COMMUNICATION STRATEGIES AND CODING FOR RELAYING*

GERHARD KRAMER†

Abstract. Information-theoretic models for relaying are developed and applied to wireline and wireless networks. For wireline networks with node constraints, data compression is shown to improve rates. For wireless networks with half-duplex constraints, decode-and-forward strategies are developed that can give substantial rate gains over no-relay transmission and traditional multihopping. The performance of the strategies is verified by designing coded modulations that approach certain information-theoretic limits. The codes used are irregular low-density parity-check (LDPC) codes.

Key words. Relay channel, capacity, code, half-duplex, low-density parity-check code, network.

AMS(MOS) subject classifications. 94A05, 94A15, 94A24

1. Introduction. Information theory usually models a communication *channel* by a conditional probability distribution. For example, a model for communicating a symbol from one point to another might involve the conditional probability distribution $P_{Y|X}(\cdot)$ that evaluates to

$$P_{Y|X}(b|a), \quad a \in \mathcal{X},\, b \in \mathcal{Y} \tag{1.1}$$

where X and Y are random variables taking on values in the discrete and finite alphabets $\mathcal{X}$ and $\mathcal{Y}$, respectively. The aim of communication is to transmit reliably a *message* index W taking on one of M values from a source to a sink. Suppose that to accomplish this task one transmits a string of n symbols $X^n = X_1, X_2, \ldots, X_n$ over the channel. The *rate* of communication is then

$$R = \log_2(M)/n \tag{1.2}$$

bits per channel use. The maximum rate C at which one can transmit reliably is called the *capacity* of the channel.

A *relay channel* is a multiterminal channel with three parties or nodes: a source (node 1), a relay (node 2), and a sink (node 3). A possible model for relaying might involve the probabilities (see [6, 20])

$$P_{Y_2Y_3|X_1X_2}(b_2, b_3|a_1, a_2) \tag{1.3}$$

where X_1 is the source's channel input, Y_3 is the sink's channel output, and X_2 and Y_2 are the relay's input and output, respectively. The idea is that

*Paper presented at the Institute for Mathematics and its Applications (IMA) summer program on Wireless Communication, University of Minnesota, Minneapolis, MN, June 22-July 1, 2005.

†Communications and Statistical Sciences Department, Bell Labs, Lucent Technologies, Murray Hill, NJ 07974.

the source and sink can only transmit and receive, respectively, but the relay can both transmit and receive. Suppose that the source and relay transmit the strings $X_1^n = X_{11}, X_{12}, \ldots, X_{1n}$ and $X_2^n = X_{21}, X_{22}, \ldots, X_{2n}$, respectively, over the channel. Suppose further that the relay can react quickly so that its input X_{2i} can be any function of its *past* outputs Y_2^{i-1}. The relay channel is said to be *memoryless* if one has (see [12])

$$\begin{aligned} &P_{Y_{2i}Y_{3i}|WX_1^iX_2^iY_2^{i-1}Y_3^{i-1}}(b_{2i}, b_{3i}|w, a_1^i, a_2^i, b_2^{i-1}, b_3^{i-1}) \\ &= P_{Y_2Y_3|X_1X_2}(b_{2i}, b_{3i}|a_{1i}, a_{2i}) \end{aligned} \tag{1.4}$$

for all a_1^i, a_2^i, b_2^i, b_3^i, and $i = 1, 2, \ldots, n$. We will consider only memoryless channels. Again, the maximum rate C at which one can transmit reliably is called the *capacity* of the channel.

A *relay network* is a generalization of a relay channel to a system with T nodes: a source (node 1), $T-2$ relays (nodes 2 to $T-1$), and a sink (node T). A model for relaying would involve the probabilities (see [12, 20])

$$P_{Y_2Y_3\ldots Y_T|X_1X_2\ldots X_{T-1}}(b_2, b_3, \ldots, b_T|a_1, a_2, \ldots, a_{T-1}). \tag{1.5}$$

The relay network is memoryless if the natural extension of the condition (1.4) is true, i.e., if the ith channel outputs Y_{ti}, $t = 2, 3, \ldots, T$, depend only on the ith channel inputs X_{ti}, $t = 1, 2, \ldots, T-1$, given the message, the present (or ith) and past channel inputs, and the past channel outputs. The capacity C is again the maximum rate at which one can transmit reliably.

2. Wireline and Wireless Networks.

2.1. Wireline Models. The memoryless relay channel defined by (1.3) models a variety of communication problems. Consider, for example, a *wireline* network with three terminals depicted in Figure 1. The idea is that the source (node 1) is *wired* to the relay (node 2) which is wired to the sink (node 3). One might therefore expect that Y_2 is a noisy function of X_1 only, and that Y_3 is a noisy function of X_2 only. In this case the channel distribution (1.3) satisfies

$$P_{Y_2Y_3|X_1X_2}(b_2, b_3|a_1, a_2) = P_{Y_2|X_1}(b_2|a_1) \cdot P_{Y_3|X_2}(b_3|a_2) \tag{2.1}$$

for all a_1, a_2, b_2, b_3. If the channels are essentially *noise-free* we might even choose to consider

$$P_{Y_2Y_3|X_1X_2}(b_2, b_3|a_1, a_2) = 1(b_2 = a_1) \cdot 1(b_3 = a_2) \tag{2.2}$$

where $1(\cdot)$ is the indicator function that takes on the value 1 if its argument is true and is zero otherwise.

Some wireline problems have constraints on the network *nodes* and not only (capacity constraints) on the network channels or *edges*. For

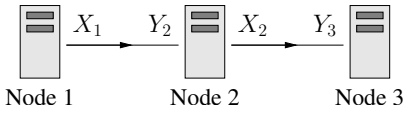


FIG. 1. *A wireline network.*

instance, suppose the relay (node 2) has limited processing power and can either transmit or receive, but not both. For noise-free networks, one might model this via the constraint

$$Y_2 = \begin{cases} X_1, & \text{if } X_2 = 0 \\ 0, & \text{if } X_2 \neq 0. \end{cases} \tag{2.3}$$

Note that (2.1) is no longer true. However, we can write

$$P_{Y_2Y_3|X_1X_2}(b_2, b_3|a_1, a_2) = P_{Y_2|X_1X_2}(b_2|a_1, a_2) \cdot P_{Y_3|X_2}(b_3|a_2) \tag{2.4}$$

for all a_1, a_2, b_2, b_3. More generally, a relay channel is said to be *physically degraded* [6] if one can write

$$P_{Y_2Y_3|X_1X_2}(b_2, b_3|a_1, a_2) = P_{Y_2|X_1X_2}(b_2|a_1, a_2) \cdot P_{Y_3|X_2Y_2}(b_3|a_2, b_2) \tag{2.5}$$

for all a_1, a_2, b_2, b_3. The channels (2.1), (2.2), and (2.4) are therefore physically degraded.

2.2. Wireless Models. Consider the *wireless* network depicted in Figure 2. For such problems one usually replaces the probability distribution (1.1) with a probability density $p_{Y|X}(\cdot)$. A commonly-studied class of probability densities is based on an additive white Gaussian noise (AWGN) model with

$$Y_2 = \frac{h_{12}}{d_{12}^{\alpha/2}} X_1 + Z_2 \tag{2.6}$$

$$Y_3 = \frac{h_{13}}{d_{13}^{\alpha/2}} X_1 + \frac{h_{23}}{d_{23}^{\alpha/2}} X_2 + Z_3 \tag{2.7}$$

where $X_1, X_2, Y_2, Y_3, Z_2, Z_3$ are complex random variables, h_{ij} and d_{ij} are the respective (fading) channel gain and distance between nodes i and j, and α is an attenuation exponent (e.g., $\alpha = 2$ for free space propagation). The average energies (or powers) of the inputs are constrained as

$$\sum_{i=1}^{n} \mathrm{E}[|X_{ti}|^2]/n \leq P_t, \quad t = 1, 2. \tag{2.8}$$

The idea of the above model is that the wireless channel permits *broadcasting* (X_1 affects both Y_2 and Y_3) but that this causes *interference* (X_1 and X_2 interfere at node 3).

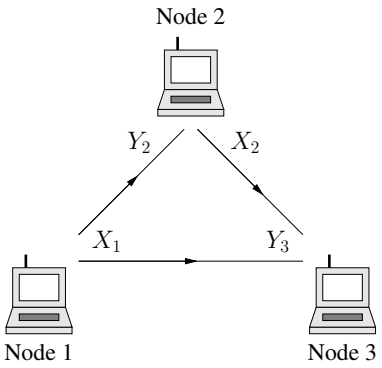


FIG. 2. *A wireless network.*

We will assume that h_{ij} is a realization of a complex random variable H_{ij}. We say that the channel exhibits *Rayleigh fading* if the H_{ij} are statistically independent, proper [15], complex, Gaussian, zero-mean, unit variance random variables. We further assume that Z_2 and Z_3 are independent, proper, complex, Gaussian, unit variance random variables that are independent of X_1, X_2, and the H_{ij} for all i, j.

We remark that the model defined by (2.6) and (2.7) implicitly permits the relay to transmit and receive at the same time in the same frequency band. However, this is often not possible due to the large differences in transmit and receive energies at the antennas of wireless devices. Most practical wireless relays operate under a *half-duplex* constraint that one can model as (see [11] and [12])

$$Y_2 = \begin{cases} \frac{h_{12}}{d_{12}^{\alpha/2}} X_1 + Z_2, & \text{if } X_2 = 0 \\ 0, & \text{if } X_2 \neq 0. \end{cases} \tag{2.9}$$

Note the similarity between (2.3) and (2.9). We remark that without or with the half-duplex constraint, the wireless models we consider do not satisfy (2.5) and are hence not physically degraded.

3. Example of a Code for Wireline Relaying. Consider the wireline network in Figure 1 with the node constraint (2.3) and $Y_3 = X_2$. Suppose for simplicity that the alphabets $\mathcal{X}_1, \mathcal{X}_2, \mathcal{Y}_2, \mathcal{Y}_3$ of the respective random variables X_1, X_2, Y_2, Y_3 are all the set $\{0, 1\}$. One might guess that the capacity of this network is $1/2$ bit per use because the relay can either receive or transmit, but not both.

Consider, however, the code tree depicted in Figure 3 that is labeled with the symbols X_2 that the relay transmits after having decoded an X_1. The idea is that after decoding $X_1 = 0$ the relay sends $X_2 = 0$, while after decoding $X_1 = 1$ the relay sends $X_{21}, X_{22} = 1, 0$. Note that every codeword in the code tree has exactly one 0, so that the source can transmit exactly

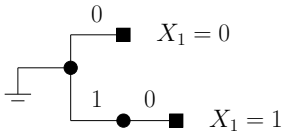


FIG. 3. *A code tree for the relay in Figure 1.*

one new message bit for every relay codeword. Note further that the tree is labeled so that the code is *prefix-free* or *instantaneously decodable* [7, Ch. 5]. This means that the sink can correctly parse its received sequence to extract the message bits.

For example, suppose that the message w has 8 bits $0, 1, 0, 0, 1, 1, 1, 0$. The transmit and receive sequences are then

$$\begin{array}{rccccccccc} i = & 1, & 2, & 3,4, & 5, & 6, & 7,8, & 9,10, & 11,12, & 13 \\ X_1^{13} = & 0, & 1, & x,0, & 0, & 1, & x,1, & x,1, & x,0, & x \\ Y_2^{13} = & 0, & 1, & 0,0, & 0, & 1, & 0,1, & 0,1, & 0,0, & x \\ X_2^{13} = & 0, & 0, & 1,0, & 0, & 0, & 1,0, & 1,0, & 1,0, & 0 \end{array} \tag{3.1}$$

where x denotes a "don't care" symbol. Note that, for this particular message, we have transmitted 8 bits in 13 channel uses which gives a rate $R(w) = 8/13$ that is larger than $1/2$.

We would like to determine the rate of the above code when the source transmits b bits for large b. The codewords have variable lengths, so we define a random variable L_2 to be the length of the codeword of any of the message bits. If the source bits are coin-tossing, we compute $\mathrm{E}[L_2] = \frac{1}{2}(1) + \frac{1}{2}(2) = \frac{3}{2}$ and

$$R = \frac{1}{\mathrm{E}[L_2]} = 2/3 \text{ bits per channel use} \tag{3.2}$$

implying that we can substantially increase the rate beyond $1/2$ bits per use. In fact, it turns out that better compression codes such as Huffman codes or arithmetic source codes (see [7, Ch. 5]) can achieve $R = 0.773$ bits per use. Furthermore, this rate is the capacity of the network, as discussed in Section 4.2 below.

4. Capacity and Coding. The capacity of a point-to-point memoryless channel (1.1) is known to be

$$C = \max_{P_X(\cdot)} I(X;Y) \tag{4.1}$$

where $I(X;Y)$ is the *mutual information* between the random variables X and Y (see [7, Ch. 2 and Ch. 8]). For the complex-alphabet AWGN model we have

$$Y = X + Z \tag{4.2}$$

where $\sum_{i=1}^{n} \mathrm{E}[|X_i|^2]/n \leq P$ and Z is proper, Gaussian, unit-variance, and independent of X. The maximization in (4.1) is now performed over all probability density functions $p_X(\cdot)$ and the result is (see [7, Ch. 10])

$$C = \log_2(1+P) \text{ bits per channel use} \tag{4.3}$$

where we recall that the channel has complex alphabets.

Consider next the relay channel (1.3). The capacity of this channel is still not known except for special cases. However, good achievable rates and upper bounds on the capacity are known. For example, a standard *cut-set* upper bound on the capacity is (see [6])

$$C \leq \max_{P_{X_1X_2}(\cdot)} \min\left\{I(X_1; Y_2Y_3|X_2),\ I(X_1X_2; Y_3)\right\}. \tag{4.4}$$

We next consider *random* coding strategies that achieve good rates for the relay channels we are interested in.

4.1. Random Codes for Relay Channels. Several types of relaying strategies have been developed and we list some of these below.

- Amplify-and-forward: the relay amplifies the most recent Y_2. More generally, the relay transmits some function of a small number of the past Y_2.
- Decode-and-forward: the relay decodes the source message, re-encodes it, and transmits the resulting codeword. The relay usually uses a different code book than the source. This method includes traditional *multihopping* where the source transmits and the relay is silent in a first block, while the source is silent and the relay transmits in a second block.
- Compress-and-forward: the relay quantizes, compresses, and channel encodes a string of Y_2 and transmits the resulting quantized values digitally to the sink. A more sophisticated quantization exploits the statistical dependence between Y_2 and Y_3 to reduce the compression rate (see [6, 12]).

We will consider only the decode-and-forward strategy and some of its variants. The coding procedure described next appeared in [5, 21].

Consider the AWGN channel with (2.6) and (2.7) and $d_{ij} = h_{ij} = 1$ for all i, j. We generate two codebooks $\mathcal{C}_1'$ and $\mathcal{C}_2$ that both have 2^{nR} codewords of length n (we assume that 2^{nR} is an integer for simplicity). Every codeword $\underline{x}_1'(w)$, $w = 1, 2, \ldots, 2^{nR}$, in $\mathcal{C}_1'$ is generated by choosing each of its n symbols independently using a proper, complex, Gaussian distribution with zero mean and variance P_1' where $P_1' \leq P_1$. The codewords $\underline{x}_2(w)$, $w = 1, 2, \ldots, 2^{nR}$, in $\mathcal{C}_2$ are generated in the same way except that the Gaussian distribution has variance P_2. The transmission protocol is depicted in Figure 4 and it operates as follows.

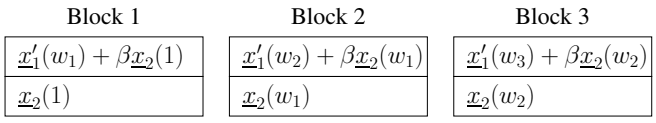


FIG. 4. *A decode-and-forward strategy. The upper and lower blocks show what the source and relay transmit, respectively.*

- Suppose W has nRB bits. We split these into B equally-sized blocks of nR bits $w_1, w_2, \ldots, w_B$. Set $w_0 = w_{B+1} = 1$.
- In block b, $b = 1, 2, \ldots, B+1$, the source transmits

$$\underline{x}_1(w_b, w_{b-1}) = \underline{x}_1'(w_b) + \beta \underline{x}_2(w_{b-1}) \tag{4.5}$$

  where $\beta = \sqrt{(P_1 - P_1')/P_2}$.
- In block b the relay transmits $\underline{x}_2(w_{b-1})$.

Note that using randomly-generated codebooks with the above transmission protocol will ensure that the power constraints (2.8) can be satisfied.

The decoding procedure is as follows.

- After block b, $b = 1, 2, \ldots, B$, the relay decodes w_b by using its bth block of channel outputs.
- After block b, $b = 2, 3, \ldots, B+1$, the sink decodes w_b by using its $(b-1)$st and bth block of channel outputs.

One can show, using virtually the same analysis as for deriving (4.3), that the above decode-and-forward strategy achieves the rates R satisfying

$$R < \log(1 + P_1') \tag{4.6}$$

$$R < \log(1 + P_1' + (1+\beta)^2 P_2) \tag{4.7}$$

where the first and second bounds arise due to the respective relay and sink decoding steps.

4.2. Generalizations. The above strategy generalizes in a natural way to other memoryless relay channels (1.3). Summarizing, we find that for large B one can achieve any rate up to

$$R = \max_{P_{X_1 X_2}(\cdot)} \min\left\{I(X_1; Y_2|X_2),\ I(X_1 X_2; Y_3)\right\} \tag{4.8}$$

where the first and second mutual information expressions in (4.8) arise due to the respective relay and sink decoding steps. We remark that (4.8) is almost the same as (4.4) except that there is no Y_3 in the first mutual information expression in (4.8). We also remark that for physically degraded relay channels satisfying (2.5) we have that

$$X_1 - [X_2, Y_2] - Y_3$$

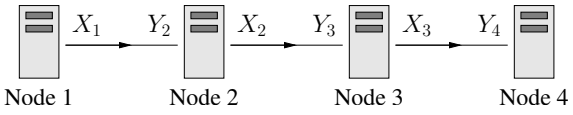


FIG. 5. *A wireline network with four nodes.*

forms a Markov chain so that $I(X_1; Y_3|X_2Y_2) = 0$. This implies that

$$I(X_1; Y_2Y_3|X_2) = I(X_1; Y_2|X_2) \tag{4.9}$$

by the chain rule for mutual information [7, p. 22]. Thus, the cut-set bound (4.4) is the same as the decode-and-forward rate (4.8) and is hence the capacity. This means that (4.8) is the capacity of the wireline models (2.1), (2.2), and (2.4). Furthermore, one can check that (4.8) gives $C = 0.773$ bits per use for the channel studied in Section 3.

A generalization of the achievable rate (4.8) is to relay networks. Let $\pi(\cdot)$ be a permutation on $\{1, 2, \ldots, T\}$ with $\pi(1) = 1$ and $\pi(T) = T$, and let $\pi(i:j) = \{\pi(i), \pi(i+1), \ldots, \pi(j)\}$. One can show that one can achieve rates up to (see [12, 22, 23])

$$R = \max_{\pi(\cdot)} \min_{1 \le t \le T-1} I\left(X_{\pi(1:t)}; Y_{\pi(t+1)} \,|\, X_{\pi(t+1:T-1)}\right) \tag{4.10}$$

where one can choose any distribution on $X_1, X_2, \ldots, X_{T-1}$. Furthermore, the rate (4.10) is the capacity of *physically degraded relay networks* that include wireline networks (see [2, p. 69] and [12, Remark 10]). For example, consider the wireline network with four nodes shown in Figure 5. Suppose that node 1 transmits a message to node 4, and that both nodes 2 and 3 have the node constraint described by (2.3). We find that (4.10) gives the capacity of this channel.

5. Wireless Networks. From here on we consider relay channels defined by (2.6) and (2.7) with Rayleigh fading, i.e., the H_{ij} are independent, proper, complex, Gaussian, zero-mean, unit variance random variables. Suppose further that the source node does not know the realizations of these random variables, the relay knows H_{12} only, and the sink knows H_{13} and H_{23} only. These restrictions on channel knowledge apply to the practical case where node j can accurately estimate its channel gains H_{ij} but it cannot (or wishes not to) synchronize its waveform with the other transmitters.

Suppose we encode by using the strategy described in Section 4.1 that is depicted in Figure 4. One can show that it is best to choose $P_1' = P_1$ or $\beta = 0$ in (4.5). The resulting protocol is depicted in Figure 6. It has recently been shown that this strategy achieves capacity if the relay is in the vicinity of the source node, but not necessarily colocated with it [12].

Block 1	Block 2	Block 3	Block 4
$\underline{x}_1(w_1)$	$\underline{x}_1(w_2)$	$\underline{x}_1(w_3)$	$\underline{x}_1(w_4)$
$\underline{x}_2(1)$	$\underline{x}_2(w_1)$	$\underline{x}_2(w_2)$	$\underline{x}_2(w_3)$

FIG. 6. *A decode-and-forward strategy for full-duplex relays.*

Block 1	Block 2	Block 3	Block 4
$\underline{x}_1(w_1)$	$\underline{x}_1(w_2)$	$\underline{x}_1(w_3)$	$\underline{x}_1(w_4)$
$\underline{0}$	$\underline{x}_2(w_1)$	$\underline{0}$	$\underline{x}_2(w_3)$

FIG. 7. *A decode-and-forward strategy for half-duplex relays.*

5.1. Half-Duplex Relays. The strategies in Figures 4 and 6 apply to *full-duplex* relays. We next wish to consider half-duplex devices satisfying (2.9). A natural approach is to modify the protocol in Figure 6 by using the protocol depicted in Figure 7. The relay now decodes only every *second* message block $w_1, w_3, w_5, \ldots$. We will design codes for such a protocol. However, we remark that there are better decode-and-forward strategies that additionally modulate the *timing* of the relay so as to transmit extra information to the sink (see [11]).

Consider, e.g., the geometry shown in Figure 8 where d is a real number, $d_{13} = 1$, $d_{12} = |d|$, and $d_{23} = |1 - d|$ (the relay is to the left of the source if d is negative). Suppose further that the sink has two antennas. We replace (2.7) with the 2×1 vector

$$\underline{Y}_3 = \frac{\underline{h}_{13}}{d_{13}^{\alpha/2}} X_1 + \frac{\underline{h}_{23}}{d_{23}^{\alpha/2}} X_2 + \underline{Z}_3 \tag{5.1}$$

where $\underline{h}_{13}$ and $\underline{h}_{23}$ are realizations of the 2×1 vectors $\underline{H}_{13}$ and $\underline{H}_{23}$ whose components are independent, proper, complex, Gaussian, zero-mean, unit-variance random variables (we thus have multiantenna Rayleigh fading [8, 18]). Similarly, $\underline{Z}_3$ is a 2×1 vector with independent, proper, complex, Gaussian, unit-variance entries.

To make the problem more realistic, suppose we use quaternary phase-shift keying (QPSK) modulation at both the source and relay. Suppose further that $\alpha = 4$, $P_1 = P_2 = 0.25$ (which we write as $E_s/N_0 = -6dB$), and the relay listens exactly half of the time. The rates (4.8) of the half-duplex decode-and-forward strategy of Figure 7 are shown in Figure 9 as a function of d. Also shown are the rates when there is no relay ($R \approx 0.54$ bits per use) and when one uses traditional multihopping (see Section 4.1) with optimized listen and transmit times for the relay.

Observe that decode-and-forward can achieve substantial rate gains over both no-relay transmission and traditional multihopping. For instance, the points marked with $*$ in Figure 9 are located at $(d, R) = (0.25, 0.5)$ and

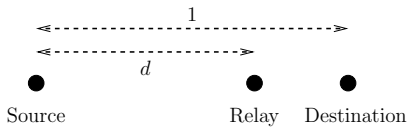


FIG. 8. *A geometry with the relay on a line going through the source and sink.*

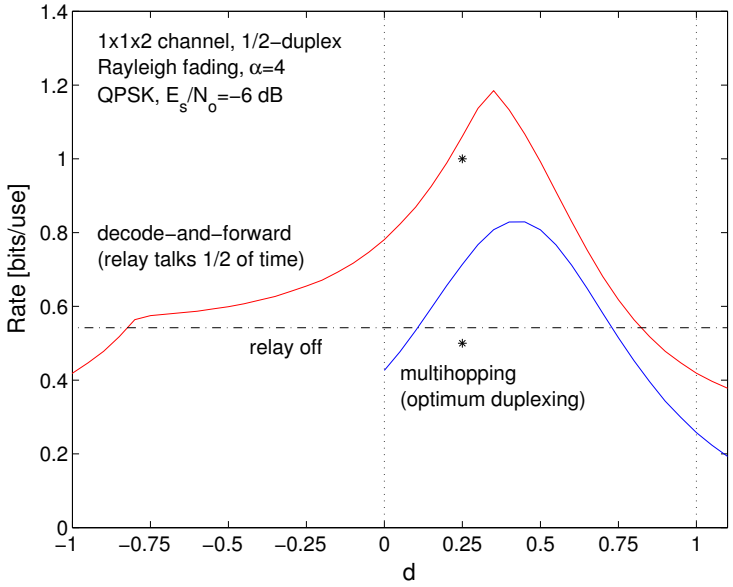


FIG. 9. *Decode-and-forward rates.*

$(d, R) = (0.25, 1)$. Decode-and-forward can thus approximately double the rate as compared to no-relay transmission at $d = 0.25$. The next section describes codes that achieve these points with low error probability.

5.2. Code Design. Consider the geometry of Figure 8 with $d = 0.25$. We design irregular low-density parity-check (LDPC) codes [9, 14] of length 16,000 for single-antenna, no fading, AWGN channels with binary phase-shift keying (BPSK) by using the curve-fitting procedure described in [4, Section III]. For the actual transmission, the coded bits are mapped to QPSK symbols by using the Gray mapping. The decoder uses the standard graph representation of an LDPC code with variable nodes on the left and check nodes on the right. The left and right nodes are connected by edges whose nodes are chosen with a random permutation that avoids 2-cycles. The decoder iterates 60 times between the left and right nodes by using an *a posteriori* probability (APP) decoder.

Consider first the case $R = 1/2$ without a relay. We design an LDPC code with rate $R_c = 1/4$ that has an (single-antenna, no fading, BPSK)

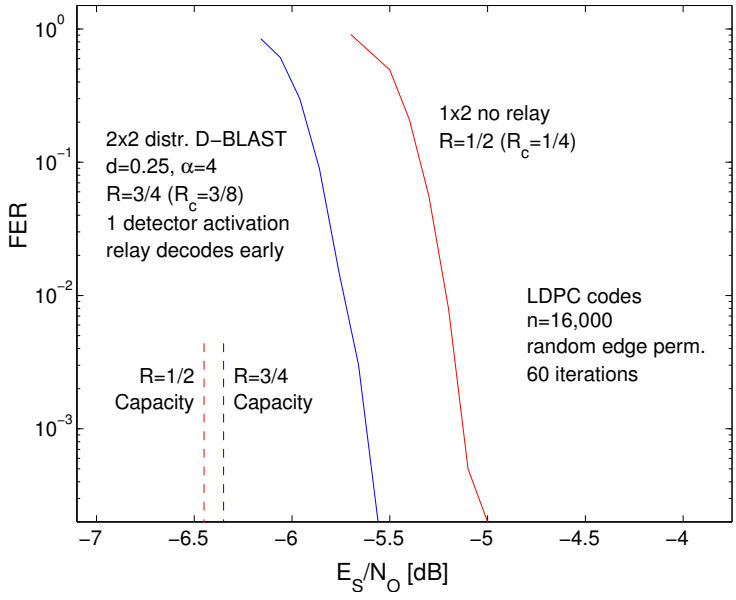


FIG. 10. *Decode-and-forward frame error rates when using LDPC codes designed for single-antenna AWGN channels.*

AWGN decoding threshold of $E_b/N_0 = -0.4$ dB which is about 0.3 dB away from the (single-antenna, no fading, BPSK) AWGN capacity. The resulting frame error rate (FER) curve is shown on the right in Figure 10. Observe that the code operates within 1.3 dB of capacity at an FER of 10^{-3}. The extra loss (as compared to 0.3 dB for the single-antenna case) can be attributed to the relatively short code length and the fading.

Consider next the case $R = 1$ with decode-and-forward. We design an LDPC code with rate $R_c = 3/8$ that has an (single-antenna, no fading, BPSK) AWGN decoding threshold of $E_b/N_0 = 0.1$ dB which is about 0.45 dB away from the (single-antenna, no fading, BPSK) AWGN capacity. The encoding and decoding procedure is as follows (see Figure 7).

- In the odd-numbered blocks $b = 1, 3, 5, \ldots$, the source transmits 4000 QPSK symbols (or 8,000 of the 16,000 codeword bits) by using the rate $R_c = 3/8$ LDPC code.
- After every odd-numbered block b, the relay decodes the information bits of the $R_c = 3/8$ code from this block. Note that the relay has received only half of this codeword's symbols.
- In the even-numbered blocks $b = 2, 4, 6, \ldots$, the source transmits using the rate $R_c = 1/4$ code described above.
- In the even-numbered blocks, the relay encodes the information bits decoded from the previous block by using the $R_c = 3/8$ en-

coder and transmits the last 4000 QPSK symbols of this codeword (or the last 8,000 of the 16,000 codeword bits).
- After every even-numbered block, the sink decodes the information bits of the rate $R_c = 3/8$ code. The sink performs only one detector activation per codeword (we remark that multiple detector activations would improve the performance a little [4]).
- The sink cancels the interference caused by the symbols of the $R_c = 3/8$ code from the even-numbered blocks.
- After every second even-numbered block, the sink decodes the information bits of the $R_c = 1/4$ code.

The overall rate is $R = 2(3/8)+2(1/4)(1/2) = 1$ bit per use, where the leading factors 2 are due to the QPSK modulation. There are three decoding steps to consider in the above procedure.

- The FER of the relay decoding step is not shown in Figure 10 because it lies far to the left of the other two curves.
- The FER of the sink decoding the information bits from the $R_c = 3/8$ code is shown as the left curve in Figure 10 (labeled "2×2 distr. D-BLAST" because our decode-and-forward strategies are closely related to a distributed version of D-BLAST [8]).
- The FER of the sink decoding the information bits from the $R_c = 1/4$ code is the same as the case where there is no relay, and is the right curve in Figure 10.

We see that the dominating FER is in both cases (without and with a relay) due to the direct link between the source and sink. The reliability of the two schemes is therefore the same. However, the decode-and-forward scheme doubles the rate. Note also that we have used only codes designed for single-antenna channels. This might be important if one needs to use "off-the-shelf" codes such as those prescribed in standards.

6. Concluding Remarks. We have developed codes for wireline and wireless relay channels. For the wireless case, we have chosen to study fast-fading channels and have designed distributed iteratively-decodable codes that approach certain information rates. We remark that it is also interesting to study slow-fading channels. This type of problem has been addressed in the context of CDMA [16], distributed space-time coding [1, 3, 13], and distributed convolutional and turbo coding [10, 17, 19, 24].

REFERENCES

[1] P. A. Anghel, G. Leus, and M. Kaveh, "Distribued space-time cooperative systems with regenerative relays," *IEEE Trans. Wireless Commun.*, submitted.

[2] M. R. Aref, *Information Flow in Relay Networks*. Ph.D. thesis, Stanford Univ., Stanford, CA, Oct. 1980.

[3] S. Barbarossa, L. Pescosolido, D. Ludovici, L. Barbetta, and G. Scutari, "Cooperative wireless networks based on distributed space-time coding," *Proc. Int. Workshop on Wireless Ad-hoc Networks (IWWAN)*, June 2004.

[4] S. ten Brink, G. Kramer, and A. Ashikhmin, "Design of low-density parity-check codes for modulation and detection," *IEEE Trans. Commun.*, 52(5): 670-678, April 2004.

[5] A. B. Carleial, "Multiple-access channels with different generalized feedback signals," *IEEE Trans. Inf. Theory*, 28(6): 841–850, Nov. 1982.

[6] T. M. Cover and A. A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inf. Theory*, 25: 572–584, Sept. 1979.

[7] T. M. Cover and J. A. Thomas, *Elements of Inform. Theory*. New York: Wiley, 1991.

[8] G. J. Foschini, "Layered space–time architecture for wireless communication in a fading environment when using multi–element antennas," *Bell Labs. Tech. J.*, 1(2): 41–59, 1996.

[9] R. G. Gallager, "Low–density parity–check codes," *IEEE Trans. Inf. Theory*, 8: 21–28, Jan. 1962.

[10] T. E. Hunter, S. Sanayei, and A. Nosratinia, "The outage behavior of coded cooperation," *Proc. IEEE Int. Symp. Inf. Theory*, Adelaide, Australia, June-July, 2004, p. 270.

[11] G. Kramer, "Models and theory for relay channels with receive constraints," *Proc. 42nd Annual Allerton Conf. on Commun., Control, and Computing*, Monticello, IL, Sept. 29–Oct. 1, 2004, pp. 1312-1321.

[12] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inf. Theory*, 51(9): 3037-3063, Sept. 2005.

[13] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inf. Theory*, 49(10): 2415–2425, Oct. 2003.

[14] M. G. Luby, M. Mitzenmacher, M. A. Shokrollahi, D. A. Spielman, "Efficient erasure correcting codes," *IEEE Trans. Inf. Theory*, 47(2): 569–584, Feb. 2001.

[15] F. D. Neeser and J. L. Massey, "Proper complex random processes with applications to information theory," *IEEE Trans. Inf. Theory*, 39(4): 1293-1302, July 1993.

[16] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity-Part II: Implementation aspects and performance analysis," *IEEE Transactions on Communications*, 51(11): 1939–1948, Nov. 2003.

[17] A. Stefanov and E. Erkip, "Cooperative coding for wireless networks," *IEEE Trans. Commun.*, 52(9): 1470-1476, Sept. 2004.

[18] I. E. Telatar, "Capacity of multi–antenna Gaussian channels," *Eur. Trans. Telecom.*, 10: 585–595, Nov. 1999.

[19] M. C. Valenti and B. Zhao, "Distributed turbo codes: towards the capacity of the relay channel," *Proc. IEEE Vehicular Tech. Conf. (VTC)*, Orlando, FL, Oct. 2003, pp. 322-326.

[20] E. C. van der Meulen, *Transmission of Information in a T-Terminal Discrete Memoryless Channel*. Ph.D. thesis, Univ. of California, Berkeley, CA, June 1968.

[21] F. M. J. Willems, *Informationtheoretical Results for the Discrete Memoryless Multiple Access Channel*. Doctor in de Wetenschappen Proefschrift, Katholieke Universiteit Leuven, Leuven, Belgium, Oct. 1982.

[22] L.-L. Xie and P. R. Kumar, "A network information theory for wireless communication: scaling laws and optimal operation," *IEEE Trans. Inf. Theory*, 50(5): 748–767, May 2004.

[23] L.-L. Xie and P. R. Kumar, "An achievable rate for the multiple level relay channel," *IEEE Trans. Inf. Theory*, 51(4): 1348–1358, April 2005.

[24] B. Zhao and M. C. Valenti, "Distributed turbo coded diversity for relay channel," *Electron. Lett.*, 39(10): 786–787, May 2003.