Correction to "Feedback Strategies for White Gaussian Interference Networks", and a Capacity Theorem for Gaussian Interference Channels with Feedback

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In the above paper [1], the following corrections are necessary.

1) On p. 1426, the second inequality of equation (16) should have no "x" on the right hand side.

2) On p. 1427, the expression immediately before equation (22) should read $\sigma_{kn}^2 = \mathrm{E}[|\epsilon_{kn}|^2]$.

3) There is a recurring error in Appendix E. On p. 1436, the upper bound on R_2 immediately before (89) should have a γ_2 rather than a γ_1 in the denominator. Equation (89) should read

$$0 \le R_k \le \frac{1}{2}\log\left(1 + \frac{P\,\alpha_k\left(\sum_{\ell=k}^{K}|\gamma_\ell|^2\right)}{1 + P\left(\sum_{\ell=1}^{k-1}\alpha_\ell\right)\left(\sum_{\ell=k}^{K}|\gamma_\ell|^2\right)}\right). \tag{89*}$$

The two equations after (89) should read, respectively,

$$P\,\alpha_1\sum_{\ell=1}^{K}|\gamma_\ell|^2 = \frac{P\,\alpha_k\left(\sum_{\ell=k}^{K}|\gamma_\ell|^2\right)}{1 + P\left(\sum_{\ell=1}^{k-1}\alpha_\ell\right)\left(\sum_{\ell=k}^{K}|\gamma_\ell|^2\right)}$$

$$\alpha_k = \alpha_1\frac{\sum_{\ell=1}^{K}|\gamma_\ell|^2}{\sum_{\ell=k}^{K}|\gamma_\ell|^2}\left[1 + P\left(\sum_{\ell=1}^{k-1}\alpha_\ell\right)\left(\sum_{\ell=k}^{K}|\gamma_\ell|^2\right)\right].$$

These corrections might have affected only the curve labeled "outer bound (Appendix E)" in Fig. 4 (see the end of Section VI-A). Fortunately, this curve was computed using the above expressions rather than those in [1].

4) Finally, equation (77) on p. 1432 should read

$$\widetilde{\rho}_{n+1} = -\frac{V_{Y_{1n}}}{V_{Y_{1n}|X_{1n}}}\left[\widetilde{\rho}_n - 2\frac{P(1+\gamma\widetilde{\rho}_n)(\gamma+\widetilde{\rho}_n)}{V_{Y_{1n}}} + \frac{P^2(1+\gamma\widetilde{\rho}_n)^2(2\gamma+\widetilde{\rho}_n+\gamma^2\widetilde{\rho}_n)}{V_{Y_{1n}}^2}\right]. \tag{77*}$$

Equation (77*) follows from [1, eq. (46)], and we are grateful to Liu Jialing of Iowa State University for making us aware of our mistake. The error might have affected the curve labeled "achievable" in Fig. 6, and therefore the discussion following (77) in Section VI-B. Fortunately, this curve was computed correctly using (46), and hence (77*), rather than (77).

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Moreover, the correction of (77) lets us solve a problem left open in [1]. As pointed out at the end of Section VI-B on p. 1433, the "outer bound" and "achievable" curves in Fig. 6 seem to touch at one point. We prove the following theorem.

Theorem: Consider a (real alphabet) interference channel with outputs

$$Y_1 = X_1 + \gamma X_2 + Z_1$$
$$Y_2 = \gamma X_1 + X_2 + Z_2$$

where Z_1 and Z_2 are independent, unit variance, Gaussian random variables, $|\gamma| < 1$, $\mathrm{E}[X_k^2] \le P$ for $k = 1, 2$, and

$$P = \frac{2}{(1-\gamma^2)^2}.$$

The feedback sum-rate capacity of this channel is

$$R_1 + R_2 = \log\left(\frac{3+\gamma^2}{1-\gamma^2}\right).$$

Proof: Consider the two upper bounds of (16) on p. 1426. The choice $\rho = \gamma$ maximizes the second bound, and it makes both bounds become

$$R_1 + R_2 \le \log\left(\frac{3+\gamma^2}{1-\gamma^2}\right).$$

Consider next (77*) with $\widetilde{\rho}_n = \gamma$. The result is $\widetilde{\rho}_{n+1} = \gamma$, so we are in a steady state with (see [1, eq. (29)])

$$R_1 + R_2 = \log\left(\frac{V_{Y_{1n}}}{V_{Y_{1n}|X_{1n}}}\right) = \log\left(\frac{3+\gamma^2}{1-\gamma^2}\right).$$

It remains to verify that we can get into this steady state. We use the approach described in Appendix C on p. 1435 for multiaccess channels, i.e., we choose the variance of the first input symbols of both terminals to be

$$P_1 = \frac{1}{1-\gamma^2}.$$

Note that we have $P_1 \le P$. Inserting $\widetilde{\rho}_1 = 0$ into (77*), and replacing P with P_1, we obtain the desired $\widetilde{\rho}_2 = \gamma$. ■

A final remark: as far as we know, the above is the first capacity theorem for Gaussian interference channels with $P > 0$, $0 < |\gamma| < 1$, and $R_k < \log(1+P)/2$ for $k = 1, 2$.

REFERENCES

[1] G. Kramer, *IEEE Trans. Inform. Theory*, vol. 48, pp. 1423–1438, June 2002.