values of source activities. The condition also defines the capacity of the cellular CDMA network.

1 INTRODUCTION

The paper addresses the challenge of integrating heterogeneous transmitting sources with a broad range of burstiness characteristics in wideband cellular CDMA networks. Provided the right power control can be devised, CDMA offers the potential of extracting significant gain from the statistical multiplexing of such sources. In this paper we propose a new distributed power control algorithm for a range of service classes in cellular CDMA wireless networks. Each service class is characterized by its mobiles' distinctive bit rates, statistical and burstiness properties, and quality of service parameters consisting of the desired minimum carrier-to-interference ratio (CIR) and the minimum probability with which this requirement on the CIR must be satisfied. A canonical service class is voice sources with silence detection, which are modelled as on-off sources. Data sources, if also present, typically have quite different burstiness characteristics and, importantly, more stringent quality of service requirements on the CIR. In the power control algorithm that we give here, the power is adapted locally based on only local measurements of the mean and variance of the interference.

The paper is in two parts. In the first part the probabilistic quality of service specification is described. Also given is an asymptotic framework for estimating the orders of magnitude of quantities which affect the quality of service. The asymptotic scaling reflects wideband characteristics, i.e., the bandwidth as well as the number of mobiles are large. The key result of the first part are expressions for the asymptotically ideal, i.e., minimal, powers at each cell site for each service class (see (3.36)). The second part of the paper gives our distributed, measurement-based algorithm for solving the aforementioned expressions for the ideal powers. The analysis of the dynamical behavior of the algorithm yields a condition for the geometric convergence of the powers to their ideal values. This condition is also important because it determines the capacity of the cellular CDMA network. The condition obtained is remarkably unburdensome. The condition is only slightly more demanding than the condition based on the mean values of source activities, i.e., where the variabilities of the sources are eliminated.

This work extends some recent results in [MMO96], where the following notions were introduced: stochastic on-off traffic sources, probabilistic quality of service specifications, an asymptotic scaling for wideband networks, a Gaussian approximation for the cell-dependent interference and, importantly, a distributed power control algorithm driven not only by the mean of the local interference, which is conventional, but also the variance. New to this paper is the concept of classes of service. The power control algorithm given here (see (4.3)) is remarkable for the simple, decoupled dependence on the classes of service. This, in turn, is due in large part to a result obtained in the analysis, which states that the dominant term in the asymptotic expansion for the ideal powers at each cell site for the various service classes has a simple product-form, in which a term common to all service classes is multiplied by the desired minimum CIR for the service class. Moreover, the common term is the ideal power at the cell of a simple cellular network construct having a single class of service.

An important limitation of the analysis of the paper is the assumption of time-invariant path gains. The conventional argument to support this assumption is based on a separation of time scales of the power control convergence and variations in the path gains. We believe that the algorithm's sensitivity to randomness renders it more effective in the presence of path gain variations and fluctuations. That is, the measurement-based terms in the power adaptation algorithm which reflect the mean and variance of the interference will take into account not only the effects of randomness in transmissions, as postulated in the model, but also time variations in the path gains. It should, however, be noted that this is not established here.

For the benefit of readers this paper has been written to be self-contained, which has necessitated some overlap with [MMO96]. Otherwise, given the complicated development, it would be excessively distracting and tedious.

Prior work on power control has been done in distributed [ZAN92(a), ZAN92(b), BPO92, FMI93, HAN93], distributed-asynchronous [MIT93, GZY95] and centralized [GVG93] frameworks. Hanly [HAN93] considers a cellular model with interference fluctuations. Early work on power control in spread spectrum networks is reported in [NAL83]. Recent work which unifies power control and cell site selection is reported by Hanly [HAN95] and Yates [YAT95]. Works which integrate power control and admission control are [BCP95] and [BCM95], the former focussing on active link protection and the latter on channel probing. On-off sources are considered in [VVI93] and [MMO94]. Recently, Everitt and Evans [EEV95] introduced the concept of effective interference to quantify the effects of variability in interference on the network capacity in the context of multiple service classes and cellular CDMA networks. However, the investigation is essentially independent of power control. Andersin and Rosberg [ARO96] develop a framework for power control in cellular networks, with some features in common with [MMO96], where the focus is mobility and time varying path gains. While this paper deals with CIR, and only indirectly with bit error rates (BER), it is possible to connect power control directly to BER, as in [KYH95]. The source for CDMA, specifically for matters related to interference and capacity, is Viterbi [VIT95].

2 MODEL

Let X_m be the activity indicator for mobile m, i.e., $X_m \in \{0,1\}$ and $X_m = 1$ if and only if the mobile is active at a point in time. An active mobile m transmits at power p_m, and the gain to cell site j is g_{jm}, so that the received power at the cell site is $g_{jm}p_m$. We assume that the received power at any cell site k from all the active mobiles of class i in that cell is common and given by P_{ki}, i.e.,

$$P_{ki} \triangleq g_{km} p_m, \quad \forall m \in \{(k,i) \mid X_m = 1\} \ . \tag{2.1}$$

Since our system is spread spectrum, the interference to an active mobile's transmission is closely related to the total received power at its local site. In order to calculate such interference, we first let $I_{jk}^{(i)}$ denote the received power at cell site j due to all the mobiles in cell k of class i, i.e.,

$$I_{jk}^{(i)} \triangleq \sum_{m \in (k,i)} g_{jm} p_m X_m = P_{ki} \sum_{m \in (k,i)} \frac{g_{jm}}{g_{km}} X_m \ . \tag{2.2}$$

We isolate the component independent of power, which depends on relative gains and the random activity indicators, and denote it by $M_{jk}^{(i)}$, i.e.,

$$M_{jk}^{(i)} \triangleq \sum_{m \in (k,i)} \frac{g_{jm}}{g_{km}} X_m \quad (1 \le j \le J, \quad 1 \le k \le J, \quad 1 \le i \le I) \ . \tag{2.3}$$

Hence, $I_{jk}^{(i)} = P_{ki} M_{jk}^{(i)}$, and we note that

$$M_{jj}^{(i)} = \sum_{m \in (j,i)} X_m \ . \tag{2.4}$$

The total power received at cell j is

$$I_j \triangleq \sum_{k=1}^{J} \sum_{\ell=1}^{I} P_{k\ell} M_{jk}^{(\ell)} + \eta_j W \ , \tag{2.5}$$

where $\eta_j W (1 \le j \le J)$ is the local receiver noise power at cell site j, and W is the spread bandwidth (and also the processing gain). Since we are considering a wideband system, W is large. For any active mobile m in cell j of class i, $X_m = 1$ and the interference to its transmission is

$$I_{ji} \triangleq I_j - P_{ji} \ . \tag{2.6}$$

The quality of service requirement in terms of the carrier-to-interference ratio (C/I) is

$$P_{ji}/I_{ji} \ge \alpha_i / W \quad (1 \le j \le J, \quad 1 \le i \le I) \ , \tag{2.7}$$

where $\{\alpha_i\}$ are prespecified numbers; the right-hand quantity is typically small, since the processing gain $W$is large. From (2.6), we may write (2.7) as

$$P_{ji} \ge \frac{\tilde{\alpha}_i}{W} I_j \qquad\qquad (1 \le j \le J, \quad 1 \le i \le I) \ , \tag{2.8}$$

where

$$\tilde{\alpha}_i = \alpha_i / \left(1 + \frac{\alpha_i}{W}\right) . \tag{2.9}$$

Typically $\tilde{\alpha}_i \approx \alpha_i$.

Now consider the stochastic traffic model of a bursty mobile m of class i to be on-off, i.e., in cell j it is on, i.e. active, with probability w_{ji}:

$$w_{ji} = \Pr\{X_m = 1\} = 1 - \Pr\{X_m = 0\}, \quad \forall m \in (j, i) . \tag{2.10}$$

Although w_{ji} may not depend on the cell index j, we are retaining the generality for possible future applications. On account of the random source behavior, the quality of service requirement has to be probabilistic, with probability of compliance at best less than unity. Let the requirement be

$$\Pr\left\{P_{ji} \geq \frac{\tilde{\alpha}_i}{W} I_j\right\} \geq 1 - L_i \qquad (1 \leq j \leq J,\ 1 \leq i \leq I) , \tag{2.11}$$

where $\{L_i\}$ are given parameters. Thus the quality of service parameters for class i are α_i and L_i.

From (2.10) we have

$$E(X_m) = w_{ki}, \quad \mathrm{var}(X_m) = w_{ki}(1 - w_{ki}), \quad \forall m \in (k, i) . \tag{2.12}$$

Hence, from (2.3),

$$E\left(M_{jk}^{(i)}\right) = \sum_{m \in (k,i)} \frac{g_{jm}}{g_{km}} w_{ki} = K_{ki} G_{jk}^{(i)} w_{ki} , \tag{2.13}$$

where K_{ki} is the number of mobiles in cell k of class i, and

$$G_{jk}^{(i)} \triangleq \frac{1}{K_{ki}} \sum_{m \in (k,i)} \frac{g_{jm}}{g_{km}} . \tag{2.14}$$

The quantity $G_{jk}^{(i)}$ represents the averaged normalized gain that is used in the determination of the mean interference at cell j due to mobiles of class i in cell k. The averaging is over the latter set of mobiles and the normalization is with respect to their local gains. Note, in particular, that $G_{jj}^{(i)} = 1$. From (2.3) and (2.12) we obtain

$$\mathrm{var}\left(M_{jk}^{(i)}\right) = \sum_{m \in (k,i)} \left(\frac{g_{jm}}{g_{km}}\right)^2 w_{ki}(1 - w_{ki}) = K_{ki} \left(H_{jk}^{(i)}\right)^2 w_{ki}(1 - w_{ki}) , \tag{2.15}$$

where

$$\left(H_{jk}^{(i)}\right)^2 = \frac{1}{K_{ki}} \sum_{m \in (k,i)} \left(\frac{g_{jm}}{g_{km}}\right)^2 . \tag{2.16}$$

The quantities $\left(H_{jk}^{(i)}\right)^2$ occur in the determination of the variance of the interference at cell j due to mobiles of class i at cell k. Their role is similar to that of $G_{jk}^{(i)}$ for the mean interference, compare (2.13) and (2.15).

We assume that g_{jm}/g_{km} are uniformly bounded for $m \in (k,i)$. Since $X_m \in \{0,1\}$, it follows that the random variables $g_{jm}X_m/g_{km}$ are uniformly bounded for $m \in (k,i)$. Hence, from the Lindeberg theorem [FEL68], the central limit theorem holds for $M_{jk}^{(i)}$ as $K_{ki} \to \infty$. Hence

$$M_{jk}^{(i)} = K_{ki}G_{jk}^{(i)}w_{ki} + H_{jk}^{(i)}\sqrt{K_{ki}w_{ki}(1-w_{ki})}\ Z_{jk}^{(i)}\ , \tag{2.17}$$

where $Z_{jk}^{(i)}$ is asymptotically normally distributed, with zero mean and unit variance as $K_{ki} \to \infty$. Moreover, $Z_{jk}^{(i)}$ $(1 \le k \le J,\ 1 \le i \le I)$ are independent random variables.

In summary, we have shown that for our model of many mobiles and class-dependent, random, on-off transmission of mobiles, the component which is independent of power in the expression for the received power at cell site j due to all mobiles of class i in cell k is a Gaussian random variable given in (2.17).

3 ASYMPTOTICS, ORDERS OF MAGNITUDE

We introduce a natural scaling, which allows us to make order of magnitude estimates and to develop meaningful and efficient approximations by dropping negligibly small terms.

3.1 Asymptotic Scaling

Inherent to wideband systems in which the bandwidth (W) is shared by a large number of users is the following scaling, in which the large parameter K is the average number of mobiles in a cell, i.e., $K \triangleq \frac{1}{J}\sum_{j=1}^{J}\sum_{i=1}^{I} K_{ji}$:

$$\frac{\alpha_i}{W} = \frac{a_i}{K}, \quad K_{ji} = \gamma_{ji}K \quad (1 \le j \le J, \quad 1 \le i \le I)\ , \tag{3.1}$$

where $a_i = O(1)$ and $\gamma_{ji} = O(1)$ as $K \to \infty$ and $W \to \infty$. Note that α_i and η_j are other $O(1)$ parameters. With a small loss of generality, consider these $O(1)$ parameters to be fixed in this scaling. Also, note that, importantly, W and K are of the same order. Let

$$P_{ji} = \widehat{P}_{ji} + \frac{1}{\sqrt{K}}Q_{ji} \quad (1 \le j \le J, \quad 1 \le i \le I)\ , \tag{3.2}$$

where $\widehat{P}_{ji}$ and Q_{ji} are $O(1)$. The orders of magnitudes of the first order (dominant) and second order terms in the above expansion of P_{ji} are dictated by consistency, as may be checked later.

3.2 Solutions to Leading Order Terms in the Asymptotic Expansion

Now let us investigate the implications of (3.1) and (3.2) on the terms P_{ji} and $\tilde{\alpha}_i I_j$ appearing in the quality of service specifications (2.11). By substituting (2.17), (3.1) and (3.2) in the expression for I_j in (2.5), and using (2.9), it may be verified that

$$\begin{aligned}\frac{\tilde{\alpha}_i}{W} I_j &= a_i \sum_k \sum_\ell G_{jk}^{(\ell)} \gamma_{k\ell} w_{k\ell} \widehat{P}_{k\ell} + \alpha_i \eta_j \\ &+ \frac{a_i}{\sqrt{K}} \sum_k \sum_\ell \left[G_{jk}^{(\ell)} \gamma_{k\ell} w_{k\ell} Q_{k\ell} + H_{jk}^{(\ell)} \sqrt{\gamma_{k\ell} w_{k\ell}(1-w_{k\ell})}\; Z_{jk}^{(\ell)} \widehat{P}_{k\ell} \right] + O\left(\frac{1}{K}\right) . \quad (3.3)\end{aligned}$$

To achieve $P_{ji} \geq \tilde{\alpha}_i I_j / W$ $(1 \leq j \leq J, \quad 1 \leq i \leq I)$, we compare first, the order 1 terms and, second, the coefficients of $1/\sqrt{K}$, and obtain the system of inequalities given below in (3.4) and (3.5), respectively. These constitute sufficient conditions to give $P_{ji} \geq \tilde{\alpha}_i I_j / W$, to within $O(1/K)$.

$$\widehat{P}_{ji} - \frac{\alpha_i}{W} \sum_k \sum_\ell G_{jk}^{(\ell)} K_{k\ell} w_{k\ell} \widehat{P}_{k\ell} \geq \alpha_i \eta_j \quad (1 \leq j \leq J, \quad 1 \leq i \leq I) , \tag{3.4}$$

and

$$Q_{ji} - \frac{\alpha_i}{W} \sum_k \sum_\ell G_{jk}^{(\ell)} K_{k\ell} w_{k\ell} Q_{k\ell} \geq a_i \xi_j \quad (1 \leq j \leq J, \quad 1 \leq i \leq I) , \tag{3.5}$$

where

$$\xi_j \triangleq \sum_k \sum_\ell H_{jk}^{(\ell)} \sqrt{\gamma_{k\ell} w_{k\ell}(1-w_{k\ell})}\; \widehat{P}_{k\ell} Z_{jk}^{(\ell)} \quad (1 \leq j \leq J) . \tag{3.6}$$

Equations (3.4) and (3.5) are systems of relations, which differ qualitatively in that (3.4) is purely deterministic while (3.5) contains random variables in its right hand side. We first treat (3.4) before returning to (3.5).

In matrix form (3.4) is

$$\widehat{\boldsymbol{P}}_i - \frac{\alpha_i}{W} \sum_\ell \boldsymbol{F}^{(\ell)} \widehat{\boldsymbol{P}}_\ell \geq \alpha_i \boldsymbol{\eta} \quad (1 \leq i \leq I) , \tag{3.7}$$

where

$$F_{jk}^{(\ell)} \triangleq G_{jk}^{(\ell)} K_{k\ell} w_{k\ell} , \tag{3.8}$$

and $\widehat{\boldsymbol{P}}_i = \{\widehat{P}_{ji}\}_j$ and $\boldsymbol{F}^{(\ell)} = \{F_{jk}^{(\ell)}\}_{j,k}$. It is helpful to recall, see (2.13), that $F_{jk}^{(\ell)}$ is the mean value of the power-independent, random component $M_{jk}^{(\ell)}$ in the interference at cell j due to all mobiles of class ℓ in cell k. Hence, it is reasonable to call $\boldsymbol{F}^{(\ell)}$ the *mean class ℓ, intercellular gain matrix.*

Now define

$$\alpha \triangleq \sum_{i=1}^{I} \alpha_i, \quad \alpha \boldsymbol{F} \triangleq \sum_{i=1}^{I} \alpha_i \boldsymbol{F}^{(i)} \ . \tag{3.9}$$

Hence $\boldsymbol{F}$ is the weighted *mean intercellular gain matrix*, where the weights are proportional to the class-dependent minimum CIR requirements, see (2.7).

From (3.7), we obtain

$$\left(\boldsymbol{I} - \frac{\alpha}{W}\boldsymbol{F}\right) \sum_{\ell} \boldsymbol{F}^{(\ell)} \widehat{\boldsymbol{P}}_\ell \geq \alpha \boldsymbol{F} \boldsymbol{\eta} \ . \tag{3.10}$$

We assume that $\boldsymbol{F}$ is an irreducible matrix and since it is also nonnegative, it has an eigenvalue of maximum modulus, called the Perron-Frobenius eigenvalue, which is real, positive and simple. We denote this eigenvalue by r_F, and we make the important assumption that

$$\frac{\alpha}{W} r_F < 1 \ . \tag{3.11}$$

Similar capacity defining constraints occur in prior studies on deterministic narrowband systems and stochastic wideband systems. The condition (3.11) is especially noteworthy for the particular composition of the mean intercellular gain matrix $\boldsymbol{F}$, see (3.8) and (3.9). On the other hand, since it is a condition applied to a matrix of mean gains, it is the least burdensome condition we may hope for in the determination of the capacity of the stochastic system. Also, note that r_F and W are $O(K)$ and hence the left hand quantity in (3.11) is $O(1)$.

We recall, see for instance [MIT93], that (3.11) is equivalent to the existence of the element-wise nonnegative matrix $[\boldsymbol{I} - \frac{\alpha}{W}\boldsymbol{F}]^{-1}$. Hence, from (3.10),

$$\sum_{\ell} \boldsymbol{F}^{(\ell)} \widehat{\boldsymbol{P}}_\ell \geqq \left(\boldsymbol{I} - \frac{\alpha}{W}\boldsymbol{F}\right)^{-1} \alpha \boldsymbol{F} \boldsymbol{\eta} \ . \tag{3.12}$$

It follows from (3.7) and (3.12) that

$$\widehat{\boldsymbol{P}}_i \geqq \alpha_i \left(\boldsymbol{I} - \frac{\alpha}{W}\boldsymbol{F}\right)^{-1} \boldsymbol{\eta} \quad (1 \leq i \leq I) \ . \tag{3.13}$$

Equivalently,

$$\widehat{\boldsymbol{P}}_i \geqq \frac{\alpha_i}{\alpha} \boldsymbol{p}^* \quad (1 \leq i \leq I) \ , \tag{3.14}$$

where

$$\boldsymbol{p}^* \triangleq \alpha \left(\boldsymbol{I} - \frac{\alpha}{W}\boldsymbol{F}\right)^{-1} \boldsymbol{\eta} \ . \tag{3.15}$$

This is an important result, which is now summarized.

Proposition 1 The dominant, leading order term in the expansion of P_{ji} is denoted by $\widehat{P}_{ji}$, which is required to satisfy the system of inequalities (3.7). We assume that the nonnegative matrix $\boldsymbol{F}$ defined in (3.9) is irreducible and that its Perron-Frobenius eigenvalue r_F satisfies (3.11). Then,

(i)
$$\widehat{\boldsymbol{P}}_i^* = \frac{\alpha_i}{\alpha}\boldsymbol{p}^* \quad (1 \le i \le I) \tag{3.16}$$

is the unique solution of the system of linear equations obtained by replacing inequalities by equalities in (3.7). Note that $\widehat{\boldsymbol{P}}_i^* > \mathbf{0}$.

(ii) The solution (3.16) is the minimal, i.e., Pareto optimal, solution to the system of inequalities in (3.7) in the following sense: any other solution to (3.7) will have every element not less and at least one element greater than the solution in (3.16). ■

The product-form in (3.16), wherein the multiplicative factor dependent on the class index i is simply α_i, is of crucial subsequent importance. Note that $\boldsymbol{p}^*$, see (3.15), is of a familiar form, see for instance [FMI93, HAN93, MIT93], since it is the minimal power solution of a network construct with a single class having CIR requirement α and mean intercellular gain matrix $\boldsymbol{F}$.

We next turn to (3.5) and (3.6), and set $\widehat{\boldsymbol{P}}_\ell = \widehat{\boldsymbol{P}}_\ell^*$ $(1 \le \ell \le I)$. But $Z_{jk}^{(\ell)}$ $(1 \le k \le J, \quad 1 \le \ell \le I)$ are independent and asymptotically normally distributed, with zero mean and unit variance. Hence ξ_j is asymptotically normally distributed, with zero mean and variance σ_j^2 where, with $\sigma_j \ge 0$,

$$\sigma_j^2 \triangleq \sum_k \sum_\ell \left(H_{jk}^{(\ell)}\right)^2 \gamma_{k\ell} w_{k\ell}(1 - w_{k\ell}) \left(\widehat{P}_{k\ell}^*\right)^2 \quad (1 \le j \le J)\ . \tag{3.17}$$

The quality of service requirement in (2.11) is satisfied asymptotically if

$$\Pr\left\{Q_{ji} - \frac{\alpha_i}{W}\sum_k \sum_\ell F_{jk}^{(\ell)} Q_{k\ell} \ge a_i \xi_j\right\} \ge 1 - L_i \quad (1 \le j \le J, \quad 1 \le i \le I)\ . \tag{3.18}$$

This condition is equivalent to the deterministic condition

$$Q_{ji} - \frac{\alpha_i}{W}\sum_k \sum_\ell F_{jk}^{(\ell)} Q_{k\ell} \ge a_i \nu_i \sigma_j \quad (1 \le j \le J, \quad 1 \le i \le I)\ , \tag{3.19}$$

where ν_i $(\nu_i > 0)$ is obtained directly from L_i and the standard Gaussian distribution thus:

$$1 - L_i = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\nu_i} e^{-y^2/2} dy \quad (1 \le i \le I)\ . \tag{3.20}$$

That is, in (3.19) ν_i is the multiple of the standard deviation of the asymptotic random variable ξ_j, which indicates how large the left-hand quantity has to be for the probabilistic requirement in (3.18) to be satisfied.

In matrix form the system of equations (3.19) is

$$\boldsymbol{Q}_i - \frac{\alpha_i}{W} \sum_{\ell} \boldsymbol{F}^{(\ell)} \boldsymbol{Q}_\ell \geq a_i \nu_i \boldsymbol{\sigma} \quad (1 \leq i \leq I)\ . \tag{3.21}$$

Then, using (3.9), we obtain

$$\left(\boldsymbol{I} - \frac{\alpha}{W}\boldsymbol{F}\right) \sum_{\ell} \boldsymbol{F}^{(\ell)} \boldsymbol{Q}_\ell \geq \sum_{\ell} a_\ell \nu_\ell \boldsymbol{F}^{(\ell)} \boldsymbol{\sigma}\ . \tag{3.22}$$

Since $(\boldsymbol{I} - \frac{\alpha}{W}\boldsymbol{F})^{-1}$ exists and is element-wise positive, it follows that

$$\sum_{\ell} \boldsymbol{F}^{(\ell)} \boldsymbol{Q}_\ell \geq \left(\boldsymbol{I} - \frac{\alpha}{W}\boldsymbol{F}\right)^{-1} \sum_{\ell} a_\ell \nu_\ell \boldsymbol{F}^{(\ell)} \boldsymbol{\sigma}\ . \tag{3.23}$$

Hence, from (3.21), we have

$$\boldsymbol{Q}_i \geq a_i \nu_i \boldsymbol{\sigma} + \frac{\alpha_i}{W} \left(\boldsymbol{I} - \frac{\alpha}{W}\boldsymbol{F}\right)^{-1} \sum_{\ell} a_\ell \nu_\ell \boldsymbol{F}^{(\ell)} \boldsymbol{\sigma} \quad (1 \leq i \leq I)\ . \tag{3.24}$$

We summarize in

Proposition 2 The second order term in the expansion of P_{ji} is denoted by Q_{ji}, which is required to satisfy the system of inequalities (3.21) that incorporates the minimal solution given in Proposition 1 for the dominant terms.

(i)
$$\boldsymbol{Q}_i^* = a_i \nu_i \boldsymbol{\sigma} + \frac{\alpha_i}{W} \left(\boldsymbol{I} - \frac{\alpha}{W}\boldsymbol{F}\right)^{-1} \sum_{\ell} a_\ell \nu_\ell \boldsymbol{F}^{(\ell)} \boldsymbol{\sigma} \quad (1 \leq i \leq I) \tag{3.25}$$

is the unique solution of the system of linear equations obtained by replacing inequalities by equalities in (3.21). Note that $\boldsymbol{Q}_i^* \geqq \boldsymbol{0}$.

(ii) The solution (3.25) is the minimal, i.e., Pareto optimal, solution to the system of linear inequalities in (3.21), i.e., $\boldsymbol{Q}_i \geqq \boldsymbol{Q}_i^*$. ■

3.3 Lifting the Veil of Asymptotics

In the final part of this section we give relations which do not make reference to the scalings in (3.1) and (3.2). Corresponding to (3.2), the asymptotic approximation to the minimal solution for the power from the leading first and second order terms in the asymptotic expansion is,

$$\boldsymbol{P}_i^* \sim \widehat{\boldsymbol{P}}_i^* + \frac{1}{\sqrt{K}} \boldsymbol{Q}_i^* \quad (1 \leq i \leq I)\ , \tag{3.26}$$

where $\widehat{\boldsymbol{P}}_i^*$ is given by (3.15) and (3.16), and $\boldsymbol{Q}_i^*$ by (3.25). Also, from (3.25),

$$\frac{1}{\sqrt{K}}\boldsymbol{Q}_i^* = \alpha_i\left(\frac{1}{\alpha}\boldsymbol{q}^* + \frac{\sqrt{K}}{W}\nu_i\boldsymbol{\sigma}\right) \quad (1 \le i \le I)\ , \tag{3.27}$$

where $\boldsymbol{q}^*$ is the solution of the following equation,

$$\left(\boldsymbol{I} - \frac{\alpha}{W}\boldsymbol{F}\right)\boldsymbol{q}^* = \frac{\sqrt{K}\alpha}{W^2}\sum_\ell \alpha_\ell\nu_\ell\boldsymbol{F}^{(\ell)}\boldsymbol{\sigma}\ . \tag{3.28}$$

We let $\boldsymbol{\pi} \triangleq \boldsymbol{p}^* + \boldsymbol{q}^*$. Then, from (3.16) and (3.26)–(3.28),

$$\boldsymbol{P}_i^* \sim \alpha_i\left(\frac{1}{\alpha}\boldsymbol{\pi} + \frac{\sqrt{K}}{W}\nu_i\boldsymbol{\sigma}\right) \quad (1 \le i \le I)\ , \tag{3.29}$$

and $\boldsymbol{\pi}$ satisfies the following equation,

$$\left(\boldsymbol{I} - \frac{\alpha}{W}\boldsymbol{F}\right)\boldsymbol{\pi} = \alpha\boldsymbol{\eta} + \frac{\sqrt{K}\alpha}{W^2}\sum_\ell \alpha_\ell\nu_\ell\boldsymbol{F}^{(\ell)}\boldsymbol{\sigma}\ . \tag{3.30}$$

Now, from (3.9) and (3.29), we obtain

$$\alpha\boldsymbol{F}\boldsymbol{\pi} = \sum_\ell \alpha_\ell\boldsymbol{F}^{(\ell)}\boldsymbol{\pi} \sim \alpha\sum_\ell \boldsymbol{F}^{(\ell)}\left(\boldsymbol{P}_\ell^* - \frac{\sqrt{K}}{W}\alpha_\ell\nu_\ell\boldsymbol{\sigma}\right)\ . \tag{3.31}$$

Hence, from (3.30),

$$\boldsymbol{\pi} \sim \frac{\alpha}{W}\sum_\ell \boldsymbol{F}^{(\ell)}\boldsymbol{P}_\ell^* + \alpha\boldsymbol{\eta}\ . \tag{3.32}$$

Then, from (3.29), we have

$$\boldsymbol{P}_i^* \sim \alpha_i\left[\frac{1}{W}\sum_\ell \boldsymbol{F}^{(\ell)}\boldsymbol{P}_\ell^* + \boldsymbol{\eta} + \frac{\sqrt{K}}{W}\nu_i\boldsymbol{\sigma}\right] \quad (1 \le i \le I)\ . \tag{3.33}$$

Now, from (3.1) and (3.17), since $\boldsymbol{P}_\ell^* \sim \widehat{\boldsymbol{P}}_\ell^*$ to lowest order, it may be verified that

$$\sqrt{K}\sigma_j \sim \psi_j(\boldsymbol{P}^*) \quad (1 \le j \le J) \tag{3.34}$$

where $\boldsymbol{P}^* = (\boldsymbol{P}_1^*, \ldots, \boldsymbol{P}_I^*)^T$ and $\boldsymbol{P} = (\boldsymbol{P}_1, \ldots, \boldsymbol{P}_I)^T$ are IJ-dimensional column vectors, and generally,

$$\psi_j(\boldsymbol{P}) \triangleq \left[\sum_k\sum_\ell \left(H_{jk}^{(\ell)}\right)^2 K_{k\ell}w_{k\ell}(1-w_{k\ell})(P_{k\ell})^2\right]^{1/2} \quad (1 \le j \le J)\ . \tag{3.35}$$

Hence, from (3.33), we obtain expressions for the asymptotically ideal powers which depend only on unscaled quantities.

$$\boxed{\boldsymbol{P}_i^* \sim \frac{\alpha_i}{W}\left[\sum_\ell \boldsymbol{F}^{(\ell)}\boldsymbol{P}_\ell^* + \boldsymbol{\eta}W + \nu_i\boldsymbol{\psi}(\boldsymbol{P}^*)\right] \quad (1 \le i \le I)\ .} \tag{3.36}$$

Note that $\nu_i\boldsymbol{\psi}(\boldsymbol{P}^*)$ is the term which represents the combined effect of the probabilistic quality of service requirement, and the variance of the interference at the cell sites, which, importantly, depends on all the powers at all the cell sites. Also, note that if $w_{k\ell} = 1$ $(1 \le k \le J, \quad 1 \le \ell \le I)$, so that all the mobiles are always active, then $\boldsymbol{\psi}(\boldsymbol{P}^*) = \boldsymbol{0}$.

4 ALGORITHM AND ITS CONVERGENCE

We now propose a distributed algorithm for power control, based on (3.36). In view of (2.5), (2.13) and (2.15), we introduce the measured quantities at cell j during time period n:

$$\overline{I}_j(n) = \sum_k \sum_\ell G_{jk}^{(\ell)} K_{k\ell} w_{k\ell} P_{k\ell}(n) + \eta_j W \quad (1 \le j \le J) \ , \tag{4.1}$$

and

$$\overline{V}_j(n) = \sum_k \sum_\ell \left(H_{jk}^{(\ell)}\right)^2 K_{k\ell} w_{k\ell}(1 - w_{k\ell}) P_{k\ell}^2(n) \quad (1 \le j \le J) \ , \tag{4.2}$$

where $P_{k\ell}(n)$ is the received power at cell site k from active mobiles of class ℓ in the cell during time period n. That is, $\overline{I}_j(n)$ and $\overline{V}_j(n)$ are, respectively, the measured values of the mean and variance of the total interference at cell j in time period n. The distributed algorithm for power control that we propose is

$$\boxed{P_{ji}(n+1) = \frac{\alpha_i}{W}\left[\overline{I}_j(n) + \nu_i\sqrt{\overline{V}_j(n)}\right] \quad (1 \le j \le J, \quad 1 \le i \le I,) \quad n = 0, 1, \ldots,} \tag{4.3}$$

where $\{\nu_i\}$, defined in (3.20), are fixed, known quantities. The adaptation of power is thus driven by *local* measurement of the mean and variance of the total interference.

In vector form, in view of (3.8) and (3.35), the algorithm is

$$\boldsymbol{P}_i(n+1) = \frac{\alpha_i}{W}\left[\sum_\ell \boldsymbol{F}^{(\ell)} \boldsymbol{P}_\ell(n) + \boldsymbol{\eta} W + \nu_i \boldsymbol{\psi}(\boldsymbol{P}(n))\right] (1 \le i \le I), \quad n = 0, 1, \cdots . \tag{4.4}$$

Hence, from (3.36),

$$\boldsymbol{P}_i(n+1) - \boldsymbol{P}_i^* = \frac{\alpha_i}{W}\left\{\sum_\ell \boldsymbol{F}^{(\ell)}\left[\boldsymbol{P}_\ell(n) - \boldsymbol{P}_\ell^*\right] + \nu_i\left[\boldsymbol{\psi}(\boldsymbol{P}(n)) - \boldsymbol{\psi}(\boldsymbol{P}^*)\right]\right\} \ (1 \le i \le I) \ . \tag{4.5}$$

Let

$$\boldsymbol{\Phi} = (\alpha_1 \boldsymbol{I}, \ldots, \alpha_I \boldsymbol{I})^T \left(\boldsymbol{F}^{(1)}, \ldots, \boldsymbol{F}^{(I)}\right) \ , \tag{4.6}$$

and

$$\boldsymbol{\tau} = (\alpha_1 \nu_1 \boldsymbol{I}, \ldots, \alpha_I \nu_I \boldsymbol{I})^T \ , \tag{4.7}$$

where $\boldsymbol{I}$ denotes the $J \times J$ identity matrix. Then,

$$\boldsymbol{P}(n+1) - \boldsymbol{P}^* = \frac{1}{W}\boldsymbol{\Phi}[\boldsymbol{P}(n) - \boldsymbol{P}^*] + \frac{1}{W}\boldsymbol{\tau}[\boldsymbol{\psi}(\boldsymbol{P}(n)) - \boldsymbol{\psi}(\boldsymbol{P}^*)], \quad n = 0, 1, \cdots . \tag{4.8}$$

Hence,

$$\|\boldsymbol{P}(n+1) - \boldsymbol{P}^*\| \le \frac{1}{W}\|\boldsymbol{\Phi}[\boldsymbol{P}(n) - \boldsymbol{P}^*]\| + \frac{1}{W}\|\boldsymbol{\tau}[\boldsymbol{\psi}(\boldsymbol{P}(n)) - \boldsymbol{\psi}(\boldsymbol{P}^*)]\| \ . \tag{4.9}$$

We will use the norm defined in terms of the element-wise positive Perron-Frobenius eigenvector $\boldsymbol{u}$ corresponding to $\boldsymbol{\Phi}$, i.e.,

$$\boldsymbol{\Phi u} = r_\Phi \boldsymbol{u}, \quad \boldsymbol{u} > \mathbf{0} \ . \tag{4.10}$$

Specifically,

$$\|\boldsymbol{x}\| = \max_{1 \le m \le IJ} |x_m|/u_m \ . \tag{4.11}$$

Let $\boldsymbol{u} = (\boldsymbol{u}_1, \ldots, \boldsymbol{u}_I)^T$. Then, from (4.6),

$$\alpha_i \sum_\ell \boldsymbol{F}^{(\ell)} \boldsymbol{u}_\ell = r_\Phi \boldsymbol{u}_i \ . \tag{4.12}$$

Using (3.9), we obtain

$$(\alpha \boldsymbol{F} - r_\Phi \boldsymbol{I}) \sum_\ell \boldsymbol{F}^{(\ell)} \boldsymbol{u}_\ell = \mathbf{0} \ . \tag{4.13}$$

The element-wise Perron-Frobenius eigenvector $\boldsymbol{v}$ of $\boldsymbol{F}$ satisfies

$$\boldsymbol{F v} = r_F \boldsymbol{v}, \quad \boldsymbol{v} > \mathbf{0} \ . \tag{4.14}$$

Hence $r_\Phi = \alpha r_F$, since r_Φ and r_F are the eigenvalues of maximum modulus. We choose $\sum_\ell \boldsymbol{F}^{(\ell)} \boldsymbol{u}_\ell = r_\Phi \boldsymbol{v}$. Then, from (4.12), $\boldsymbol{u}_i = \alpha_i \boldsymbol{v}$. Also,

$$\|\boldsymbol{\Phi}[\boldsymbol{P}(n) - \boldsymbol{P}^*]\| \le r_\Phi \|\boldsymbol{P}(n) - \boldsymbol{P}^*\| = \alpha r_F \|\boldsymbol{P}(n) - \boldsymbol{P}^*\| \ . \tag{4.15}$$

It is shown in the Appendix that

$$\|\boldsymbol{\tau}[\boldsymbol{\psi}(\boldsymbol{P}(n) - \boldsymbol{\psi}(\boldsymbol{P}^*)]\| \le \beta \|\boldsymbol{P}(n) - \boldsymbol{P}^*\| \ , \tag{4.16}$$

where

$$\beta = \delta \sqrt{IJ} \sqrt{\sum_{i=1}^{I} \alpha_i^2 \nu_i^2} \ \frac{\max\limits_{1 \le i \le I} \alpha_i \ \max\limits_{1 \le k \le J} v_k}{\min\limits_{1 \le i \le I} \alpha_i \ \min\limits_{1 \le k \le J} v} \ , \tag{4.17}$$

and

$$\delta^2 = \max_{\substack{1 \le k \le J \\ 1 \le \ell \le I}} \sum_{j=1}^{J} \left(H_{jk}^{(\ell)} \right)^2 K_{k\ell} w_{k\ell} (1 - w_{k\ell}), \quad \delta \ge 0 \ . \tag{4.18}$$

Hence, from (4.9), (4.15) and (4.16),

$$\|\boldsymbol{P}(n+1) - \boldsymbol{P}^*\| \le \frac{1}{W} (\alpha r_F + \beta) \|\boldsymbol{P}(n) - \boldsymbol{P}^*\|, \quad n = 0, 1, \cdots . \tag{4.19}$$

It follows, by induction, that

$$\|\boldsymbol{P}(n) - \boldsymbol{P}^*\| \le \left[\frac{1}{W} (\alpha r_F + \beta) \right]^n \|\boldsymbol{P}(0) - \boldsymbol{P}^*\| \ . \tag{4.20}$$

Proposition 3 If $(\alpha r_F + \beta)/W < 1$, then $\boldsymbol{P}(n)$ converges geometrically to $\boldsymbol{P}^*$. ∎

We note that in our asymptotic analysis, with the scalings in (3.1), $\beta/W = O(1/\sqrt{K})$. Since, as noted in Sec. 3.2, $\alpha r_F/W = O(1)$, the above condition for geometric convergence is only slightly more stringent than $\alpha r_F/W < 1$, the condition which arises from consideration of just mean activity values. The condition in Proposition 3 is also the condition which determines the capacity of the cellular CDMA system.

5 CONCLUSIONS

We have given an algorithm in (4.3) for distributed power control in cellular CDMA networks which supports heterogeneous transmitting sources having diverse statistical/burstiness characteristics and quality of service requirements. The algorithm is derived from an asymptotic analysis based on a scaling appropriate for wideband systems. The analysis of the dynamical behavior of the algorithm considers the geometrically fast convergence of the adapted power levels to their optimum values.

A topic for future work is the extension of the approach developed in [MMO96] and here to correctly incorporate random fluctuations in the transmission medium. Since we are already taking into account statistical fluctuations in the interference due to transmissions, it should be possible to extend the calculations to take into account explicitly fluctuations in the medium.

APPENDIX

We here derive the inequality (4.16). If $x_{k\ell}$ and $y_{k\ell}$ $(1 \leq k \leq J, \quad 1 \leq \ell \leq I)$ are real, then Cauchy's inequality implies that

$$\left(\sum_{k=1}^{J} \sum_{\ell=1}^{I} x_{k\ell}^2\right)^{1/2} \left(\sum_{k=1}^{J} \sum_{\ell=1}^{I} y_{k\ell}^2\right)^{1/2} \geq \sum_{k=1}^{J} \sum_{\ell=1}^{I} x_{k\ell} y_{k\ell} \ . \tag{A1}$$

It follows that

$$\left[\left(\sum_{k=1}^{J} \sum_{\ell=1}^{I} x_{k\ell}^2\right)^{1/2} - \left(\sum_{k=1}^{J} \sum_{\ell=1}^{I} y_{k\ell}^2\right)^{1/2}\right]^2 \leq \sum_{k=1}^{J} \sum_{\ell=1}^{I} (x_{k\ell} - y_{k\ell})^2 \ . \tag{A2}$$

Hence, from (3.35), we obtain

$$\sum_{j=1}^{J} [\psi_j(\boldsymbol{P}) - \psi_j(\boldsymbol{P}^*)]^2 \leq \sum_{j=1}^{J} \sum_{k=1}^{J} \sum_{\ell=1}^{I} \left(H_{jk}^{(\ell)}\right)^2 K_{k\ell} w_{k\ell}(1 - w_{k\ell}) (P_{k\ell} - P_{k\ell}^*)^2 \ . \tag{A3}$$

Thus, with δ defined by (4.18),

$$\sum_{j=1}^{J} [\psi_j(\boldsymbol{P}) - \psi_j(\boldsymbol{P}^*)]^2 \leq \delta^2 \sum_{k=1}^{J} \sum_{\ell=1}^{I} (P_{k\ell} - P_{k\ell}^*)^2 \ . \tag{A4}$$

Now, from (4.7),

$$(\|\boldsymbol{\tau}[\boldsymbol{\psi}(\boldsymbol{P}) - \boldsymbol{\psi}(\boldsymbol{P}^*)]\|_2)^2 = \sum_{i=1}^{I} \alpha_i^2 \nu_i^2 \sum_{j=1}^{J} \left[\psi_j(\boldsymbol{P}) - \boldsymbol{\psi}_j(\boldsymbol{P}^*)\right]^2 \ , \tag{A5}$$

where

$$\|\boldsymbol{x}\|_2 = \left(\sum_{m=1}^{IJ} x_m^2\right)^{1/2} \ . \tag{A6}$$

From (A4)–(A6) we obtain

$$\|\boldsymbol{\tau}[\boldsymbol{\psi}(\boldsymbol{P}) - \boldsymbol{\psi}(\boldsymbol{P}^*)]\|_2 \leq \delta \left(\sum_{i=1}^{I} \alpha_i^2 \nu_i^2\right)^{1/2} \|\boldsymbol{P} - \boldsymbol{P}^*\|_2 \ . \tag{A7}$$

But,

$$\max_{1 \leq m \leq IJ} |x_m| \leq \|\boldsymbol{x}\|_2 \leq \sqrt{IJ} \max_{1 \leq m \leq IJ} |x_m| \ . \tag{A8}$$

Hence, from (4.11), we have

$$\min_{1 \leq m \leq IJ} u_m \ \|\boldsymbol{x}\| \leq \|\boldsymbol{x}\|_2 \leq \sqrt{IJ} \max_{1 \leq m \leq IJ} u_m \ \|\boldsymbol{x}\| \ . \tag{A9}$$

Note that, since $\boldsymbol{u} = (\alpha_1, \ldots, \alpha_I)^T \boldsymbol{v}$,

$$\min_{1 \leq m \leq IJ} u_m = \min_{1 \leq i \leq I} \alpha_i \ \min_{1 \leq k \leq J} v_k \ , \tag{A10}$$

and

$$\max_{1 \le m \le IJ} u_m = \max_{1 \le i \le I} \alpha_i \max_{1 \le k \le J} v_k \ . \tag{A11}$$

The inequality (4.16) follows from (A7) and (A9)–(A11), where β is defined by (4.17) and (4.18).

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