Outer Bound and Noisy-interference Sum-rate Capacity for Symmetric Gaussian Interference Channels

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***Abstract*— A new outer bound on the capacity region of Gaussian interference channels is developed. The bound combines and improves existing genie-aided methods and is shown to give the sum-rate capacity for Gaussian interference channel with *noisy interference* as defined in this paper. Specifically, it is shown that if the channel coefficients and power constraints satisfy a simple condition then single-user detection at each receiver is sum-rate optimal, i.e., treating the interference as noise incurs no loss in performance. Finally, we generalize the noisy interference sum-rate capacity to m-user interference channels.**

Index terms — capacity, Gaussian noise, interference.

I. Introduction

The interference channel (IC) models communication systems where transmitters communicate with their respective receivers while causing interference to all other receivers. For a two-user Gaussian IC, the channel output can be written in the standard form [1]

$$\begin{aligned} Y_1 &= X_1 + \sqrt{a}X_2 + Z_1, \\ Y_2 &= \sqrt{b}X_1 + X_2 + Z_2, \end{aligned}$$

where $\sqrt{a}$ and $\sqrt{b}$ are channel coefficients, X_i and Y_i are the transmit and receive signals. The user/channel input sequence $X_{i1}, X_{i2}, \cdots, X_{in}$ is subject to the power constraint $\sum_{j=1}^{n} \mathcal{E}(X_{ij}^2) \leq nP_i$, $i = 1, 2$. The transmitted signals X_1 and X_2 are statistically independent. The channel noises Z_1 and Z_2 are possibly correlated unit variance Gaussian random variables, and (Z_1, Z_2) is statistically independent of (X_1, X_2).

The capacity region of an IC is defined as the closure of the set of rate pairs (R_1, R_2) for which both receivers can decode their own messages with arbitrarily small positive error probability. The capacity region of a Gaussian IC is known only for three cases:

- $a = 0$, $b = 0$.
- $a \geq 1$, $b \geq 1$: see [2]–[4].
- $a = 0$, $b \geq 1$; or $a \geq 1$, $b = 0$: see [5]

For the second case both receivers can decode the messages of both transmitters. Thus this IC acts as two multiple access channels (MACs), and the capacity region for the IC is the intersection of the capacity region of the two MACs. However, when the interference is weak or moderate, the capacity region is still unknown. The best inner bound of the capacity region is obtained in [4] by using superposition coding and joint decoding. A simplified form of the Han-Kobayashi region was given by Chong-Motani-Garg-El Gamal [6], [7]. Various outer bounds have been developed in [8]–[12]. Sato's outer bound in [8] is derived by allowing the receivers to cooperate. Carleial's outer bound in [9] is derived by decreasing the noise power. Kramer in [10] presented two outer bounds. The first is obtained by providing each receiver with just enough information to decode both messages. The second outer bound is obtained by reducing the IC to a degraded broadcast channel. Both of these two bounds dominate the bounds by Sato and Carleial. The recent outer bounds by Etkin, Wang, and Tse in [11] are also based on genie-aided methods, and they show that Han and Kobayashi's inner bound is within one bit or a factor of two of the capacity region. This result can also be established by the methods of Telatar and Tse [12]. We remark that neither of the bounds of [10] and [11] implies each other. But as a rule of thumb, our numerical results show that the bounds of [10] are better at low SNR while those of [11] are better at high SNR. The bounds of [12] are not amenable to numerical evaluation since the optimal distributions of the auxiliary random variables are unknown.

In this paper, focusing on the symmetric Gaussian IC, we present a new outer bound on the capacity region of Gaussian ICs that improves on the bounds of [10], [11]. The bound generalizes to asymmetric channels which will be treated in a subsequent paper. The new bounds are based on a genie-aided approach and a recently proposed extremal inequality [13]. Unlike the genie-aided method used in [10, Theorem 1], neither receiver is required to decode the messages from the other transmitter. Based on this outer bound, we obtain new sum-rate capacity results (Theorem 2) for ICs satisfying some channel coefficient and power constraint conditions. We show that the sum-rate capacity can be achieved by treating the interference as noise when both the channel gain and the power constraint are weak. We say that such channels have *noisy interference*. For this class of interference, the simple single-user transmission and detection strategy is sum-rate optimal.

This paper is organized as follows. In Section II, we present a new genie-aided outer bound. The resulting sum-rate capacity for certain Gaussian ICs is shown in Section III. Numerical examples are given in Section IV, and Section V concludes the paper.

Before proceeding, we introduce some notation. We write vectors and matrices by using a bold font (e.g., $\mathbf{X}$ and $\mathbf{S}$). When useful we also write vectors with length n using the notation X^n. Random variables are written as uppercase letters (e.g., X_i) and their realizations as the corresponding lowercase letter (e.g., x_i). The notation $h(\mathbf{X})$ and $\mathrm{Cov}(\mathbf{X})$ refers to the respective differential entropy and covariance matrix of $\mathbf{X}$.

II. A Genie-aided Outer Bound

The symmetric Gaussian IC is defined as the IC with $a = b$ and $P_1 = P_2$. In the following we denote this symmetric IC as IC(a, P). The following is a new outer bound on the capacity region of Gaussian ICs.

Theorem 1: If the rates (R_1, R_2) are achievable for IC(a, P) with $0 < a < 1$, they must satisfy the following constraints (1)-(3) for $\mu > 0$, $\frac{1+aP}{a+aP} \le \eta_1 \le \frac{1}{a}$ and $a \le \eta_2 \le \frac{a+aP}{1+aP}$, where

$$\Sigma = \left\{ \begin{array}{l} \left\{ \left(\sigma_1^2, \sigma_2^2\right) \,|\sigma_1^2 > 0, 0 < \sigma_2^2 \le \frac{1-\rho_1^2}{a} \right\}, \text{ if } \mu \ge 1, \\ \left\{ \left(\sigma_1^2, \sigma_2^2\right) \,|0 < \sigma_1^2 \le \frac{1-\rho_2^2}{a}, \sigma_2^2 > 0 \right\}, \text{ if } \mu < 1, \end{array} \right. \quad (4)$$

and if $\mu \ge 1$ we define

$$P_1^* = \left\{ \begin{array}{ll} P, & \sigma_1^2 \le \left[\frac{(1-\mu)P}{\mu} + \frac{1-\rho_2^2}{a\mu} \right]^+, \\ \frac{1-\rho_2^2-a\mu\sigma_1^2}{a\mu-a}, & \left[\frac{(1-\mu)P}{\mu} + \frac{1-\rho_2^2}{a\mu} \right]^+ < \sigma_1^2 \le \frac{1-\rho_2^2}{a\mu}, \\ 0, & \sigma_1^2 > \frac{1-\rho_2^2}{a\mu}, \end{array} \right. \quad (5)$$

$$P_2^* = P, \qquad \sigma_2^2 \le \frac{1-\rho_1^2}{a}, \quad (6)$$

where $(x)^+ \triangleq \max\{x, 0\}$, and if $0 < \mu < 1$ we define

$$P_1^* = P, \qquad \sigma_1^2 \le \frac{1-\rho_2^2}{a}, \quad (7)$$

$$P_2^* = \left\{ \begin{array}{ll} P, & \sigma_2^2 \le \left[(\mu-1)P + \frac{\mu\left(1-\rho_1^2\right)}{a} \right]^+, \\ \frac{\mu\left(1-\rho_1^2\right)-a\sigma_2^2}{a-a\mu}, & \left[(\mu-1)P + \frac{\mu\left(1-\rho_1^2\right)}{a} \right]^+ \\ & \quad < \sigma_2^2 \le \frac{\mu\left(1-\rho_1^2\right)}{a}, \\ 0, & \sigma_2^2 > \frac{\mu\left(1-\rho_1^2\right)}{a}. \end{array} \right. \quad (8)$$

Proof: Here is the sketch of the proof. A genie is assumed to provide the two receivers with side information $X_1 + N_1$ and $X_2 + N_2$ respectively, where N_i is Gaussian distributed with variance σ_i^2 and $\mathcal{E}(N_i Z_i) = \rho_i \sigma_i$, $i = 1, 2$. Starting from Fano's inequality, we have that reliable communication requires

$$\begin{aligned} & n(R_1 + \mu R_2) - n\epsilon \\ & \le I\left(X_1^n; Y_1^n\right) + \mu I\left(X_2^n; Y_2^n\right) \\ & \le I\left(X_1^n; Y_1^n, X_1^n + N_1^n\right) + \mu I\left(X_2^n; Y_2^n, X_2^n + N_2^n\right) \\ & = \left[h\left(X_1^n + N_1^n\right) - \mu h\left(\sqrt{a}X_1^n + Z_2^n | N_2^n\right)\right] - h\left(N_1^n\right) \\ & \quad + \left[\mu h\left(X_2^n + Z_2^n\right) - h\left(\sqrt{a}X_2^n + Z_1^n | N_1^n\right)\right] - \mu h\left(N_2^n\right) \\ & \quad + h\left(Y_1^n | X_1^n + N_1^n\right) + \mu h\left(Y_2^n | X_2^n + N_2^n\right). \end{aligned} \quad (9)$$

where $\epsilon \to 0$ as $n \to \infty$. For $h\left(Y_1^n | X_1^n + N_1^n\right)$, zero-mean Gaussian X_1^n and X_2^n with covariance matrices $P_1\mathbf{I}$ and $P_2\mathbf{I}$ are optimal, and we have

$$\begin{aligned} & h\left(Y_i^n | X_i^n + N_i^n\right) \\ & \le \frac{n}{2} \log \left[2\pi e \left(1 + aP + P - \frac{\left(P + \rho_i\sigma_i\right)^2}{P + \sigma_i^2} \right) \right]. \end{aligned} \quad (10)$$

From the extremal inequality introduced in [13, Theorem 1, Corollary 4], we have

$$\begin{aligned} & h\left(X_1^n + N_1^n\right) - \mu h\left(\sqrt{a}X_1^n + Z_2^n | N_2^n\right) \\ & \le \frac{n}{2} \log\left[2\pi e\left(P_1^* + \sigma_1^2\right)\right] - \frac{n\mu}{2} \log\left[2\pi e\left(aP_1^* + 1 - \rho_2^2\right)\right] \end{aligned} \quad (11)$$

and

$$\begin{aligned} & \mu h\left(X_2^n + Z_2^n\right) - h\left(\sqrt{a}X_2^n + Z_1^n | N_1^n\right) \\ & \le \frac{n\mu}{2} \log\left[2\pi e\left(P_2^* + \sigma_2^2\right)\right] - \frac{\mu}{2} \log\left[2\pi e\left(aP_2^* + 1 - \rho_1^2\right)\right] \end{aligned} \quad (12)$$

where equalities hold when X_1^n and X_2^n are both Gaussian with covariance matrices $P_1^*\mathbf{I}$ and $P_2^*\mathbf{I}$ respectively. From (9)-(12) we obtain the outer bound (1).

The outer bounds in (2) and (3) are obtained by letting the genie provide side information X_2 to receiver one and X_1 to receiver two respectively, and applying the extremely inequality.

$$\begin{aligned} & n(R_1 + \eta_1 R_2) - n\epsilon \\ & \le I\left(X_1^n; Y_1^n, X_2^n\right) + \eta_1 I\left(X_2^n; Y_2^n\right) \\ & = h\left(X_1^n + Z_1^n\right) - \eta_1 h\left(\sqrt{a}X_1^n + Z_2^n\right) - h\left(Z_1^n\right) + \eta_1 h\left(Y_2^n\right) \\ & \le \frac{n}{2} \log\left(\tilde{P}_1 + 1\right) - \frac{n\eta_1}{2} \log\left(a\tilde{P}_1 + 1\right) \\ & \quad + \frac{n\eta_1}{2} \log\left(1 + aP + P\right), \end{aligned} \quad (13)$$

where $\tilde{P}_1 = \frac{a\eta_1 - 1}{a - a\eta_1}$ for $\frac{1+aP}{a+aP} \le \eta_1 \le \frac{1}{a}$. This is the bound in (2). Similarly, we obtain bound (3). ■

Remark 1: The bounds (1)-(3) are obtained by providing different genie-aided signals to the receivers. There is overlap of the range of μ, η_1, and η_2, and none of the bounds uniformly dominates the other two bounds. Which one of them is active depends on the channel conditions and the rate pair.

Remark 2: Equations (2) and (3) are outer bounds for the capacity region of a Z-IC, and a Z-IC is equivalent to a degraded IC [5]. For such channels, it can be shown that (2) and (3) are the same as the outer bounds in [14]. For $0 \le \eta_1 \le \frac{1+aP}{a+aP}$ and $\eta_2 \ge \frac{a+aP}{1+aP}$, the bounds in (2) and (3) are tight for a Z-IC (or degraded IC) because there is no power sharing between the transmitters. Consequently, $\frac{1+aP}{a+aP}$ and $\frac{a+aP}{1+aP}$ are the negative slopes of the tangent lines for the capacity region at the corner points.

$$
\begin{aligned}
R_1+\mu R_2 &\le \min_{\substack{\rho_i\in(0,1)\\(\sigma_1^2,\sigma_2^2)\in\Sigma}} \frac{1}{2}\log\left(1+\frac{P_1^*}{\sigma_1^2}\right)-\frac{1}{2}\log\left(aP_2^*+1-\rho_1^2\right)+\frac{1}{2}\log\left(1+P+aP-\frac{(P+\rho_1\sigma_1)^2}{P+\sigma_1^2}\right)\\
&\quad+\frac{\mu}{2}\log\left(1+\frac{P_2^*}{\sigma_2^2}\right)-\frac{\mu}{2}\log\left(bP_1^*+1-\rho_2^2\right)+\frac{\mu}{2}\log\left(1+P+aP-\frac{(P+\rho_2\sigma_2)^2}{P+\sigma_2^2}\right), \quad (1)\\
R_1+\eta_1 R_2 &\le \frac{1}{2}\log\left(1+\frac{a\eta_1-1}{a-a\eta_1}\right)-\frac{\eta_1}{2}\log\left(1+\frac{a\eta_1-1}{1-\eta_1}\right)+\frac{\eta_1}{2}\log\left(1+aP+P\right), \quad (2)\\
R_1+\eta_2 R_2 &\le \frac{1}{2}\log\left(1+P+aP\right)-\frac{1}{2}\log\left(1+\frac{a-\eta_2}{\eta_2-1}\right)+\frac{\eta_2}{2}\log\left(1+\frac{a-\eta_2}{a\eta_2-a}\right), \quad (3)
\end{aligned}
$$

Remark 3: The bounds in (2)-(3) turn out to be the same as the bounds in [10, Theorem 2]. This can be shown by rewriting the bounds in [10, Theorem 2] in the form of the weighted sum rate.

Remark 4: The bounds in [10, Theorem 2] are obtained by getting rid of one of the interference links to reduce the IC into a Z interference channel. In addition, the proof in [10] allowed the transmitters to share their power, which further reduces the Z-IC into a degraded broadcast channel. Then the capacity region of this degraded broadcast channel is an outer bound for the capacity region of the original IC. The bounds in (2) and (3) are also obtained by reducing the IC to a Z-IC. Although we do not explicitly allow the transmitters to share their power, it is interesting that these bounds are equivalent to the bounds in [10, Theorem 2] with power sharing. In fact, a careful examination of our new derivation reveals that power sharing is implicitly assumed. For example, for the term $h\left(X_1^n+Z_1^n\right)-\eta_1 h\left(\sqrt{a}X_1^n+Z_2^n\right)$ of (13), user 1 uses power $P_1^*=\frac{a\eta_1-1}{a-a\eta_1}\le P$, while for the term $\eta_1 h\left(Y_2^n\right)$ user 1 uses all the power P. This is equivalent to letting user 1 use the power P_1^* for both terms, and letting user 2 use a power that exceeds P. To see this, consider (13) and write

$$
\begin{aligned}
n(R_1+\eta_1 R_2)-n\epsilon &\le \frac{n}{2}\log\left(P_1'+1\right)-\frac{n\eta_1}{2}\log\left(aP_1'+1\right)\\
&\quad+\frac{n\eta_1}{2}\log\left(1+aP_1'+P_2'\right),
\end{aligned}
$$

where $P_1'\triangleq P_1^*$, and $P_2'\triangleq P+a(P-P_1^*)$. Therefore, one can assume that user 2 uses extra power provided by user 1.

Remark 5: Theorem 1 improves [11, Theorem 3]. Specifically, the bound in (2) is tighter than the first sum-rate bound of [11, Theorem 3]. Similarly, the bound in (3) is tighter than the second sum-rate bound of [11, Theorem 3]. The third sum-rate bound in [11, Theorem 3] is a special case of (1).

Remark 6: Our outer bound is not always tighter than that of [11] for all rate points. The reason is that in [11, last two equations of (39)], different genie-aided signals are provided to the same receiver. Our outer bound can also be improved in a similar and more general way by providing different genie-aided signals to the receivers. Specifically the starting point of the bound is

$$
\begin{aligned}
n\left(R_1+\mu R_2\right) &\le \sum_{i=1}^{k}\lambda_i I\left(X_1^n;Y_1^n,U_i\right)\\
&\quad+\sum_{j=1}^{m}\mu_i I\left(X_2^n;Y_2^n,W_j\right)+n\epsilon, \quad (14)
\end{aligned}
$$

where $\sum_{i=1}^{k}\lambda_i=1,\sum_{j=1}^{m}\mu_j=\mu,\lambda_i>0,\mu_j>0$.

III. Sum-rate Capacity for Noisy Interference

The outer bound in Theorem 1 is in the form of an optimization problem. Four parameters $\rho_1,\rho_2,\sigma_1^2,\sigma_2^2$ need to be optimized for different choices of the weights μ,η_1,η_2. When $\mu=1$, Theorem 1 leads directly to the following sum-rate capacity result.

Theorem 2: For the symmetric IC(a,P) satisfying

$$
P \le \frac{\sqrt{a}-2a}{2a^2}, \quad (15)
$$

the sum-rate capacity is

$$
C=\log\left(1+\frac{P}{1+aP}\right). \quad (16)
$$

Proof: By choosing

$$
\sigma_1^2=\sigma_2^2=\frac{1}{2a}\left\{1\pm\sqrt{1-4a(aP+1)^2}\right\} \quad (17)
$$

$$
\rho_1=\rho_2=\sqrt{1-a\sigma_1^2} \quad (18)
$$

the bound (1) with $\mu=1$ is

$$
R_1+R_2\le\log\left(1+\frac{P}{1+aP}\right). \quad (19)
$$

But one can achieve equality in (19) by treating the interference as noise at both receivers. Therefore if $1-4a(aP+1)^2\ge 0$, the sum-rate capacity is (16). ■

Remark 7: Consider the bound in (1) with $\mu=1$, $\rho_1=\rho_2=\rho$, $\sigma_1=\sigma_2=\sigma$. We further let

$$
1-\rho^2\ge a\sigma^2, \quad (20)
$$

so $P_1^*=P_2^*=P$, then we have

$$
\begin{aligned}
&R_1+R_2\\
&\le\log\left(1+\frac{P}{\sigma^2}\right)-\log(aP+1-\rho^2)\\
&\quad+\log\left[1+aP+P-\frac{(P+\rho\sigma)^2}{P+\sigma^2}\right]
\end{aligned}
$$

$$= \log\left[\frac{P(1+aP)}{1+aP-\rho^2}\left(\frac{1}{\sigma}-\frac{\rho}{1+aP}\right)^2+1+\frac{P}{1+aP}\right]$$
$$\triangleq f(\rho,\sigma). \quad (21)$$

Therefore, for any given ρ, when

$$\sigma = \frac{1+aP}{\rho}, \quad (22)$$

$f(\rho,\sigma)$ achieves its minimum which is the sum-rate capacity for noisy interference. Since the constraint in (20) must be satisfied, we have

$$\frac{1+aP}{\rho} \le \sqrt{\frac{1-\rho^2}{a}}. \quad (23)$$

As long as there exists a $\rho \in (0,1)$ such that (23) is satisfied, we can choose σ to be (22) and hence the bound in (21) is tight. Clearly, we have

$$P \le \frac{\sqrt{a} * 2\rho\sqrt{1-\rho^2} - 2a}{2a^2} \le \frac{\sqrt{a}-2a}{2a^2}. \quad (24)$$

Remark 8: Among all the $\rho \in (0,1)$, the most special one is in (18), since (11) and (12) become

$$\begin{aligned} &h\left(X_1^n + N_1^n\right) - h\left(\sqrt{a}X_1^n + Z_2^n | N_2^n\right) \\ &= h\left(X_1^n + N_1^n\right) - h\left(\sqrt{a}X_1^n + \sqrt{a}N_1^n\right) \\ &= -n\log\sqrt{a} \end{aligned} \quad (25)$$

and

$$h\left(X_2^n + N_2^n\right) - h\left(\sqrt{a}X_2^n + Z_1^n | N_1^n\right) = -n\log\sqrt{a}. \quad (26)$$

Therefore, we do not need the extremal inequality [13] to prove Theorem 2.

Remark 9: The sum-rate capacity for a Z-IC with $a = 0$, $0 < b < 1$ is a special case of Theorem 2 since (15) is satisfied. The sum capacity is therefore given by (16).

Theorem 2 follows directly from Theorem 1 with $\mu = 1$. It is remarkable that a genie-aided bound is tight if (15) is satisfied since the genie provides extra signals to the receivers without increasing the rates. This situation is reminiscent of the recent capacity results for vector Gaussian broadcast channels (see [15]). Furthermore, the sum-rate capacity (16) is achieved by treating the interference as noise. We therefore refer to channels satisfying (15) as ICs with *noisy interference*. Note that (15) involves both channel gain a and power P. The constraint (15) implies that

$$a \le \frac{1}{4}. \quad (27)$$

Noisy interference is therefore *weaker* than *weak interference* as defined in [5] and [16], namely $a \le \frac{\sqrt{1+2P}-1}{2P}$ or

$$P \le \frac{1-2a}{2a^2}. \quad (28)$$

Recall that [16] showed that for *weak interference* satisfying (28), treating interference as noise achieves larger sum rate than time- or frequency-division multiplexing (TDM/FDM), and [5] claimed that in "weak interference" the largest known achievable sum rate is achieved by treating the interference as noise.

The sum-rate capacity for Gaussian interference channels with noisy interference can be generalized to the m-user case where the receive signal at user i is defined as

$$Y_i = X_i + \sqrt{a}\sum_{j=1, j\neq i}^{m} X_j + Z_i, \quad i,j = 1,\ldots,m. \quad (29)$$

Z_i is unit Gaussian noise and the block power constraint with length n is $\sum_{t=1}^{n}\mathcal{E}(X_{i,t}^2) \le nP$. We denote this IC as IC(m,a,P). We have the following sum-rate capacity result.

Theorem 3: For the IC(m,a,P) satisfying

$$P \le \frac{\sqrt{(m-1)a} - 2(m-1)a}{2(m-1)^2a^2}, \quad (30)$$

the sum-rate capacity is

$$C = \frac{m}{2}\log\left[1+\frac{P}{1+(m-1)aP}\right]. \quad (31)$$

Theorem 2 is a special case of Theorem 3. One can also show that (30) implies

$$a \le \frac{1}{4(m-1)}. \quad (32)$$

We skip proof of Theorem 3 due to space limit. We remark here that this result can be generalized to the asymmetric case and will be reported in a subsequent paper.

IV. Numerical examples

The lower and upper bounds for the sum-rate capacity of the symmetric IC are shown in Figs. 1-3 for different power levels. For all of these cases, the upper bounds are tight up to point A. The bound in [11, Theorem 3] approaches to the bound in Theorem 1 when the power becomes large, but there is still a gap. Fig. 3 also provides a definitive answer to a question from [16, Fig. 2]: whether the sum-rate capacity of symmetric Gaussian IC is a decreasing function of a, or there exists a bump like the lower bound when a varies from 0 to 1. In Fig. 3, our proposed upper bound and Sason's inner bound explicitly show that the sum capacity is not a monotone function of a (this result also follows by the bounds of [11]).

V. Conclusions, extensions and parallel work

We derived an outer bound for the capacity region of symmetric Gaussian ICs by a genie-aided method. From this outer bound, the sum-rate capacities for ICs that satisfy (15) are obtained. We note that all the results for symmetric IC in this paper can be readily extended to the asymmetric IC.

Finally, we wish to acknowledge parallel work. After submitting our 2-user bounds and capacity results on the arXiv.org e-Print archive [17], two other teams of researchers - Motahari and Khandani from the University of Waterloo, Annapureddy and Veeravalli from the University of Illinois at Urbana-Champaign - let us know that they had derived the same 2-user sum-rate capacity results.

Acknowledgement

The work of X. Shang and B. Chen was supported in part by the NSF under Grants 0546491 and 0501534, and by the AFOSR under Grant FA9550-06-1-0051, and by the AFRL under Agreement FA8750-05-2-0121.

G. Kramer gratefully acknowledges the support of the Board of Trustees of the University of Illinois Subaward no. 04-217 under NSF Grant CCR-0325673 and the Army Research Office under ARO Grant W911NF-06-1-0182.

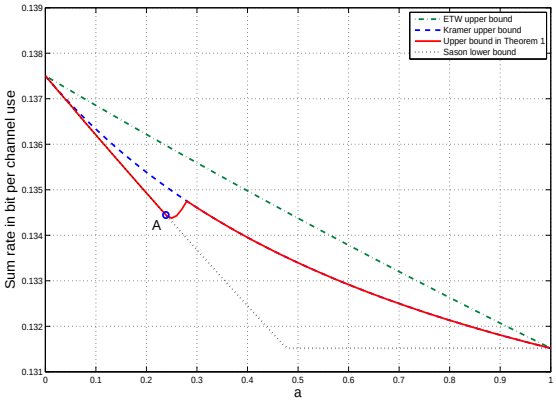


Fig. 1. Lower and upper bounds for the sum-rate capacity of symmetric Gaussian ICs with $a = b, P_1 = P_2 = 0.1$. Sason's bound is an inner bound obtained from Han and Kobayashi's bound by a special time sharing scheme [16, Table I]. The channel gain at point A is $a = 0.2385$.

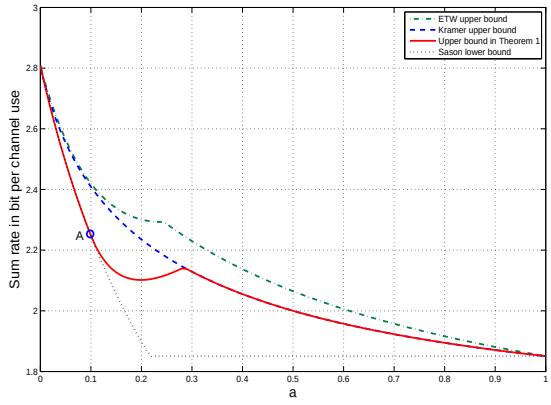


Fig. 2. Lower and upper bounds for the sum-rate capacity of symmetric Gaussian ICs with $a = b, P_1 = P_2 = 6$. The channel gain at point A is $a = 0.0987$.

REFERENCES

[1] A.B. Carleial, "Interference Channels," *IEEE Trans. Inform. Theory*, vol. 24, pp. 60–70, Jan. 1978.

[2] A.B. Carleial, "A case where interference does not reduce capacity," *IEEE Trans. Inform. Theory*, vol. 21, pp. 569–570, Sep. 1975.

[3] H. Sato, "The capacity of the Gaussian interference channel under strong interference," *IEEE Trans. Inform. Theory*, vol. 27, pp. 786–788, Nov. 1981.

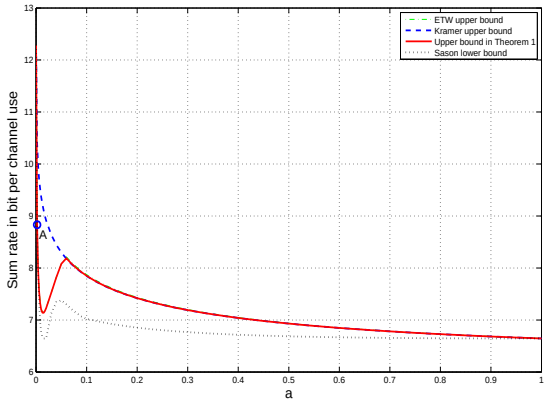


Fig. 3. Lower and upper bounds for the sum-rate capacity of symmetric Gaussian ICs with $a = b, P_1 = P_2 = 5000$. The channel gain at point A is $a = 0.002$.

[4] T.S. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inform. Theory*, vol. 27, pp. 49–60, Jan. 1981.

[5] M.H.M. Costa, "On the Gaussian interference channel," *IEEE Trans. Inform. Theory*, vol. 31, pp. 607–615, Sept. 1985.

[6] H.F. Chong, M. Motani, H.K. Garg, and H.E. Gamal, "On the Han-Kobayashi Region for the interference channel," *submitted to the IEEE Trans. Inform. Theory*, 2006.

[7] G. Kramer, "Review of rate regions for interference channels," in *International Zurich Seminar*, Zurich, Feb.22-24 2006, pp. 162–165.

[8] H. Sato, "Two-user communication channels," *IEEE Trans. Inform. Theory*, vol. 23, pp. 295–304, May 1977.

[9] A.B. Carleial, "Outer bounds on the capacity of interference channels," *IEEE Trans. Inform. Theory*, vol. 29, pp. 602–606, July 1983.

[10] G. Kramer, "Outer bounds on the capacity of Gaussian interference channels," *IEEE Trans. on Inform. Theory*, vol. 50, pp. 581–586, Mar. 2004.

[11] R. H. Etkin, D. N. C. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *submitted to the IEEE Trans. Inform. Theory*, 2007.

[12] E. Telatar and D. Tse, "Bounds on the capacity region of a class of interference channels," in *Proc. IEEE International Symposium on Information Theory 2007*, Nice, France, Jun. 2007.

[13] T. Liu and P. Viswanath, "An extremal inequality motivated by multiterminal information-theoretic problems," *IEEE Trans. Inform. Theory*, vol. 53, no. 5, pp. 1839–1851, May 2006.

[14] H. Sato, "On degraded Gaussian two-user channels," *IEEE Trans. Inform. Theory*, vol. 24, pp. 634–640, Sept. 1978.

[15] H. Weingarten, Y. Steinberg, and S. Shamai (Shitz), "The capacity region of the Gaussian multiple-input multiple-output broadcast channel," *IEEE Trans. Inform. Theory*, vol. 52, no. 9, pp. 3936–3964, Sep. 2006.

[16] I. Sason, "On achievable rate regions for the Gaussian interference channels," *IEEE Trans. Inform. Theory*, vol. 50, pp. 1345–1356, June 2004.

[17] X. Shang, G. Kramer, and B. Chen, "A new outer bound and the noisy-interference sum-rate capacity for Gaussian interference channels," *submitted to the IEEE Trans. Inform. Theory. http://arxiv.org/abs/0712.1987*, 2007.