On the Profitability of Remanufactured Products

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We investigate the profitability of offering remanufactured products for a monopoly firm in a single period setting. We characterize a threshold for the remanufacturing cost below which it is optimal to offer remanufactured products, and focus on analyzing its dependence on the consumer profile. This analysis allows us to identify when it is customer segmentation, and not the cost difference, that drives the remanufacturing decision. That is, we find conditions under which it is optimal to offer remanufactured products even if they are as costly to produce as new products. Consequently, it becomes cost effective to substitute them and offer new products under the remanufactured label when the latter are scarce. These results are also applicable to the evaluation of the common marketing practices of branding, i.e, offering virtually identical products under different brands, and generics, i.e. offering generic versions of a landmark label.

Key words: Market segmentation, product remanufacturing, product substitution

1. Introduction

During the last decade, remanufacturing has become more and more popular in various industries. Progress in ecological design, strict regulations (such as the European Union's WEEE directive (2003)) and the increasing number of environmentally conscious consumers are bringing the handling of used products to the forefront of business priorities. Examples abound: Dell Outlet (www.delloutlet.com) and USA notebook (www.usanotebook.com) sell refurbished computers. ReCellular (www.recellular.net) sells refurbished cell phones. Saint Vincent de Paul (www.humstvincentdepaul.org/mattress.html) remanufactures mattresses. CycleTires (www.cycletires.com/about.htm) remanufactures aircraft tires.

Remanufacturing operations begin with the collection of products that have either been discarded after use or found to be defective. These products are disassembled and their components cleaned and checked to determine which ones to bring up to specification and which ones to replace. The product is then reassembled and tested, ready for a second cycle of use with practically the same level of performance (Hauser and Lund (2003)). Remanufacturing is beneficial for the environment. It reduces the consumption of raw materials and energy from the environment, and the waste stream back to the environment, such as solid waste that ends up at landfill sites.

Moreover, remanufacturing can also be profitable for the firms. The cost to produce a remanufactured unit of a particular product is in general lower than the cost to produce a new unit because of the savings in raw materials, energy, and manufacturing plant and equipment in the remanufacturing process. Furthermore, firms can attract new buyers into a market where new product prices have been prohibitively high for them, by providing like-new products at prices that typically range from 45% to 65% of comparable new products (Hauser and Lund (2003)). The overall size of the market is thus increased.

Consider a monopoly firm that has the option to either offer only new products or offer both new and remanufactured products to the market. Customers value new and remanufactured products differently; that is, customers have different utilities on the same new product and a lower relative utility for the remanufactured counterpart. This can be described by a utility distribution function over the entire population of customers and a relative utility function for remanufactured products as in Debo et al. (2005). Consumer utility for remanufactured products is modeled as a function of the utility the consumer places on new products, which we refer to as a relative utility function, and is thus identical for all customers that share the same utility for new products. The ratio of the utility of the remanufactured product to the utility of the new product may vary as the utility of the new product decreases. For example, a high-end customer may be willing to pay \$100 for the new product and \$50 for the remanufactured one. The ratio is 50%. However, a price-sensitive, low-end customer may be willing to pay \$50 for the new product but \$35 for the remanufactured one. The ratio is 70%. As the example shows, the relative utility function is not necessarily linear;

in general, it is expected to be concave. The combination of the utility distribution function for new products and the relative utility function for remanufactured products is referred to as a *consumer profile*. Customers will buy a product only if it is priced below their utilities. In addition, customers decide whether to buy new or remanufactured products by comparing their surpluses, i.e., their utilities minus the price of the product, from either of the two products. In turn, the firm has to decide whether to offer only new products or offer both new and remanufactured products, and at what prices.

In some situations, the amount available for remanufacture may be limited by a low stream of returned products. This is particularly true at the early start of the remanufacturing operation, when very few of the new products sold have reached the end-of-life stage. Given a scarce supply of remanufactured products, the firm may consider labeling a certain amount of new products as remanufactured (or simply including a minimal amount of used components into the remanufactured product) to reap the full benefits of segmenting the market into high-paying new product customers and low-paying remanufactured product customers. The consumer, oblivious to this substitution taking place and to the extent to which used components are used, cannot tell the difference between the substituted and the genuinely remanufactured products.

In the present paper, we address the problems of (1) whether or not to offer remanufactured products in addition to new products and (2) whether or not to offer new products as substitutes for remanufactured products, in a single-period monopoly setting. We focus on understanding the impact of the consumer profile on the optimal decisions and profits of the firm. Our major departure from the previous literature is the explicit study of nonlinear relative utility functions for remanufactured products. We show that the concavity of these functions is a necessary condition to efficiently segment the market under no cost differences in the production of the two products.

These problems are thus closely related to the classical literature on market segmentation. Mussa and Rosen (1978) and Moorthy (1984) study the optimal pricing of independent products that are differentiated by quality in a market of heterogeneous consumers whose valuations of quality vary. Mussa and Rosen (1978) approximate the customer utility to be linear with the product quality.

Moorthy (1984) assumes a finite number of consumer types. An extensive body of literature on product assortment and pricing has followed their seminal work; see for instance Aydin and Ryan (2000) and the references therein. More recently, new research has focused on the study of customer segmentation in the remanufacturing context. Ferrer (1996) solves the market segmentation problem for new and remanufactured products faced by a monopoly firm in a steady-state market. The author examines the trade-offs between the customers utility and production cost variables and identifies conditions under which the remanufactured product may coexist with the brand new product. New and remanufactured product pricing decisions are linked not only by the product substitution effects common in market segmentation, but also by the need of used components to allow for remanufacturing of the same steady-state amount with the desired used part content in the next period. The customer utility for remanufactured products is assumed to be linear in the customer utility for new products. The customer valuation of new products is assumed to have a distribution with increasing hazard rate when the used part content in remanufactured products is fixed, and to be uniformly distributed when optimizing the used part content. Debo et al. (2005) consider the problem of jointly determining the level of remanufacturability to invest in and the prices at which to offer both new and remanufactured products over an infinite horizon to maximize the net present value. In their general model, the utility for new products is given by a strictly increasing and continuous function F on $[0,1]$ and the utility of remanufactured products is a nonnegative monotonically increasing function $\eta(\theta)$ of the utility for new products θ. Most of their analysis and all of their numerical work and managerial conclusions, however, rely on the assumption that the relative utility for remanufactured products is linear, $\eta(\theta) = (1-\delta)\theta$, what they refer to as a linear consumer profile. They first characterize the optimal solutions to the problem and then use this to give conditions that render remanufacturing profitable. Under a linear consumer profile, they show that the factors driving the remanufacturing potential are the manufacturing and remanufacturing costs, the incremental (fixed and variable) cost of providing remanufacturability and the discount factor. They also point out the importance of the consumer profile (F, η) on the remanufacturing potential, i.e., on the additional profit to be reaped from offering a remanufactured product. Vorasayan and Ryan (2006) similarly consider a linear consumer profile, but with stochastic demand and service times, in a queueing model that captures both manufacturing and remanufacturing operations.

Our paper builds upon these findings and complements the previous studies by focusing on the impact of the functional form of the relative consumer utility for remanufactured products on the optimal decisions and profits of the firm. Our goal is to identify the role of customer segmentation as a driver of the remanufacturing decision. We show that whether or not remanufacturing is profitable is highly dependent on the consumer profile (F, η). This contrasts the finding in Debo et al. (2005), under the assumption of a linear consumer profile, that the distribution of consumer types, $F(\theta)$, impacts the sign of the remanufacturing potential only when remanufacturing requires a positive investment. The reason is that for nonlinear relative utility functions, the distribution of consumer utilities, F, takes a more prominent role in determining the profitability of remanufacturing. Moreover, we discuss the option of a firm to offer new products as substitutes for remanufactured products, when the latter are scarce. This one-way substitution has been studied before in Baymdir et al. (2003, 2005) in an inventory control setting where prices and demand distributions for new and remanufactured products are exogenously given. In this case, the benefits of substitution are derived from risk pooling effects. The impact of this substitution at the strategic level to allow for customer segmentation, to our knowledge, has not been previously addressed.

Some papers study the substitution option in different contexts. Bassok et al. (1999) study a single period multi-product inventory problem with substitution and proportional costs and revenues. They consider N products and N corresponding demand classes with full downward substitution, i.e., excess demand for product i can be satisfied using a higher quality product j. For example, a car rental company uses mid-sized cars to satisfy unexpectedly high demand for compact cars. They focus on finding the optimal inventory replenishment policy that maximizes the firm's profit. They show that the benefits of the substitution option are higher with high demand variability, low substitution cost, low profit margins (or low price to cost ratio), high salvage values, and similarity of products in terms of prices and costs. Netessine et al. (2000) consider the capacity investment problem for a firm that provides multiple services using both specialized and flexible capacity. The demand can again be satisfied with downward substitution. They analytically characterize the effects of increasing demand correlation on the optimal capacity investment solution. For the case with two customer classes, they show that increasing correlation induces a shift from flexible to dedicated capacity. Khouja (1999) offers a literature review on the single-period (news-vendor) problem, including the case of multiple products with substitution, and suggests some future directions for research.

The remainder of the paper is structured as follows. In Section 2, we introduce the profit maximization problems faced by a firm offering either both new and remanufactured products or only a single product to the market. In Section 3, we analyze these models and derive the conditions under which the firm should offer both new and remanufactured products. In Section 4, we allow the firm to offer new products as substitutes for remanufactured products and derive conditions under which this substitution is beneficial. In Section 5, we identify the form of the relative utility function that leads to highest profits from customer segmentation. This provides a tight upper bound on the potential profits that can be reaped through offering remanufactured products. Section 6 presents the major results of an extensive computational study to illustrate the effect of the consumer profile on remanufacturing decisions. Finally, in Section 7, we discuss the major managerial insights and directions for future research.

2. Models

In this section, we model the pricing problems faced by the monopoly firm when offering either both new and remanufactured products or only one of them to the market.

Throughout the paper we refer to a function as increasing (decreasing) when it is non-decreasing (non-increasing). We will explicitly point out when a function is strictly increasing (decreasing), otherwise.

As in Debo et al (2005), we consider a general utility distribution function defined on $[0, 1]$ to model the customer's willingness to pay for the new product. Let $F(\theta)$ denote the cumulative distribution function, that is the volume of customers that are willing to pay θ or less for the new product, with $F(0) = 0$ and $F(1) = 1$. We use $\eta(\theta)$ to denote the relative customer utility for remanufactured products when the customer utility for new products is θ. We assume $\eta(\theta)$ to be increasing with $\eta(0) = 0$ and $\eta'(\theta) \leq 1$; that is, $\theta - \eta(\theta)$ is increasing, to reflect the lower relative preference for the remanufactured product associated with high-end, less price-sensitive consumers, as discussed in Section 1. This assumption is weaker than concavity and is easily justifiable in practice: if a consumer who is willing to pay \$50 for the new product will switch to the remanufactured one for a \$10 price break, then a consumer that values the new product at \$100 will require at least a \$10 break to switch.

We refer to the combination of the utility distribution function and the relative utility function for remanufactured products as a consumer profile. The customer chooses between new and remanufactured products by comparing his surplus, i.e., his utility minus the product price, from either of the products. This type of model is referred to as a self-selection model in the literature (Moorhty (1984)).

Let p_n and p_r denote the price for new and remanufactured products, respectively. Let c_n and c_r denote the unit cost to produce new and remanufactured products, respectively. We ignore the investment required to acquire the capability to produce a remanufactured product. The cost should be compared to the discounted profits over an appropriate time horizon, which is beyond the scope of our single period problem. The problems faced by a profit maximizing firm offering one or both of the products are described in the following subsections.

2.1 When the Firm Offers Only One Product

If the firm only offers new products, the customer purchases the product only when his utility exceeds the price for new products, that is, $\theta \geq p_n$. Let n denote the percent of customers that purchase products in this case. We have

$$n(p_n) = \int_{p_n}^{1} dF(\theta) = 1 - F(p_n) \tag{1}$$

The profit maximization problem for the firm is

Problem N

$$\max_{p_n} \quad (p_n - c_n)n(p_n) = (p_n - c_n)[1 - F(p_n)] \tag{2}$$

We denote by Π^N the optimal solution to Problem N, and by p^N and n^N the optimal price and sales volume, respectively.

Similarly, if the firm only offers remanufactured products, the customer purchases products only when his utility exceeds the price for the products, that is, $\eta(\theta) \geq p_r$. Let r denote the percent of customers that purchase the remanufactured products. We have

$$r(p_r) = \int_{\eta^{-1}(p_r)}^{1} dF(\theta) = 1 - F(\eta^{-1}(p_r)) \tag{3}$$

The profit maximization problem for the firm is

Problem R

$$\max_{p_r} \quad (p_r - c_r)r(p_r) = (p_r - c_r)[1 - F(\eta^{-1}(p_r))] \tag{4}$$

To facilitate the comparison with the model when both new and remanufactured products are offered, define $\theta_r = \eta^{-1}(p_r)$ so that $\eta(\theta_r) = p_r$. The new variable θ_r represents the indifference point between purchasing the remanufactured product or nothing at all, i.e., the lowest utility consumer that will purchase the remanufactured product. The new formulation for Problem R is

Problem R

$$\max_{\theta_r} \quad [\eta(\theta_r) - c_r][1 - F(\theta_r)] \tag{5}$$

We denote by Π^R the optimal solution to Problem R, and the optimal decision variables by θ^R, p^R and r^R.

2.2 When the Firm Offers Both New and Remanufactured Products

Let $\gamma(\theta)$ denote the difference between the customer's utility for new and remanufactured products; that is, $\gamma(\theta) = \theta - \eta(\theta)$, an increasing function of θ. If the customer surplus from new products is positive and no lower than that from remanufactured products, that is, $\theta - p_n \geq \eta(\theta) - p_r$ (or $\gamma(\theta) \geq p_n - p_r$), the customer chooses new products. Otherwise, the customer chooses remanufactured products. Let n and r denote again the percent of customers that purchase new and remanufactured products respectively. We have

$$n(p_n, p_r) = \int_{\gamma^{-1}(p_n - p_r)}^{1} dF(\theta) = 1 - F(\gamma^{-1}(p_n - p_r)) \tag{6}$$

$$r(p_n, p_r) = \int_{\eta^{-1}(p_r)}^{\gamma^{-1}(p_n - p_r)} dF(\theta) = F(\gamma^{-1}(p_n - p_r)) - F(\eta^{-1}(p_r)) \tag{7}$$

The lower limit in the integration in Equation (7) is $\eta^{-1}(p_r)$ because the customer chooses remanufactured products only if his utility exceeds the price for remanufactured products, that is $\eta(\theta) \geq p_r$. Observe also that for the equation to be valid we need to impose $\eta^{-1}(p_r) \leq \gamma^{-1}(p_n - p_r)$ or, equivalently, $p_r \leq \eta(p_n)$. That is, the price of the remanufactured product must be lower than the value the last consumer that has positive surplus for the new product (the one with utility exactly p_n) places on the remanufactured product in order to have a positive remanufacturing market. When $p_r = \eta(p_n)$ the size of the remanufacturing market is 0, representing the case when the firm offers only the new product.

Similarly, the lower limit in the integration in Equation (6) has to be greater than or equal to the price of new products p_n for the customers to extract positive utility from the new products. We omit that requirement since the current lower limit of integration $\gamma^{-1}(p_n - p_r)$ is always greater than or equal to p_n under our conditions:

$$p_r \leq \eta(p_n) \rightarrow p_n - p_r > p_n - \eta(p_n) \rightarrow \gamma^{-1}(p_n - p_r) \geq p_n.$$

The profit maximization problem for the firm can then be written as follows.

Problem B

$$\begin{aligned} \max_{p_n, p_r} \quad \Pi(p_n, p_r) &= (p_n - c_n) n(p_n, p_r) + (p_r - c_r) r(p_n, p_r) \\ &= (p_n - c_n)[1 - F(\gamma^{-1}(p_n - p_r))] + (p_r - c_r)[F(\gamma^{-1}(p_n - p_r)) - F(\eta^{-1}(p_r))] \\ &= [(p_n - p_r) - (c_n - c_r)][1 - F(\gamma^{-1}(p_n - p_r))] + (p_r - c_r)[1 - F(\eta^{-1}(p_r))] \end{aligned}$$

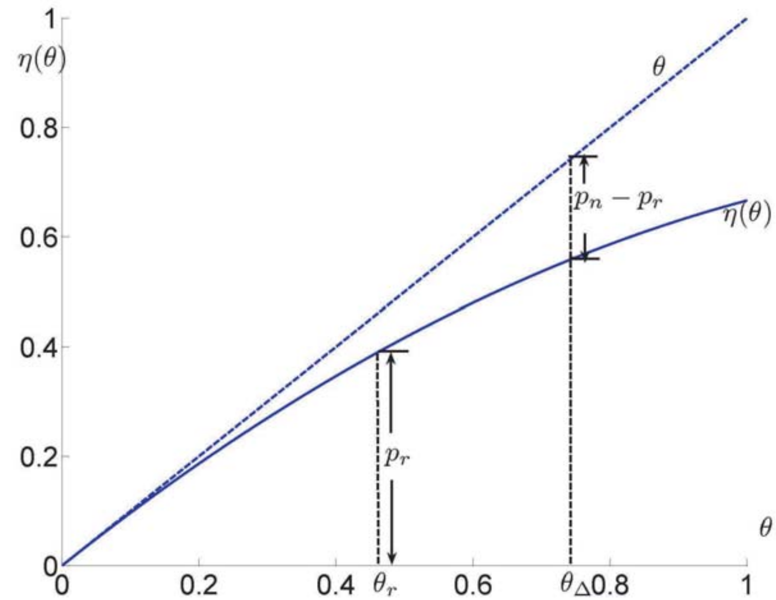

Figure 1 **Reformulation**

subject to the constraints $p_r \leq \eta(p_n)$, $0 \leq p_n \leq 1$, and $0 \leq p_r \leq 1$. The last expression of the objective function suggests a reformulation to simplify the problem. Define $\theta_\Delta = \gamma^{-1}(p_n - p_r)$, $\theta_r = \eta^{-1}(p_r)$ and $c_\Delta = c_n - c_r$. If the function η (or γ) is not strictly increasing, we can still use this transformation by defining $\eta^{-1}(p_r) = \min\{\theta : \eta(\theta) \geq p_r\}$ (or $\gamma^{-1}(p_n - p_r) = \min\{\theta : \gamma(\theta) \geq p_n - p_r\}$. Observe that θ_Δ represents the utility of a customer that obtains the same surplus from, and is thus indifferent between, new and remanufactured products. Figure 1 provides an illustration of the graphical interpretation of the new variables. The percent of customers that purchase new and remanufactured products after the variable substitution is as follows:

$$n = 1 - F(\theta_\Delta) \tag{8}$$

$$r = F(\theta_\Delta) - F(\theta_r) \tag{9}$$

Customers with utility for new products higher than or equal to θ_Δ purchase new products. Customers with utility for new products higher than or equal to θ_r but lower than θ_Δ purchase remanufactured products.

The problem can then be formulated using these new variables:

Problem B

$$\max_{\theta_\Delta,\theta_r} \quad \Pi(\theta_\Delta,\theta_r) = [\gamma(\theta_\Delta) - c_\Delta][1 - F(\theta_\Delta)] + [\eta(\theta_r) - c_r][1 - F(\theta_r)]$$
$$s.t. \quad 0 \le \theta_r \le \theta_\Delta \le 1 \tag{10}$$

We denote the optimal solution by Π^* and the optimal decision variables by θ_Δ^*, θ_r^*, p_n^*, p_r^*, n^* and r^*.

In this formulation, the maximization can be performed separately on each of the variables, θ_Δ and θ_r. If the constraint is violated, then it is optimal to only offer new products. Observe that the second term in the objective function is identical to the objective in Problem R. If the unconstrained maximizers θ_Δ^* and θ_r^* satisfy the constraint $\theta_\Delta^* \ge \theta_r^*$, then $\theta_r^* = \theta^R$, $p_r^* = p^R$, and the total market size in Problem B is the same as that in Problem R, that is, $1 - F(\theta^R)$.

3. Model Analysis

In this section, we derive necessary and sufficient conditions under which it is optimal for the firm to offer both new and remanufactured products.

3.1 Assumptions

First, let's discuss the assumptions we make in our analysis. In order for the problem to be of interest, we impose the following conditions on the parameters: $c_n < 1$, $c_r < \eta(1)$, and $c_\Delta < \gamma(1)$. Clearly, for the firm to obtain profits from new products, we must have $c_n < 1$, and to obtain profits from remanufacturing products $c_r < \eta(1)$. For the firm to offer a mixture of the two, we must also require $\gamma(1) > c_\Delta$, since otherwise $\theta - \eta(\theta) = \gamma(\theta) \le \gamma(1) \le c_n - c_r \Rightarrow$ the difference in cost is larger than the loss in value associated with the remanufactured product in the eyes of all consumers and thus offering only the remanufactured product would be optimal. Observe that this option is problematic in practice because the monopoly firm needs to build an inventory of remanufacturable products through the sell of new products. In general we should require $r < qn$ where q is the portion of new products that make their way back to the firm and are suitable for remanufacturing (see Ferrer (1996)). This, however, is beyond the scope of our single period model. The condition $\gamma(1) < c_\Delta$ is analogous to the condition in Theorem 1 of Ferrer (1996) under which only remanufactured products should be offered.

We assume that $F(\theta)$ is strictly increasing and has an increasing hazard rate. That is, $\frac{F'(\theta)}{1-F(\theta)}$ is increasing in θ. This is true for a great variety of distributions including Uniform, Normal (as well as truncated Normal at zero), Exponential, Gamma (with shape parameter $s \geq 1$), Beta (with parameters (r, s) being both ≥ 1), and Weibull distribution (with shape parameter $s \geq 1$) as indicated by Yao, Chen and Yan (2006). For illustration we will consider the consumer profile $F(\theta) = 1 - (1-\theta)^\kappa$ used in Debo et al (2005).

From now onwards, we also assume that $\eta(\theta)$ is increasing, differentiable and $\frac{\eta'(\theta)}{\eta(\theta)-c_r}$ is decreasing in θ. This is clearly satisfied when $\eta(\theta)$ is also concave in θ, since we have that $\eta(\theta) - c_r$ is increasing in θ and $\eta'(\theta)$ is decreasing in θ. In addition, we assume that $\gamma(\theta)$ is increasing and $\frac{\gamma'(\theta)}{\gamma(\theta)-c_\Delta}$ is decreasing in θ, over the range of interest (where $\gamma(\theta) = \theta - \eta(\theta) > c_\Delta$ in order to have positive offering of new products). This condition is satisfied by all the practical examples we have constructed. In what follows, we consider both concave and convex functions that satisfy the stated conditions.

3.2 Optimal Solution

To characterize the optimal solutions to Problems N and B, we calculate the necessary first-order conditions and show that they are also sufficient for optimality by proving that the objective functions are unimodal.

For Problem N, the first-order condition is:

$$[1 - F(p^N)] - (p^N - c_n)F'(p^N) = 0$$

That is

$$\frac{1 - F(p^N)}{F'(p^N)} = p^N - c_n \tag{11}$$

The economic interpretation is as follows. The optimal price p^N is the point at which the percent decrease in demand ($\frac{F'(p^N)}{1-F(p^N)} \times 100$) per percent increase in profit margin ($\frac{1}{p^N - c_n} \times 100$) is equal to 1.

For Problem B, the first-order conditions are:

$$\gamma'(\theta_\Delta)[1 - F(\theta_\Delta)] - [\gamma(\theta_\Delta) - c_\Delta]F'(\theta_\Delta) = 0$$

$$\eta'(\theta_r)[1-F(\theta_r)] - [\eta(\theta_r)-c_r]F'(\theta_r) = 0$$

That is

$$\frac{F'(\theta_\Delta)}{1-F(\theta_\Delta)} = \frac{\gamma'(\theta_\Delta)}{\gamma(\theta_\Delta)-c_\Delta} \tag{12}$$

$$\frac{F'(\theta_r)}{1-F(\theta_r)} = \frac{\eta'(\theta_r)}{\eta(\theta_r)-c_r} \tag{13}$$

These two conditions have a similar economic interpretation. The second necessary condition (13) states that the optimal price for the remanufactured product, $p_r^* = \eta(\theta_r^*)$, is such that the percent decrease in total demand in the system $\frac{F'(\theta_r)}{\eta'(\theta_r)(1-F(\theta_r))} \times 100$ per percent increase in the profit margin of the remanufactured products $\frac{1}{\eta(\theta_r)-c_r} \times 100$ is equal to 1. Interestingly, this occurs at the point θ_r^* where the marginal decrease in demand $\frac{F'(\theta_r)}{(1-F(\theta_r))}$ is equal to the marginal increase in relative utility surplus (i.e., utility minus cost), $\frac{\eta'(\theta_r)}{\eta(\theta_r)-c_r}$. The first condition (12) requires that the optimal price difference $p_n^* - p_r^* = \gamma(\theta_\Delta^*)$ be such that the percent decrease in demand for new products $\frac{F'(\theta_\Delta)}{\gamma'(\theta_\Delta^*)(1-F(\theta_\Delta))} \times 100$ per percent increase in the profit margin difference $\frac{1}{\gamma(\theta_\Delta)-c_\Delta} \times 100$ be equal to 1. (The profit margin difference is $(p_n - c_n) - (p_r - c_r) = p_n - p_r - c_\Delta = \gamma(\theta_\Delta) - c_\Delta$.) Optimality occurs at the point θ_Δ^* where the marginal decrease in demand is equal to the marginal increase in the difference in utility surplus of new versus remanufactured products $\frac{\gamma'(\theta_\Delta)}{\gamma(\theta_\Delta)-c_\Delta}$.

Let's now show that the objective functions are unimodal. The first-order derivative of $[\eta(\theta_r) - c_r][1-F(\theta_r)]$ is

$$\eta'(\theta)F'(\theta)\left(\frac{1-F(\theta)}{F'(\theta)} - \frac{\eta(\theta)-c_r}{\eta'(\theta)}\right)$$

Under the assumptions previously discussed, this derivative is positive to the left of the solution to the first order condition (13), say θ_r^*, and negative to the right. So the objective function $[\eta(\theta_r) - c_r][1-F(\theta_r)]$ is increasing to the left of θ_r^* and decreasing to the right. A similar argument follows for the function $[\gamma(\theta_\Delta) - c_\Delta][1-F(\theta_\Delta)]$ as well. Both functions are unimodal and thus the necessary conditions become sufficient for optimality.

We are now ready to characterize the relationship between the optimal solutions to the two models. Observe that:

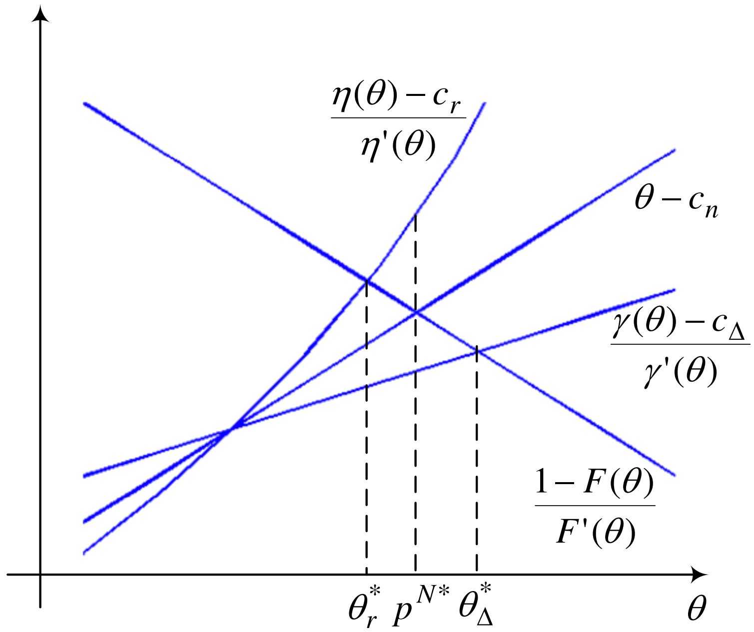

Figure 2 **The Optimal Solution**

1. If

$$\frac{\eta(p^N)-c_r}{\eta'(p^N)} > p^N - c_n$$

and consequently $\frac{\gamma(p^N)-c_\Delta}{\gamma'(p^N)} < p^N - c_n$, we must have $\theta_r^* < p^N$ to satisfy the first-order condition (13) under our assumptions, as shown in Figure 2. Similarly, we must have $\theta_\Delta^* > p^N$ to satisfy the first-order condition (12). The unconstrained maximizers θ_Δ^* and θ_r^* satisfy the constraint $\theta_\Delta > \theta_r$ since $\theta_\Delta^* > p^N > \theta_r^*$. In this case, it is easy to show that $\Pi^* > \Pi^N$.

2. If

$$\frac{\eta(p^N)-c_r}{\eta'(p^N)} = p^N - c_n$$

and consequently $\frac{\gamma(p^N)-c_\Delta}{\gamma'(p^N)} = p^N - c_n$, we have $\theta_\Delta^* = \theta_r^* = p^N$ and $\Pi^* = \Pi^{N*}$.

3. If

$$\frac{\eta(p^N)-c_r}{\eta'(p^N)} < p^N - c_n$$

and consequently $\frac{\gamma(p^N)-c_\Delta}{\gamma'(p^N)} > p^N - c_n$, the unconstrained maximizers $\theta_\Delta^* < p^N < \theta_r^*$. The solution violates the constraint $\theta_\Delta \geq \theta_r$. Given the unimodality of the objective function in θ_Δ and θ_r, the constrained optimal solution will be found at a point such that $\theta_\Delta = \theta_r$; therefore only new products will be offered, and thus $\theta_\Delta^* = \theta_r^* = p^N$ and $\Pi^* = \Pi^N$.

In conclusion, whether or not to offer both new and remanufactured products depends on the relationship between $\frac{\eta(p^N)-c_r}{\eta'(p^N)}$ and $p^N - c_n$. We now use this result to characterize the threshold for the remanufacturing cost below which it is optimal to offer both products to the market.

3.3 Results

From the first-order condition (11) for Problem N, we have

$$c_n = p^N - \frac{1-F(p^N)}{F'(p^N)} \tag{14}$$

Using this equation and the fact that F has an increasing failure rate, it is easy to see that as the cost of producing the product, c_n, increases, the optimal price, p^N, strictly increases and the profit margin, $p^N - c_n$, decreases (strictly if F has strictly increasing failure rate). Define $G(\theta) = \theta - \frac{1-F(\theta)}{F'(\theta)}$. We have $c_n = G(p^N)$, and consequently, $p^N = G^{-1}(c_n)$. Note that G is a strictly increasing function.

Similarly, the first-order conditions (12) and (13) under our assumptions on F, η and γ, lead us to the following properties of the optimal prices and market sizes.

Theorem 1 *The optimal solution to Problem B satisfies the following properties:*

- *As the remanufacturing cost c_r increases, θ_r^* and thus p_r^* increase, θ_Δ^* and thus $p_n^* - p_r^*$ decrease, and therefore the sales of remanufactured products, $F(\theta_\Delta^*) - F(\theta_r^*)$, decrease while those for new products, $1 - F(\theta_\Delta^*)$, increase.*

- *As the new product cost c_n increases, θ_r^* and thus p_r^* remain the same, θ_Δ^* and thus $p_n^* - p_r^*$ increases, and therefore p_n^* and the sales of remanufactured products increase while the sales of new products decrease. The total sales from both products, $1 - F(\theta_r^*)$, remain the same.*

- *As the cost difference $c_\Delta = c_n - c_r$ increases, θ_Δ^* and thus $p_n^* - p_r^*$ increase and the sales of new products decreases.*

- *The optimal sales of new products $1 - F(\theta_\Delta^*)$ will not vary as long as the cost difference c_Δ remains the same, even if both c_r and c_n increase, up to a point where $\theta_r^* = \theta_\Delta^*$, when it is no longer optimal to offer remanufactured products to the market and the sales of new products become $1 - F(p^N)$, which is declining in c_n.*

Theorem 1 implies that if $\theta_r^* < \theta_\Delta^*$ then this relationship will always continue to hold as we increase c_n. On the other hand, the difference $\theta_\Delta^* - \theta_r^*$ will decrease as c_r increases up to the point where $\theta_\Delta^* = \theta_r^*$, after which the firm no longer offers remanufactured products. Using the relationship between the optimal solutions to Problems B and N established in the previous section, that threshold value of the remanufacturing cost is given by

$$\frac{\eta(p^N) - c_r}{\eta'(p^N)} = p^N - c_n. \tag{15}$$

Thus, we have

$$c_r = \eta(p^N) - \eta'(p^N)(p^N - c_n) \tag{16}$$

Substituting p^N with $G^{-1}(c_n)$, we have a threshold on the remanufacturing cost c_r as a function of the cost of producing a new product:

$$c_r^0(c_n) \equiv \eta(G^{-1}(c_n)) - \eta'(G^{-1}(c_n))(G^{-1}(c_n) - c_n) \tag{17}$$

Observe that c_r^0 is increasing in c_n, as a consequence of Theorem 1.

Theorem 2 *There is a threshold c_r^0 for the remanufacturing cost c_r, as given in Equation (17), such that the firm optimally offers both new and remanufactured products if and only if $c_r < c_r^0$. This threshold, c_r^0, is increasing in the cost c_n of manufacturing the new product and the difference $c_n - c_r^0$ is increasing in c_n.*

Proof. It only remains to show that the difference $c_n - c_r^0$ is increasing in c_n. We do this using the properties in Theorem 1. We use superindex $i = 1, 2$ to distinguish the parameters and solutions associated with the two sets of costs. Suppose to the contrary that there exists $0 \le c_n^1 < c_n^2 \le 1$ such that $c_n^1 - c_r^{01} > c_n^2 - c_r^{02}$. Note that, by definition of the threshold, for those remanufacturing costs, c_r^{0i}, and new product costs, c_n^i, for $i = 1, 2$, we must have $\theta_\Delta^{i*} = \theta_r^{i*} = p^{Ni}$. Observe also that since $c_\Delta^1 = c_n^1 - c_r^{01} > c_n^2 - c_r^{02} = c_\Delta^2$, Theorem 1 implies that $\theta_\Delta^{1*} \ge \theta_\Delta^{2*}$. At the same time, observe that p^N is strictly increasing in c_n and therefore $p^{N1} < p^{N2}$. Consequently, we have $\theta_r^{1*} = p^{N1} < p^{N2} = \theta_r^{2*} =$

$\theta_\Delta^{2*} \leq \theta_\Delta^{1*}$. This contradicts the fact that at the threshold the equality $\theta_\Delta^{1*} = \theta_r^{1*}$ must be satisfied. ∎

Observe that in Theorem 1, the behavior of the optimal price for new products, p_n, as c_r increases is not clear. In the Online Appendix we prove the following.

Proposition 1 *For $F_\kappa = 1-(1-\theta)^\kappa$ and $c_r < c_r^0$:*

- *if η is concave, p_n decreases in c_r and thus $p_n \geq p^N$,*
- *if η is convex, p_n increases in c_r and thus $p_n \leq p^N$,*
- *if η is linear, p_n does not depend on c_r and thus $p_n = p^N$.*

Furthermore, this behavior holds locally, i.e. for $c_r < c_r^0$ and sufficiently close to it, for any general utility function F, and a relative utility function η that is continuously double differentiable.

Using the threshold in Theorem 2 we can now study the impact of the consumer utility functions on the attractiveness of remanufacturing, which we interpret as the maximum cost of producing a remanufactured product that renders remanufacturing economically viable.

First, let's consider the linear consumer profile studied in Ferrer (1996) and Debo et al. (2005).

Corollary 1 *If $\eta(\theta)$ is linear, i.e., $\eta(\theta) = \delta\theta$, then remanufacturing is economically viable if and only if $c_r < \delta c_n$.*

Proof. It suffices to substitute $\eta(\theta) = \delta\theta$ in equation (16). ∎

The Corollary matches the results in Theorem 1 of Ferrer (1996) and Proposition 2 of Debo et al. (2005) applied to the particular setting under consideration. In this case, whether or not remanufacturing is profitable does not depend on the distribution of consumer utilities $F(\theta)$.

Corollary 2 *If $\eta(\theta)$ is convex, a necessary condition for remanufacturing to be economically viable is $c_r \leq \eta'(p^N)c_n$.*

Proof. Since η is increasing and convex on $[0,1]$, we have that $\eta(\theta) \leq \eta'(\theta)\theta$ for all θ. It suffices to substitute $\eta(p^N) \leq \eta'(p^N)p^N$ in equation (16) to find that the threshold must satisfy $c_r^0 < \eta'(p^N)c_n$. ∎

The above two corollaries show that when the value associated with the remanufacturing product is linear or convex in the utility of the new product, the remanufacturing cost has to be, in general, well below the new product manufacturing cost in order for remanufacturing to be profitable. A parallel argument shows that in the case of concave η the remanufacturing cost threshold satisfies $c_r^0 \geq \eta'(p^N)c_n$. The question remains as to whether the remanufacturing cost can ever exceed the manufacturing cost. We explore this issue in the example below and in the following section.

As a first step, we study the impact of the characteristics of the market valuation for the new product. For that purpose, we consider consumer utilities given by the family of cumulative distribution functions $F_\kappa(\theta) = 1-(1-\theta)^\kappa$, where higher values of κ correspond to higher concentrations on the low end of the market (Debo et al. (2005)).

Corollary 3 *If the consumer profile is given by a utility function $F_\kappa(\theta) = 1-(1-\theta)^\kappa$ for the new product and a relative valuation $\eta(\theta)$ for the remanufactured product that is increasing and concave, then the threshold remanufacturing cost c_r^0 that makes remanufacturing economically viable decreases as κ increases (i.e., as the consumer profile has a higher concentration in the low end of the market).*

Proof.

$$G(p^N) = p^N - \frac{1-F(p^N)}{F'(p^N)} = p^N - (1-p^N)/\kappa = \frac{(\kappa+1)p^N - 1}{\kappa}$$

$$c_r^0 = \eta(G^{-1}(c_n)) - \eta'(G^{-1}(c_n))(G^{-1}(c_n) - c_n) = \eta(\frac{\kappa c_n + 1}{\kappa+1}) - \eta'(\frac{\kappa c_n + 1}{\kappa+1})(\frac{1-c_n}{\kappa+1})$$

Taking derivative with respect to κ,

$$\frac{\delta c_r^0}{\delta \kappa} = \eta''(\frac{\kappa c_n + 1}{\kappa+1})\frac{(1-c_n)^2}{(\kappa+1)^3} < 0,$$

shows that the remanufacturing threshold decreases with κ. ∎

An analogous argument shows that when η is convex the threshold c_r^0 increases with κ.

To understand the magnitude of this threshold relative to the cost of new products, c_n, consider the following example.

Example:

Let $F(\theta) = 1 - (1-\theta)^\kappa$ and $\eta(\theta) = (1-\theta/m)\theta$ (see Figure 6). Then

$$c_r^0 = c_n - \frac{1}{m}\left[c_n^2 - \left(\frac{1-c_n}{\kappa+1}\right)^2\right]$$

If $c_r < c_r^0$, it's beneficial to offer both new and remanufactured products. Otherwise, the optimal solution is to offer new products only. Observe that $c_n < c_r^0$ if and only if $c_n^2 < \left(\frac{1-c_n}{\kappa+1}\right)^2$, that is, if and only if $c_n < \frac{1}{\kappa+2}$. Thus, when the cost of manufacturing the new product is below that value, it is optimal to offer remanufactured products even when the cost of remanufacturing is greater than the cost of producing new items.

As pointed out in Debo et al. (2005), while most of the previous operations literature on remanufacturing assumes that pure production cost savings $c_n - c_r$ drive remanufacturing activities (Klausner et al. (1998), Savaşkan et al. (2004) and Ferrer (1996)), the consumer profile given by F and η is key in assessing the value of remanufacturing. Furthermore, this example shows that remanufacturing can be economically attractive even when the marginal cost of remanufacturing *exceeds* that of producing a new unit. In this case, it is customer segmentation alone that drives the benefits of remanufacturing. The higher the κ (i.e. the higher the concentration of consumers in the low end of the market) the lower the production cost, c_n, required for this to happen. Finally note that the threshold on the production cost that makes $c_n < c_r^0$ does not depend on the parameter m of the relative utility function, but the magnitude of the allowed cost difference $c_r^0 - c_n$ does.

This finding leads to our next research question. Given that it may be optimal to offer relatively high cost remanufactured products to the market, should we sell new products as remanufactured when our supply of remanufacturable products is not sufficient to achieve the optimal levels? In the next section we study the market segmentation problem when new products can be used to substitute remanufactured products. The advantages of substituting a specific product by a higher level product have been previously studied in inventory control settings (Bassok et al. (1999) and Baymdir et al. (2003, 2005), and in the service sector (Netessine et al. (2000)).

4. Substitution

In this section we investigate the option of offering new products as substitutes for remanufactured ones when there is a limited supply of the latter. In practice, these could be products manufactured with a minimal, symbolic amount of used components; we will refer to them as new throughout this section. To retain the advantages achieved by segmenting the customer base, the substitute products will be labeled and sold as "remanufactured" and be no different from the genuinely remanufactured products in the customer eyes. Furthermore, such a practice could enhance the value consumers place in the remanufactured product and result in a further increase of the overall profits of the firm.

In the previous sections, we implicitly assume that there is ample supply of remanufacturable products. Let Q^R be the number of remanufacturable products available. If the supply of remanufacturable products is sufficient, the amount of remanufactured products sold is $F(\theta_\Delta^*) - F(\theta_r^*) < Q^R$ where θ_Δ^* and θ_r^* are the optimal solutions to Problem B. Otherwise, the amount of remanufactured products sold must be equal to Q^R and the profit maximization problem needs to incorporate the supply constraint $F(\theta_\Delta) - F(\theta_r) = Q^R$. In this case, the firm also has the option to offer some new products as substitutes for remanufactured products.

When the substitution strategy is implemented, all available remanufactured products will be sold and, in addition, some new products may be sold under the "remanufactured" label. The profit maximization problem for the firm, after the reformulation introduced in Section 2.2, can be written as

Problem S

$$\max_{\theta_\Delta, \theta_r} \ \gamma(\theta_\Delta)[1 - F(\theta_\Delta)] + [\eta(\theta_r) - c_n][1 - F(\theta_r)] + c_\Delta Q^R \tag{18}$$

$$s.t. \ \ F(\theta_\Delta) - F(\theta_r) \geq Q^R \tag{19}$$

$$0 \leq \theta_r \leq \theta_\Delta \leq 1$$

In Problem S, the term $c_\Delta Q^R$ is constant. We can drop it when seeking the optimal solution. Thus, the problem reduces to Problem B with $c_r = c_n$, and thus $c_\Delta = 0$, with the additional constraint (19). If the optimal solution to the corresponding Problem B, $(\theta_r^*, \theta_\Delta^*)$, satisfies the constraint, i.e., $F(\theta_\Delta^*) - F(\theta_r^*) > Q^R$, then it is optimal to offer $F(\theta_\Delta^*) - F(\theta_r^*) - Q^R$ new units as remanufactured to complement the available ones. Otherwise, no new units should be offered as remanufactured. Observe that Problem B, with $c_n = c_r$, dictates the total number $F(\theta_\Delta^*) - F(\theta_r^*)$ of products to offer under the "remanufactured" label, independently of how many remanufacturable products are available or the original remanufacturing cost. That number may very well be under Q^R (due to the higher production cost associated with the new but labeled "remanufactured" products), in which case Q^R remanufactured units are offered and no substitution of new units for remanufactured ones occurs.

The question of whether to offer new products as substitutes for remanufactured products is thus equivalent to the question of whether to offer both new and remanufactured products when the production costs are the same for new and remanufactured products. This can be decided by comparing c_n with the threshold c_r^0 derived in Section 3. Let c_n^0 denote the solution to the equation

$$c_n = c_r^0(c_n) = \eta(G^{-1}(c_n)) - \eta'(G^{-1}(c_n))(G^{-1}(c_n) - c_n) \tag{20}$$

Recall that $c_n - c_r^0$ is an increasing function of c_n as shown in the previous section. Consequently, we have that $c_n \le c_r^0$ if and only if $c_n \le c_n^0$. Thus, if $c_n < c_n^0$ then it is optimal to offer new products as substitutes for remanufactured ones. Otherwise, the firm should not offer new products as substitutes for remanufactured ones. Observe that c_r^0 is a function of c_n while c_n^0 is a constant value.

Let $r^*(c_n)$ be the total sales of remanufactured products when the unit cost of remanufacturing is c_n. Similarly, in the following discussion we emphasize the dependence of the optimal variables on c_n by explicitly writing them as functions of c_n, when necessary.

Theorem 3 *There is a threshold for the production cost of new products c_n^0 as given by the solution to Equation (20) such that if $c_n < c_n^0$, then it's optimal for the firm to offer new products as substitutes for remanufactured ones to bring the total of remanufactured products available to the* *market up to the optimal offering in a supply unconstrained problem with $c_r = c_n$; otherwise, it's optimal not to make use of the substitution option. That is, new products will be used as substitutes for remanufactured products if and only if $c_n < c_n^0$ and $Q^R < r^*(c_n)$; in that case, $r^*(c_n) - Q^R$ units will be substituted.*

Note again that whether or not to offer the new products as remanufactured depends on the number of genuine remanufactured products (with cost $c_r < c_n$) available. If that amount is beyond the optimal quantity of remanufactured products to offer if the remanufacturing cost were equal to c_n, then substitution will not take effect.

Theorem 1 shows that as the remanufacturing cost c_r decreases the company will reduce the number of new products offered and increase that of remanufactured ones. It is thus expected to result in reduced disposal volumes of end of life products. In the presence of profitable substitution, however, the optimal number of new units produced to be sold under both categories, new and remanufactured, increases as the number of available remanufacturable products decreases, resulting in an increase in production of new units of up to $(1 - F(\theta_r^*) - (1 - F(p^N))$ (positive since $\theta_r^* < p^N < \theta_\Delta^*$ is necessary for substitution to be viable, where recall that $(\theta_r^*, \theta_\Delta^*)$ is the optimal solution to Problem B with $c_r = c_n$). Consequently, the offering of remanufactured products could have the unexpected effect of increasing the production of new products due to substitution. Furthermore, Corollary 4 below shows that the lower the c_n, the larger the amount of substitution that will take place, thus increasing the volume of new products manufactured.

Using the first and last properties in Theorem 1 together with the fact that the solution to Problem S is identical to that of Problem B for $c_\Delta = 0$, we get the following result. For simplicity of exposition, we assume that there are no returned products available for remanufacture.

Corollary 4 *As $c_n < c_n^0$ increases, the optimal number of new units to offer to the market at price p_n^*, $1 - F(\theta_\Delta^*)$, and the price difference $p_n^* - p_r^*$ remain the same, while the optimal number of new units to sell as remanufactured decreases, the price charged for 'remanufactured' products p_r^* increases, and the optimal price for new products p_n^* increases.*

Corollary 4 shows that the number of new units to offer to the market does not change as c_n increases to c_n^0. Therefore, it must be equal to the number of new components to offer when the firm only carries new products and the cost of production is c_n^0. That is, when the firm offers remanufactured products with no cost advantage, the number of new units to offer at price $p_n^*(c_n)$ is $1-F(p^N(c_n^0))$, regardless of the production cost $c_n < c_n^0$.

In the next section we determine the functional form of the relative utility function that leads to the highest benefits associated with remanufacturing and, as a byproduct, with substitution. The computational section complements this analysis with the study of various other forms of the relative utility function.

5. Maximum Benefits from Customer Segmentation

In this section, we identify the form of the relative utility function $\eta(\theta)$ for the lower quality (remanufactured) product that leads to highest benefits from customer segmentation regardless of the form of the utility function for new products.

Proposition 2 *For given prices of new and remanufactured products, (p_n, p_r), $p_n > p_r$, the maximum profit obtained through market segmentation in Problem B is achieved when the relative utility function for remanufactured products is of the form:*

$$\eta^*(\theta) = \begin{cases} \theta, & \text{if } 0 \le \theta < p_r; \\ p_r, & \text{otherwise.} \end{cases} \tag{21}$$

Mathematically, the proposition shows that $\max_\eta \Pi(\gamma^{-1}(p_n - p_r), \eta^{-1}(p_r))$, where Π represents the objective function in Problem B, is achieved by the given function η^*.

Proof.

Since (p_n, p_r) are given, maximizing the objective in Problem B is equivalent to maximizing the overall market size, $1 - F(\theta_r)$, and the market size charged with the extra premium, $1 - F(\theta_\Delta)$. Consequently, our problem reduces to finding the function η that minimizes θ_r and θ_Δ, where recall that $\eta(\theta_r) = p_r$ and $\gamma(\theta_\Delta) = p_n - p_r$.

Observe that $\theta_r \geq \eta(\theta_r) = p_r$. Thus, the minimum feasible value of θ_r is p_r. This minimum is achieved for the given relative utility η^*.

In addition, observe that $\theta_\Delta \geq p_n$, since the customers with valuation higher than θ_Δ buy the higher premium product. Therefore, the minimum possible value of $\theta_\Delta = \gamma^{-1}(p_n - p_r)$ is p_n. The relative utility function η^* achieves this minimum value, since $\eta^*(p_n) = p_r$ and thus $\gamma^*(p_n) = p_n - \eta^*(p_n) = p_n - p_r$.

We have shown that the function η^* leads to the minimum possible values of θ_r and θ_Δ, and thus to the highest market sizes and profits associated with a pair of prices (p_n, p_r). ∎

As a result, an upper bound on the benefits of customer segmentation is given by the solution to the pricing problem using relative utility function η^*, which reduces to:

Problem UB

$$\max_{p_n \geq p_r} \quad (p_n - p_r - c_\Delta)(1 - F(p_n)) + (p_r - c_r)(1 - F(p_r)) \tag{22}$$

This is intuitive; the best case occurs when customers can be perfectly segmented. Under η^*, there is no cannibalization of high-premium consumers switching to the lower quality product and low-end consumers have an equal preference for both products. Our analysis shows that this naive upper bound is tight, that is, achievable when consumer preferences are captured by the relative utility function η^*.

The solution to Problem UB provides a benchmark to the profits possible through customer segmentation. If the utility function F for the currently offered product is well known, this benchmark can be used to make a first rough assessment of the value of considering to offer an alternative product geared to the low-end of the market. If the possible profits are sufficiently high, then market research is needed to estimate the relative utility function and get an accurate picture of the business case. The closer the shape of the actual relative utility is to that of η^*, the higher the profits to be expected from offering the second product.

6. Computational Study

One of the major contributions of this paper is to show that in order for customer segmentation alone to drive the profits of remanufacturing, under no cost differences, the relative utility function must be concave. In general, the threshold under which remanufacturing is viable is $c_r^0 = \eta(G^{-1}(c_n)) - \eta'(G^{-1}(c_n))(G^{-1}(c_n) - c_n)$, strongly dependent on the functions η and F (since G is uniquely linked to F), which highlights the important role played by the consumer profile. To further understand the impact of the consumer profile, namely the utility distribution function $F(\theta)$ and the relative utility function for remanufactured products $\eta(\theta)$, on the benefit of offering both new and remanufactured products and on the benefit of offering new products as substitutes for remanufactured products, we carry out an extensive computational study.

6.1 Impact of the Utility Distribution Function

We start by analyzing the impact of the utility distribution function $F(\theta)$ on the effectiveness of the substitution option. For this purpose, we consider utility distribution functions $F(\theta)$ of the form $F(\theta) = 1 - (1-\theta)^\kappa$ for varying κ, $\kappa = \{0.1, 0.2, \cdots, 3.9, 4.0\}$ (see Figure 3) and no available remanufactured products, i.e., $Q^R = 0$. This is equivalent to considering Problem B with $c_r = c_n$. In this case, all the "remanufactured" products sold are actually new products. This provides an upper bound on the benefit obtained from the substitution option since the maximum amount of new products would be offered as substitutes for remanufactured products when appropriate.

We first calculate the maximum possible profit improvement by solving Problem UB (see Section 5). Figure 4 illustrates the relative profit improvement, that is $\frac{\Pi^{UB} - \Pi^N}{\Pi^N} \times 100$, as a function of κ. The percent profit gain is identical for all values of the production cost c_n (thus omitted in the figure) and increases in a concave fashion as κ increases, that is, as consumers are more concentrated on the low-end of the market. The major take-away from this study is the fact that customer segmentation alone, under no cost differences in the two products, can result in up to a 40% profit improvement.

Let's now fix the relative utility function for remanufactured products to $\eta(\theta) = (1-\theta/3)\theta$. The production cost for new products is $c_n = \{0.05, 0.1, \cdots, 0.35, 0.4\}$. Recall that the largest c_n for

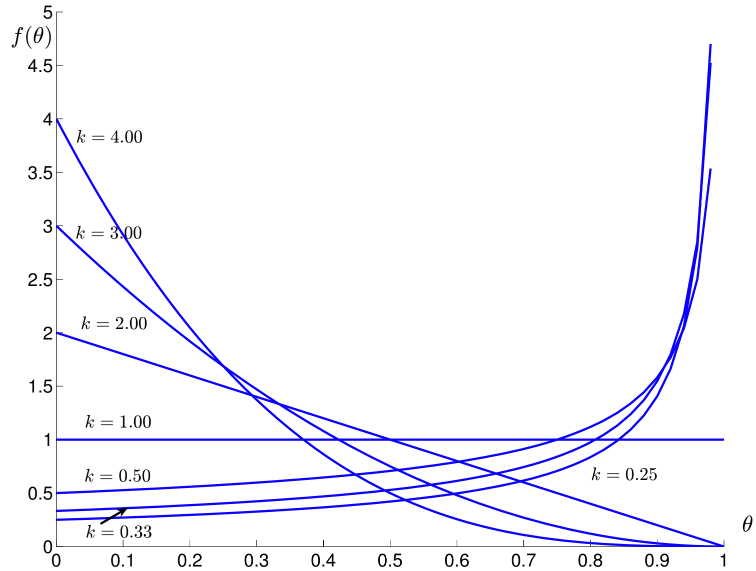

Figure 3 **Utility Density Function** $f(\theta)$ **(**$F(\theta) = 1 - (1-\theta)^{\kappa}$**)**

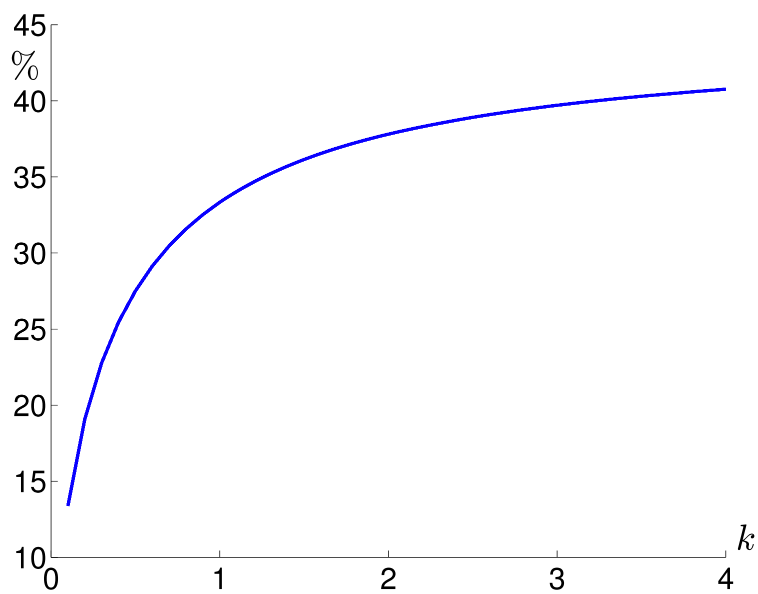

Figure 4 **Potential Relative Profit Gains through Customer Segmentation**

which substitution is profitable is $c_n^0 = 1/(\kappa + 2)$, as calculated in the example at the end of Section 3. Consequently, for most of the set $\kappa = \{0.1, 0.2, \cdots, 3.9, 4.0\}$, the substitution option is not viable beyond $c_n = 0.4$; the largest κ that makes the substitution option viable with $c_n = 0.4$ is $\kappa = 0.5$. Thus, we make $0.05 \leq c_n \leq 0.4$ so that we focus on comparing solutions for which the substitution option is viable.

Figure 5 illustrates the relative profit increase achieved through the substitution option. As

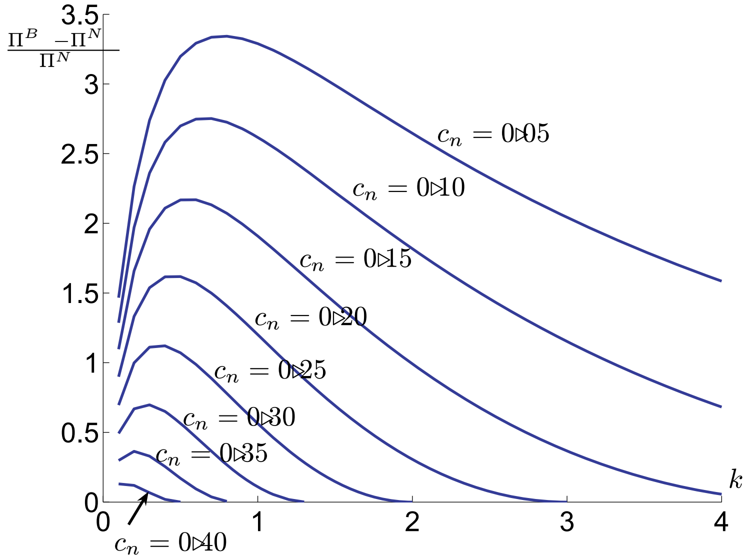

Figure 5 Relative Profit Increase as Customers are More Concentrated on the Low End

c_n increases, the relative profit improvement declines. This is consistent with intuition. As the production cost for new products increases, it becomes less advantageous to offer the substitution option because the profit margin of the "remanufactured" products decreases while their price increases making them affordable only to consumers that place a lower relative value on them. On the other hand, observe that the relative profit gain first increases, and then decreases, as κ increases. How much additional profit the substitution option brings in really depends on the ability to segment the market. We would like to reach out to the low-end customers by offering remanufactured products and at the same time extract a fat premium from high-end customers by keeping them purchasing new products. It is only when κ is at the medium level (when the utility function is close to uniform) that the market can be segmented adequately and thus it is in this case that the firm will reap the largest profit from the substitution option. These results reinforce the findings of Debo et al. (2005), where substitution is not allowed and the relative utility is linear. Please refer to the online Appendix for more details on the computational results.

6.2 Impact of the Relative Utility Function

To further investigate the impact of the consumer profile, and in particular the relative utility for remanufactured products, on the decision of whether or not to offer both new and remanufactured products, we study the solutions to Problem B and Problem N with a relative utility function of the form $\eta(\theta) = (1-\theta/m)\theta$ with $m = \{2,3,4,5\}$. Figure 6 shows the utility function for remanufactured products. As m increases, $\eta(\theta)$ approaches the utility for new products, θ. The difference in the ratio $\theta/\eta(\theta)$ for high-end versus low-end consumers decreases as m increases. The utility distribution function is set to $F(\theta) = 1-(1-\theta)^3$ and the unit production cost for new products to $c_n = 0.3$ for all the cases studied in this section. The unit production cost for remanufactured products is $c_r = \{0.01, 0.02, \cdots, 0.39, 0.4\}$. Figure 7 gives the relative profit improvement obtained through remanufacturing, $\frac{\Pi^B - \Pi^N}{\Pi^N} \times 100$. As expected, when the remanufacturing cost c_r increases, the gains from offering both products decrease. When the remanufacturing cost c_r is small, the larger the value of m, the higher the relative profit improvement. However, beyond a certain value of c_r, the lower values of m lead to higher relative profit improvements. (When comparing $m = 2$ and $m = 3$, this threshold value of c_r is out of the range [0.01, 0.4]). The intuition behind this behavior is as follows. When c_r is small, the major driver for remanufacturing profits is the lower cost of the remanufactured products; the higher the m, the higher the value consumers place on the remanufactured product and the firm can charge a higher price for these products. When c_r is large, on the other hand, customer segmentation becomes the driver of the profit gains. Since lower values of m represent a more disparate relative valuation of the remanufactured product between low and high end consumers, the firm can segment the market more effectively. This results in a proportionally higher profit. Observe that in Figure 7, the curves for each m start at different levels of c_r because the requirement $\gamma(1) > c_n - c_r$ is imposed to prevent the optimality of offering solely remanufactured products. We refer the reader to the online Appendix for further details on the market sizes and prices for new and remanufactured products in the optimal solutions, and for the study of other relative utility functions.

7. Conclusions and Extensions

In this paper we have characterized, under mild conditions, a threshold for the remanufacturing cost under which offering remanufacturing products is profitable, and focused on analyzing its

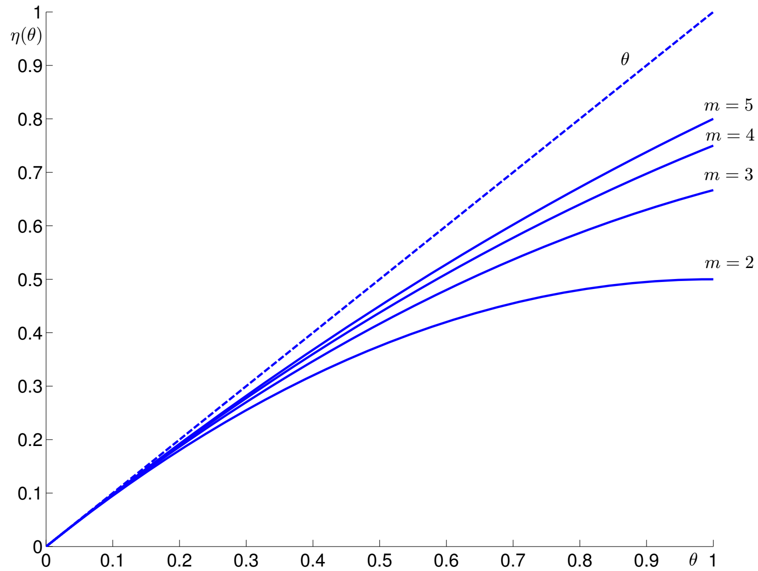

Figure 6 **Utility Function for Remanufactured Products** $\eta(\theta) = (1 - \theta/m)\theta$

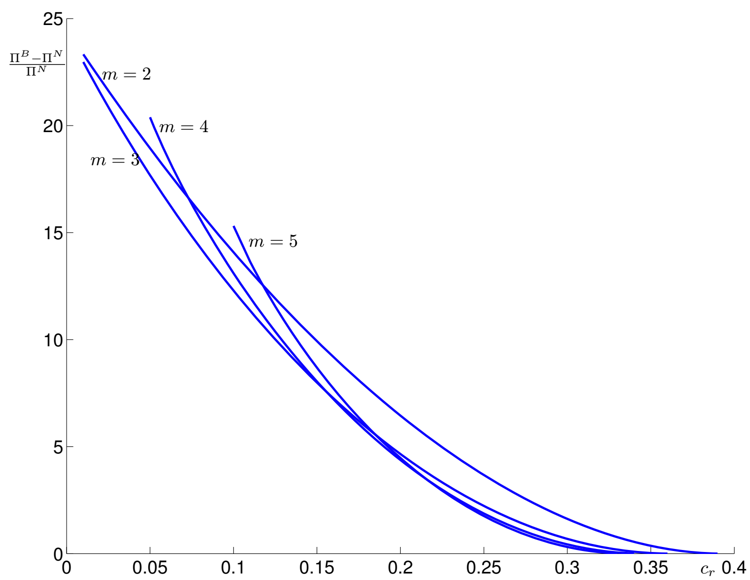

Figure 7 **Relative Profit Increase as a Function of Remanufacturing Cost**

dependence on the consumer profile. In particular, we show that it is viable to offer remanufactured products even when their production cost is higher than that of new products in some situations. While this can never happen when the consumer profile is linear or convex, it leads to a significant increase in profits for intuitively plausible concave profiles. Furthermore, we identify the form of the relative utility function that leads to highest profits from customer segmentation and thus provides a tight upper bound on the potential profits (as high as 40%) that can be reaped through the substitution option.

These results also apply to the market segmentation problem faced by retailers offering merely identical products under different brand labels. This is the case, for instance, of supermarkets offering virtually the same product under the national name brand and their own supermarket brand. Our analysis shows how this strategy may make sense even if the cost to acquire the product with their label on it is higher than that of acquiring it under the national name brand. Similarly, it applies to the problem arising in pharmaceutical and other companies considering whether or not to offer a generic version of a successful drug or product.

In all these applications, it would be important to extend the analysis to competitive settings. Competition in remanufacturing operations has been studied in Majumder and Groenevelt (2001), Debo et al. (2005), Ferrer and Swaminathan. The impact of the shape of the relative utility function, however, has not been addressed. As our current paper shows, it is a critical factor and thus its effect in competitive settings needs to be understood.

Some of our results have important policy implications. When substitution of new for remanufactured products is profitable and the availability of remanufacturable products limited, the offering of remanufactured products leads to an increase in the total production of new products (see Theorem 3). The lower the production cost, the larger the increase. This effect suggests that policies aimed at reducing disposal volume by promoting the sell of remanufactured products or providing subsidies to reduce remanufacturing costs may have a negative environmental impact unless the stream of returned products is sufficiently large. See also Baker et al. (2006) where we show that increasing the information available on the quality of the returned product, through information acquisition technologies such as data-logging devices or RFID, can have the undesired effect of reducing the fraction of units that will be remanufactured. For their joint market segmentation and technology selection setting, Debo et al. (2005) show that new product sales may either increase or decrease as the remanufacturing cost is reduced, also pointing to the potentially negative effect of remanufacturing subsidies. In conclusion, new policy should focus mainly on facilitating product recovery operations to guarantee an ample supply of remanufacturable products, rather than subsidizing the actual remanufacturing step.

The most notable extension to make our models more accurate representations of reality is the consideration of the heterogeneity in customer tastes for remanufactured products within those that value the new product equally. That is, $\eta(\theta)$ should be a distribution and not a single value. Many other extensions are also worthwhile. In particular, considering multiple periods in a finite or infinite horizon setting and the interaction between sales in one period and availability of remanufacturable products in future periods is important. The excellent analysis of the joint determination of the investment in product remanufacturability and the prices over an infinite horizon, in Debo et al. (2005) could be extended in view of our results to determine the impact of the remanufactured product valuation η and to identify when new products should be offered as "remanufactured." Ferrer and Swaminathan (2006) consider finite and infinite horizon models, in both monopoly and competitive settings, but they assume that lower-cost remanufactured products are sold at the same price as new products; the major decision then is the quantity of each to offer at a single price.

Finally, this paper highlights the need for marketing research on consumer preferences in order to identify the form of the willingness to pay or utility functions for different industries.

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Appendix A: Behavior of p_n vs. c_r

In this section we explore how the optimal price at which to offer new products, p_n, changes as the unit cost of remanufacturing, c_r, increases. From Theorem 1, we know that increasing c_r results in an increase in the optimal price for remanufactured products (p_r) and the amount of new products sold, and a decrease in the difference in prices ($p_n - p_r$), the amount of remanufactured products sold and the total market size. The effect on the optimal price for new products, however, is not clear. Here we show that it depends on the specific consumer profile (F, η). Furthermore, we prove that if F is of the form $F(\theta) = 1-(1-\theta)^k$ then, as c_r increases, the optimal price for new products decreases when η is concave, increases when η is convex, and remains constant for a linear consumer profile.

This result also allows us to determine how the price of new products is affected by the offering of the remanufactured product. As shown in the paper, when c_r hits the boundary c_r^0, the remanufactured product is no longer profitable and hence, the problem reduces to the case where only new products are offered; that is, $p_n = p^N$ for $c_r = c_r^0$ and p_n converges to p^N as $c_r < c_r^0$ increases. Thus, for $F(\theta) = 1-(1-\theta)^k$ we have that for $c_r < c_r^0$:

1. if $\eta(\theta)$ is concave, p_n decreases to p^N, and thus $p_n \geq p^N$,
2. if $\eta(\theta)$ is convex, p_n increases to p^N, and thus $p_n \leq p^N$.

When the consumer profile is linear the price for the new products is always identical to that charged when remanufactured products are not offered, i.e., $p_n = p^N$.

In the following sections, we assume all first and second order derivatives exist.

A.1 General $F(\theta)$ and $\eta(\theta)$

At the point $(\theta_\Delta^*, \theta_r^*)$ of maximum firm profit, the first order and second order optimality conditions must be satisfied. That is,

$$\begin{cases} \gamma'(\theta_\Delta^*)\,[1-F(\theta_\Delta^*)] - [\gamma(\theta_\Delta^*) - c_\Delta]\,F'(\theta_\Delta^*) = 0 \\ \eta'(\theta_r^*)\,[1-F(\theta_r^*)] - [\eta(\theta_r^*) - c_r]\,F'(\theta_r^*) = 0 \end{cases} \tag{23}$$

and

$$\begin{cases} \gamma''(\theta_\Delta^*)\,[1-F(\theta_\Delta^*)] - 2\gamma'(\theta_\Delta^*)F'(\theta_\Delta^*) - [\gamma(\theta_\Delta^*) - c_\Delta]\,F''(\theta_\Delta^*) \leq 0 \\ \eta''(\theta_r^*)\,[1-F(\theta_r^*)] - 2\eta'(\theta_r^*)F'(\theta_r^*) - [\eta(\theta_r^*) - c_r]\,F''(\theta_r^*) \leq 0 \end{cases} \tag{24}$$

Taking the first order derivative of Eq. (23) with regard to c_r and rearranging the terms, we have

$$\begin{cases} \frac{\partial \theta_\Delta^*}{\partial c_r} = \frac{F'(\theta_\Delta^*)}{\gamma''(\theta_\Delta^*)\left[1-F(\theta_\Delta^*)\right] - 2\gamma'(\theta_\Delta^*)F'(\theta_\Delta^*) - \left[\gamma(\theta_\Delta^*) - c_\Delta\right]F''(\theta_\Delta^*)} \\ \frac{\partial \theta_r^*}{\partial c_r} = \frac{-F'(\theta_r^*)}{\eta''(\theta_r^*)[1-F(\theta_r^*)] - 2\eta'(\theta_r^*)F'(\theta_r^*) - [\eta(\theta_r^*) - c_r]F''(\theta_r^*)} \end{cases} \tag{25}$$

Therefore,

$$\begin{aligned}\frac{\partial p_n}{\partial c_r} &= \frac{\partial[\eta(\theta_r^*)+\gamma(\theta_\Delta^*)]}{\partial c_r}\\ &= \eta'(\theta_r^*)\frac{\partial\theta_r^*}{\partial c_r}+\gamma'(\theta_\Delta^*)\frac{\partial\theta_\Delta^*}{\partial c_r}\\ &= \frac{1}{\{\gamma''(\theta_\Delta^*)\left[1-F(\theta_\Delta^*)\right]-2\gamma'(\theta_\Delta^*)F'(\theta_\Delta^*)-\left[\gamma(\theta_\Delta^*)-c_\Delta\right]F''(\theta_\Delta^*)\}}\\ &\times \frac{1}{\{\eta''(\theta_r^*)\left[1-F(\theta_r^*)\right]-2\eta'(\theta_r^*)F'(\theta_r^*)-\left[\eta(\theta_r^*)-c_r\right]F''(\theta_r^*)\}}\\ &\times \left\{\frac{1-F(\theta_r^*)}{F'(\theta_r^*)}\left[\frac{\eta''(\theta_r^*)}{\eta'(\theta_r^*)}-\frac{F''(\theta_r^*)}{F'(\theta_r^*)}\right]-\frac{1-F(\theta_\Delta^*)}{F'(\theta_\Delta^*)}\left[\frac{\gamma''(\theta_\Delta^*)}{\gamma'(\theta_\Delta^*)}-\frac{F''(\theta_\Delta^*)}{F'(\theta_\Delta^*)}\right]\right\}\end{aligned} \tag{26}$$

From Eq. (24), we know that the product of the first two terms in Eq. (26) is positive and hence, the sign depends solely on the third term, which in turn depends on the local information of F and η at the optimal solution.

Observe that at the point $c_r = c_r^0$, we have that $\eta(\theta_r^*) = \eta(\theta_\Delta^*) = p^N$ and the third term in equation (26) simplifies to:

$$\left\{\frac{1-F(p^N)}{F'}\left[\frac{\eta''(p^N)}{\eta'(p^N)}-\frac{\gamma''(p^N)}{\gamma'(p^N)}\right]\right\} \tag{27}$$

Recall that $\gamma'' = -\eta''$ and that both γ and η are increasing. Thus, at $c_r = c_r^0$, the derivative is positive for η concave and negative for η convex. Consequently, in the neighborhood of c_r^0, that is, for $c_r < c_r^0$ sufficiently close to c_r^0, we have that:

- if η is concave, p_n decreases in c_r and thus $p_n \geq p^N$,
- if η is convex, p_n increases in c_r and thus $p_n \leq p^N$,
- if η is linear, p_n does not depend on c_r and thus $p_n = p^N$.

We are now ready to show that this local behavior of p_n with respect to c_r, valid in the neighborhood of c_r^0, holds globally for the consumer utility function F_κ used to illustrate our results.

A.2 Arbitrary $\eta(\theta)$ with $F(\theta) = 1-(1-\theta)^k$

For this specific form of $F(\theta)$, we have

$$\begin{cases}\frac{1-F(\theta_\Delta^*)}{F'(\theta_\Delta^*)} = \frac{1-\theta_\Delta^*}{k}\\ \frac{1-F(\theta_r^*)}{F'(\theta_r^*)} = \frac{1-\theta_r^*}{k}\end{cases} \tag{28}$$

and Eq. (26) reduces to

$$\begin{aligned}\frac{1}{k}\frac{\partial p_n}{\partial c_r} &= \frac{\frac{\eta''(\theta_r^*)(1-\theta_r^*)}{\eta'(\theta_r^*)}-\frac{\gamma''(\theta_\Delta^*)(1-\theta_\Delta^*)}{\gamma'(\theta_\Delta^*)}}{\left[\frac{\gamma''(\theta_\Delta^*)(1-\theta_\Delta^*)}{\gamma'(\theta_\Delta^*)}-(k+1)\right]\left[\frac{\eta''(\theta_r^*)(1-\theta_r^*)}{\eta'(\theta_r^*)}-(k+1)\right]}\\ &= \frac{\frac{\eta''(\theta_r^*)(1-\theta_r^*)}{\eta'(\theta_r^*)}+\frac{\eta''(\theta_\Delta^*)(1-\theta_\Delta^*)}{\gamma'(\theta_\Delta^*)}}{\left[\frac{\gamma''(\theta_\Delta^*)(1-\theta_\Delta^*)}{\gamma'(\theta_\Delta^*)}-(k+1)\right]\left[\frac{\eta''(\theta_r^*)(1-\theta_r^*)}{\eta'(\theta_r^*)}-(k+1)\right]}\end{aligned} \tag{29}$$

(Note the fact $\gamma''(\theta) = -\eta''(\theta)$)

The denominator is actually the product of the RHS of the two equations in Eq. (24), and hence, always positive. Therefore, it is clear that

1. If $\eta(\theta)$ is concave, p_n decreases as c_r increases.
2. If $\eta(\theta)$ is convex, p_n increases as c_r increases.
3. If $\eta(\theta)$ is linear, p_n does not change as c_r increases.

Appendix B: Further Computational Study

Here we provide additional details on our computational study.

B.1 Impact of the Utility Distribution Function

We start by providing more detail on the results presented in Section 6.1. Figure 8 and Figure 9 show the amount of customers who purchase new and remanufactured products respectively in the optimal solution for the given relative utility function, $\eta(\theta) = (1 - \theta/3)\theta$. Figure 10 and Figure 11 present the corresponding optimal product prices, illustrating the behavior described in Corollary 4. In Figure 8, given the utility function $F_\kappa(\theta)$, the volume of customers who purchase the new products remains the same as c_n increases, as shown in Corollary 4. Observe nonetheless that for the larger c_n, substitution will only be viable when the utility function F_k represents consumers highly concentrated on the high end of the market, i.e., for low values of κ. The curve for each c_n starts at the same point, $\kappa = 0.1$, but ends at the circle indicated with its corresponding c_n value. When κ is small, most of the customers are concentrated in the high end of the utility range and value new products highly. If the "remanufactured" product is offered, there is not much of a market for it and it will erode some of the higher-profit new product sales. As a result, sales barely increase and the relative profit gained from the substitution option is small. On the other hand, when κ is large, most of the customers are concentrated in the low end of the utility range, which means most of them place low value on the new products and have a relatively higher outlook for the remanufactured ones. If the "remanufactured" product is offered, the market size will increase at the cost of having a good portion of high-premium customers switch to the lower-margin product. The firm fails to seize the higher profit margins for the new product. The relative profit gained from the substitution option is not large in this case either. For medium values of κ, the utility function is close to uniform reflecting a very diverse market. The market can be segmented more adequately and thus higher benefits will be obtained from the substitution option.

Utility distribution functions of the form $F(\theta) = 1 - (1 - \theta)^\kappa$ represent situations where customer utilities of new products are concentrated at either the high or the low ends of the market, or equally distributed across the utility spectrum. To further investigate the impact of the utility distribution, we consider a new form of the utility distribution where customer utilities for new products are concentrated on both ends of the market: $F(\theta) = (1 - b/3)\theta - b(1 - 2\theta)^3/6 + b/6$ where $b = \{1/3, 1/2, 1, 4/3, 3/2\}$. Figure 12 shows the density function $f(\theta)$ of these utility distributions. As b increases, the distribution becomes more polarized. The distribution function of this form does not have an increasing hazard rate. But we can still derive the

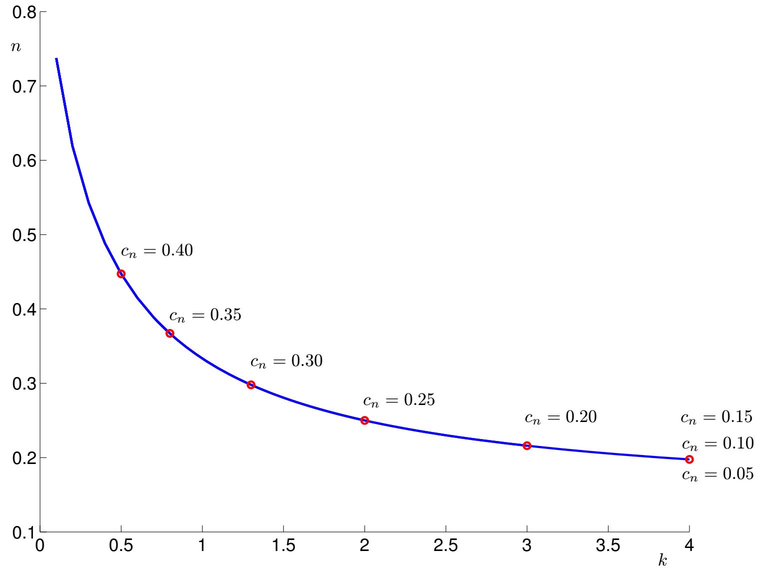

Figure 8 Proportion of customers that purchase new products, where for each κ the number is the same regardless of the value of c_n and each circle indicates the maximum point beyond which substituting is no long profitable for the c_n listed besides

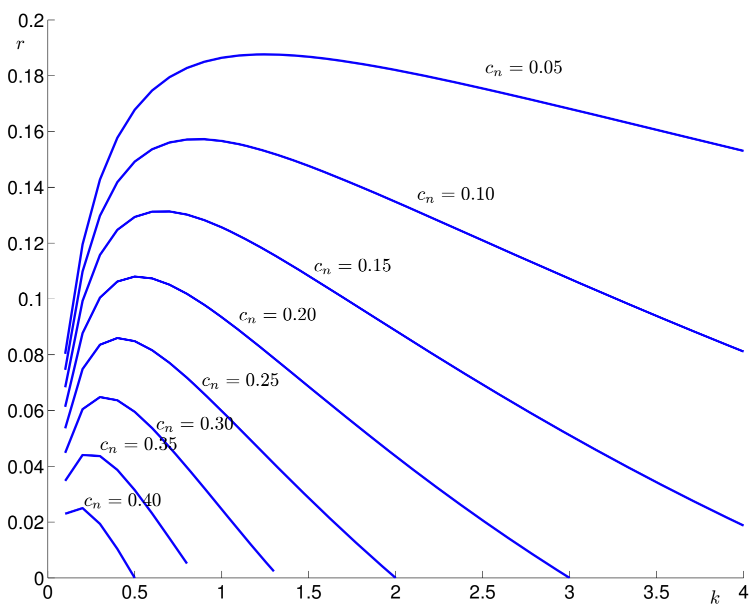

Figure 9 Proportion of customers that purchase remanufactured products

optimal solutions based on the first-order conditions. All the other parameters are kept the same. Figure 13 illustrates the benefits from the substitution option. The x axis is the parameter b in the utility distribution function $F(\theta)$. The y axis is relative profit improvement as defined above. Once again, the relative profit gains decrease as c_n increases. Moreover, this relative profit gains tend to decrease as b increases for a relatively

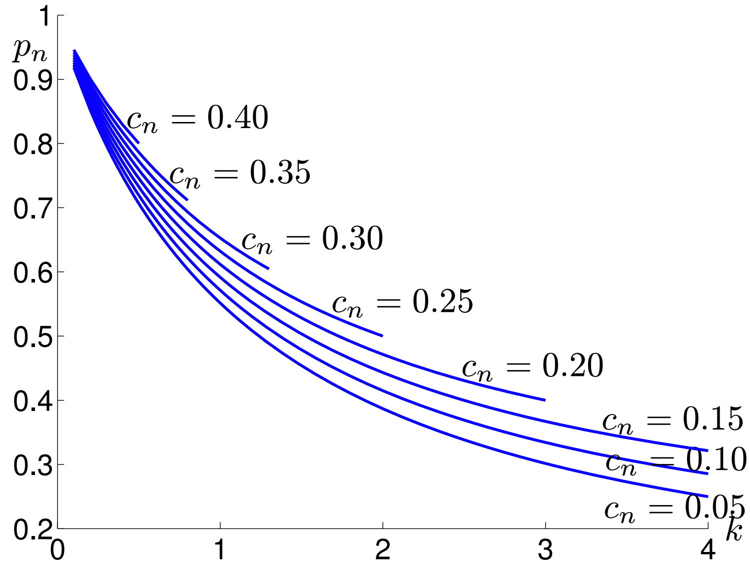

Figure 10 **New product price as a function of κ and c_n**

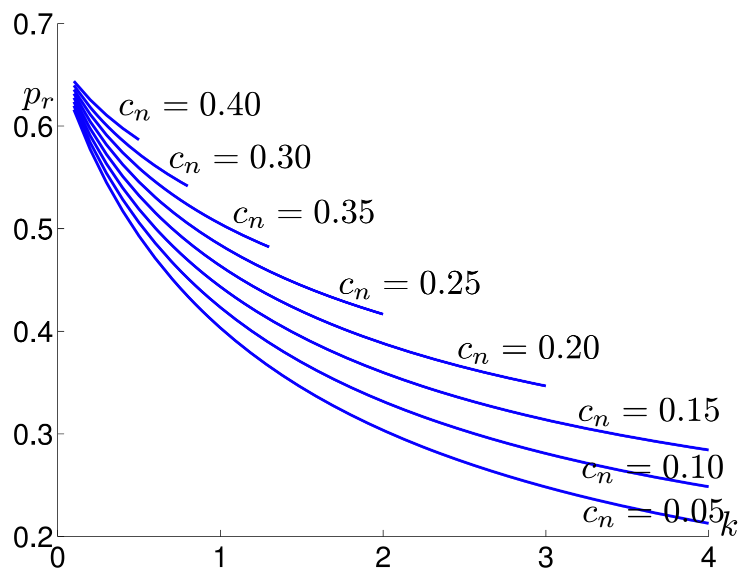

Figure 11 **Remanufactured product price as a function of κ and c_n**

low production cost c_n because with the more evenly spread product valuations for low b the remanufactured product will capture a larger portion of the market. Volume is the driver here. As the utility distribution becomes more polarized, however, more customers are concentrated on either the low or high end of the market. This is useful when c_n is high because the larger concentration of high-end, little price-sensitive consumers allows for better segmenting of the high end of the market with steeper premiums. The prices of new and remanufactured products are higher than 0.42 in all the cases, and go up as b increases.

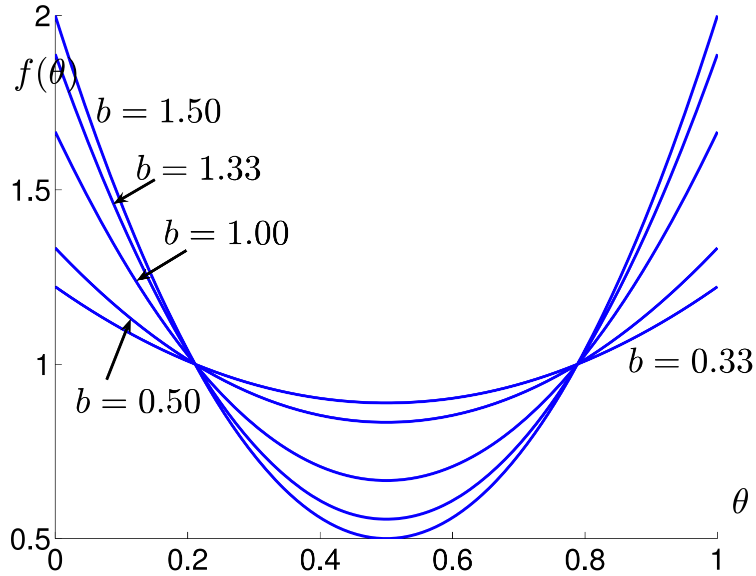

Figure 12 **Utility Density Function** $f(\theta)$ **(**$F(\theta) = (1 - b/3)\theta - b(1 - 2\theta)^3/6 + b/6$**)**

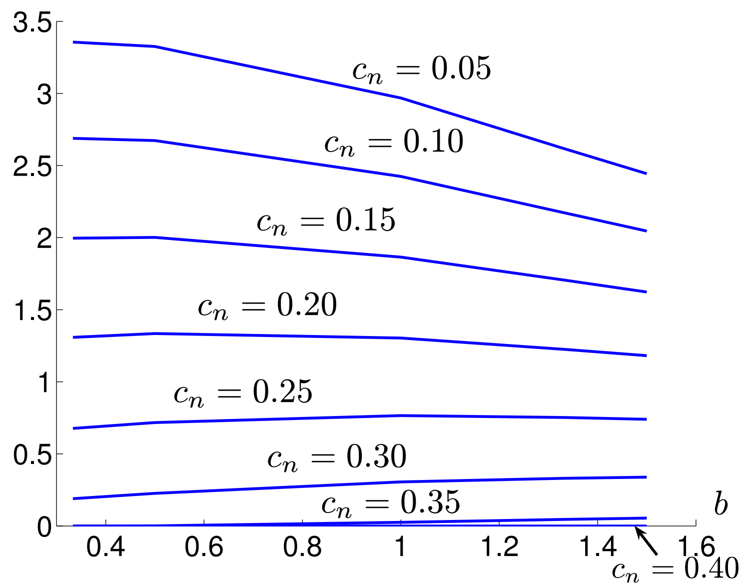

Figure 13 **Relative Profit Improvement (**$Q^R = 0$**)**

B.2 Impact of the Relative Utility Function

For the cases run in Section 6.2, Figure 14, Figure 15, Figure 16, and Figure 17, provide the optimal market sizes and prices for new and remanufactured products in the optimal solutions.

In the relative utility functions, $\eta(\theta) = (1 - \theta/m)\theta$, studied thus far, the relative utility for remanufactured products is very close to the utility for new products for small values of θ; that is, the ratio of the utility of remanufactured products over the utility of new products, $\eta(\theta)/\theta$, is very close to 1 in this case, suggesting

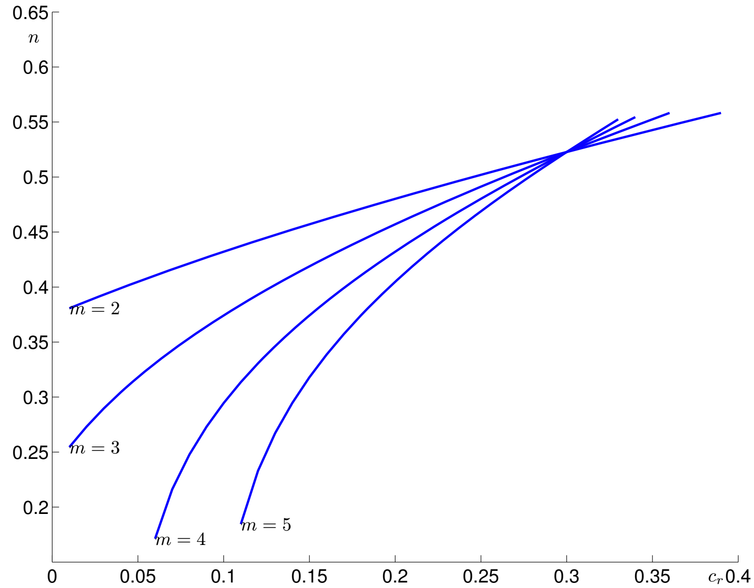

Figure 14 **Proportion of customers that purchase new products**

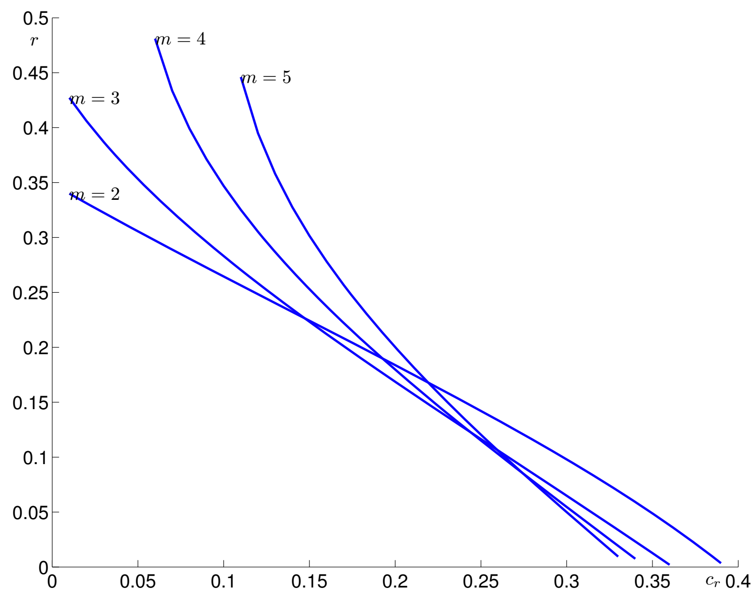

Figure 15 **Proportion of customers that purchase remanufactured products**

that low-end customers place almost the same values for new and remanufactured products. In Section 5 we saw that this was a desirable characteristic of the relative utility. Customers, however, may not behave this way; they may value the new and remanufactured products pretty differently over the entire range. To study the impact of such consumer behavior, we test our model on this form of the relative utility: $\eta(\theta) = a(1-\theta/3)\theta$ where $a = \{0.55, 0.65, \cdots, 1.00\}$. Observe that with these functions the ratio of remanufactured to new product utilities, $\eta(\theta)/\theta$, is always kept below the value a. Figure 18 shows this utility function for

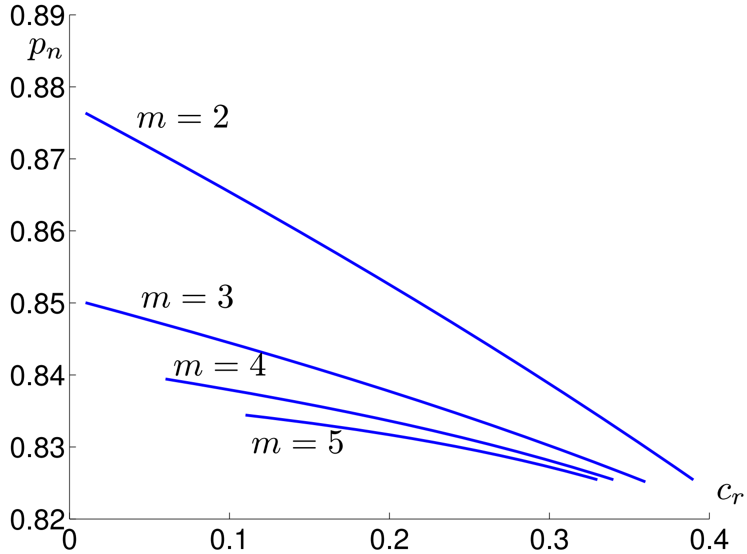

Figure 16 **Price of new products**

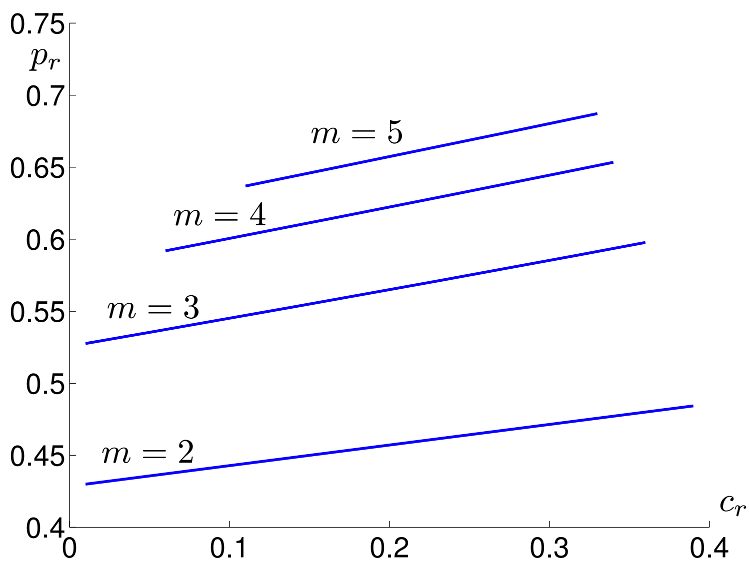

Figure 17 **Price of remanufactured products**

remanufactured products. As a increases, $\eta(\theta)$ becomes closer to the utility for new products, θ. All the other parameters are kept the same. Figure 19 gives the relative profit improvement as a varies. The profit gains increase as a increases, since customers place a higher utility value on the remanufactured product, and they decrease at a similar rate for all a as the remanufacturing cost c_r increases. The threshold cost at which remanufacturing stops being viable thus decreases as a decreases, making the substitution strategy unprofitable for all except the three highest levels of a.

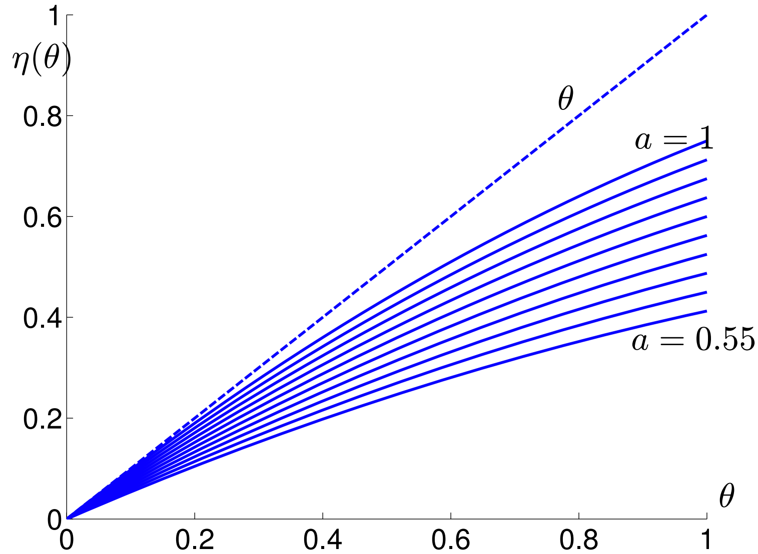

Figure 18 Utility Function for Remanufactured Products $\eta(\theta) = a(1 - \theta/3)\theta$

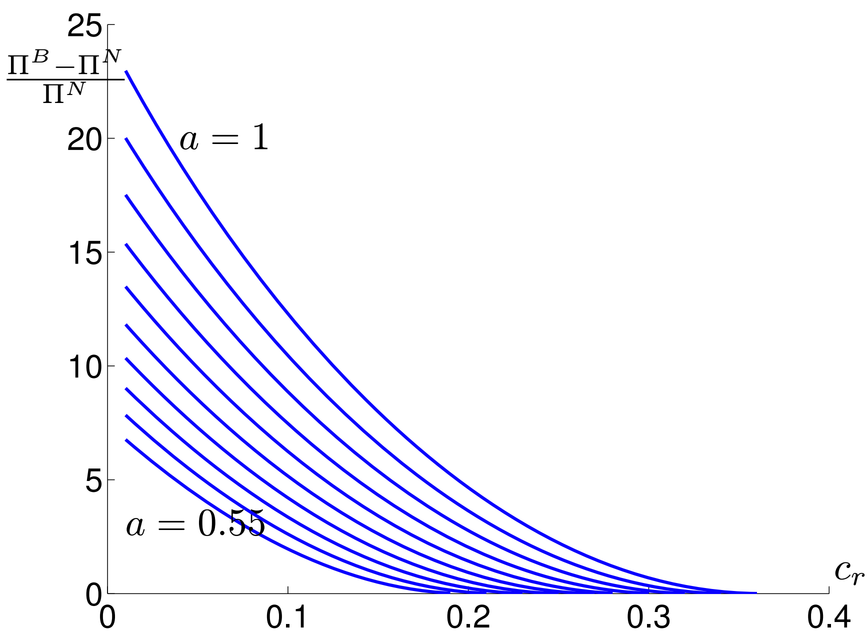

Figure 19 Relative Profit Improvement