New Outer Bounds on the Capacity Region of Gaussian Interference Channels

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***Abstract*— An outer bound on the capacity region of the two-user Gaussian interference channel is derived. The bound shows that for low power and small crosstalk coefficients the sum-rate capacity is achieved by treating interference as noise. The results are generalized to multiuser channels.**

Index terms — capacity, Gaussian noise, interference.

I. Introduction

The interference channel (IC) models communication systems where transmitters communicate with their respective receivers while causing interference to all other receivers. For a two-user Gaussian IC, the channel output can be written in the standard form [1]

$$\begin{aligned} Y_1 &= X_1 + \sqrt{a}X_2 + Z_1, \\ Y_2 &= \sqrt{b}X_1 + X_2 + Z_2, \end{aligned}$$

where $\sqrt{a}$ and $\sqrt{b}$ are channel coefficients, X_i and Y_i are the transmit and receive signals. The user/channel input sequence $X_{i1}, X_{i2}, \cdots, X_{in}$ is subject to the power constraint $\sum_{j=1}^{n} \mathcal{E}(X_{ij}^2) \leq nP_i$, $i = 1, 2$. The transmitted signals X_1 and X_2 are statistically independent. The channel noises Z_1 and Z_2 are possibly correlated unit variance Gaussian random variables, and (Z_1, Z_2) is statistically independent of (X_1, X_2). In the following, we denote this Gaussian IC as $\text{IC}(a, b, P_1, P_2)$.

The capacity region of an IC is defined as the closure of the set of rate pairs (R_1, R_2) for which both receivers can decode their own messages with arbitrarily small positive error probability. The capacity region of a Gaussian IC is known only for three cases:

- $a = 0$, $b = 0$.
- $a \geq 1$, $b \geq 1$: see [2]–[4].
- $a = 0$, $b \geq 1$; or $a \geq 1$, $b = 0$: see [5].

For the second case both receivers can decode the messages of both transmitters. Thus this IC acts as two multiple access channels (MACs), and the capacity region for the IC is the intersection of the capacity region of the two MACs. However, when the interference is weak or moderate, the capacity region is still unknown. The best inner bound is obtained in [4] by using superposition coding and joint decoding. A simplified form of the Han-Kobayashi region was given by Chong-Motani-Garg-El Gamal [6], [7]. Various outer bounds have been developed in [8]–[12]. Kramer in [10] presented two outer bounds. The first is obtained by providing each receiver with just enough information to decode both messages. The second is obtained by reducing the IC to a degraded broadcast channel. Both bounds dominate the bounds by Sato [8] and Carleial [9]. The recent outer bounds by Etkin, Tse, and Wang in [11] are also based on genie-aided methods, and they show that Han and Kobayashi's inner bound is within one bit or a factor of two of the capacity region. This result can also be established by the methods of Telatar and Tse [12]. We remark that neither of the bounds of [10] and [11] implies each other. Numerical results show that the bounds of [10] are better at low SNR while those of [11] are better at high SNR. The bounds of [12] are not amenable to numerical evaluation since the optimal distributions of the auxiliary random variables are unknown.

In this paper, we present new outer bounds on the capacity region of Gaussian ICs that generalize results of [13]. The new bound is based on a genie-aided approach and an extremal inequality proposed by Liu and Viswanath [14]. Based on this outer bound, we obtain new sum-rate capacity results for ICs satisfying some channel coefficient and power constraint conditions. We show that the sum-rate capacity can be achieved by treating the interference as noise when both the crosstalk gains and transmit powers are weak. We say that such channels exhibit *noisy interference*. For this class of interference, the simple single-user transmission and detection strategy is sum-rate optimal.

This paper is organized as follows. In Section II, we present an outer bound and the resulting sum-rate capacity for certain 2-user Gaussian ICs. An extension of the sum-rate capacity under noisy interference to m-user ICs is provided in Section III. Numerical examples are given in Section IV, and Section V concludes the paper.

II. A Genie-aided Outer Bound

A. General outer bound

The following is a new outer bound on the capacity region of Gaussian ICs. Note that in contrast to [13] these bounds permit $P_1 \neq P_2$ and $a \neq b$.

Theorem 1: If the rates (R_1, R_2) are achievable for $\text{IC}(a, b, P_1, P_2)$ with $0 < a < 1, 0 < b < 1$, they must satisfy

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$$
\begin{aligned}
R_1+\mu R_2 &\le \min_{\substack{\rho_i\in[0,1]\\(\sigma_1^2,\sigma_2^2)\in\Sigma}} \frac{1}{2}\log\left(1+\frac{P_1^*}{\sigma_1^2}\right)-\frac{1}{2}\log\left(aP_2^*+1-\rho_1^2\right)+\frac{1}{2}\log\left(1+P_1+aP_2-\frac{(P_1+\rho_1\sigma_1)^2}{P_1+\sigma_1^2}\right)\\
&\quad+\frac{\mu}{2}\log\left(1+\frac{P_2^*}{\sigma_2^2}\right)-\frac{\mu}{2}\log\left(bP_1^*+1-\rho_2^2\right)+\frac{\mu}{2}\log\left(1+P_2+bP_1-\frac{(P_2+\rho_2\sigma_2)^2}{P_2+\sigma_2^2}\right) \quad (1)\\
R_1+\eta_1 R_2 &\le \frac{1}{2}\log\left(1+\frac{b\eta_1-1}{b-b\eta_1}\right)-\frac{\eta_1}{2}\log\left(1+\frac{b\eta_1-1}{1-\eta_1}\right)+\frac{\eta_1}{2}\log\left(1+bP_1+P_2\right) \quad (2)\\
R_1+\eta_2 R_2 &\le \frac{1}{2}\log\left(1+P_1+aP_2\right)-\frac{1}{2}\log\left(1+\frac{a-\eta_2}{\eta_2-1}\right)+\frac{\eta_2}{2}\log\left(1+\frac{a-\eta_2}{a\eta_2-a}\right). \quad (3)
\end{aligned}
$$

the following constraints (1)-(3) for $\mu>0$, $\frac{1+bP_1}{b+bP_1}\le\eta_1\le\frac{1}{b}$ and $a\le\eta_2\le\frac{a+aP_2}{1+aP_2}$, where

$$
\Sigma=\begin{cases}\left\{(\sigma_1^2,\sigma_2^2)\ |\sigma_1^2>0,0<\sigma_2^2\le\frac{1-\rho_1^2}{a}\right\}, & \text{if } \mu\ge 1,\\ \left\{(\sigma_1^2,\sigma_2^2)\ |0<\sigma_1^2\le\frac{1-\rho_2^2}{b},\sigma_2^2>0\right\}, & \text{if } \mu<1,\end{cases} \quad (4)
$$

and if $\mu\ge 1$ we define

$$
P_1^*=\begin{cases}P_1, & \sigma_1^2\le\left[\frac{(1-\mu)P_1}{\mu}+\frac{1-\rho_2^2}{b\mu}\right]^+,\\ \frac{1-\rho_2^2-b\mu\sigma_1^2}{b\mu-b}, & \left[\frac{(1-\mu)P_1}{\mu}+\frac{1-\rho_2^2}{b\mu}\right]^+<\sigma_1^2\le\frac{1-\rho_2^2}{b\mu},\\ 0, & \sigma_1^2>\frac{1-\rho_2^2}{b\mu},\end{cases} \quad (5)
$$

$$
P_2^*=P_2, \qquad \sigma_2^2\le\frac{1-\rho_1^2}{a}, \quad (6)
$$

where $(x)^+\triangleq\max\{x,0\}$, and if $0<\mu<1$ we define

$$
P_1^*=P_1, \qquad \sigma_1^2\le\frac{1-\rho_2^2}{b}, \quad (7)
$$

$$
P_2^*=\begin{cases}P_2, & \sigma_2^2\le\left[(\mu-1)P_2+\frac{\mu(1-\rho_1^2)}{a}\right]^+,\\ \frac{\mu(1-\rho_1^2)-a\sigma_2^2}{a-a\mu}, & \left[(\mu-1)P_2+\frac{\mu(1-\rho_1^2)}{a}\right]^+\\ & \qquad<\sigma_2^2\le\frac{\mu(1-\rho_1^2)}{a},\\ 0, & \sigma_2^2>\frac{\mu(1-\rho_1^2)}{a}.\end{cases} \quad (8)
$$

Proof: We give a sketch of the proof. A genie provides the two receivers with side information X_1+N_1 and X_2+N_2 respectively, where N_i is Gaussian distributed with variance σ_i^2 and $\mathcal{E}(N_iZ_i)=\rho_i\sigma_i$, $i=1,2$. Starting from Fano's inequality, we have

$$
\begin{aligned}
&n(R_1+\mu R_2)-n\epsilon\\
&\le I\left(X_1^n;Y_1^n\right)+\mu I\left(X_2^n;Y_2^n\right)\\
&\le I\left(X_1^n;Y_1^n,X_1^n+N_1^n\right)+\mu I\left(X_2^n;Y_2^n,X_2^n+N_2^n\right)\\
&=\left[h\left(X_1^n+N_1^n\right)-\mu h\left(\sqrt{b}X_1^n+Z_2^n|N_2^n\right)\right]-h\left(N_1^n\right)\\
&\quad+\left[\mu h\left(X_2^n+N_2^n\right)-h\left(\sqrt{a}X_2^n+Z_1^n|N_1^n\right)\right]-\mu h\left(N_2^n\right)\\
&\quad+h\left(Y_1^n|X_1^n+N_1^n\right)+\mu h\left(Y_2^n|X_2^n+N_2^n\right), \quad (9)
\end{aligned}
$$

where $\epsilon\to 0$ as $n\to\infty$. For $h\left(Y_1^n|X_1^n+N_1^n\right)$, zero-mean Gaussian X_1^n and X_2^n with covariance matrices $P_1\mathbf{I}$ and $P_2\mathbf{I}$ are optimal, and we have

$$
\begin{aligned}
&h\left(Y_1^n|X_1^n+N_1^n\right)\\
&\le\frac{n}{2}\log\left[2\pi e\left(1+aP_2+P_1-\frac{(P_1+\rho_1\sigma_1)^2}{P_1+\sigma_1^2}\right)\right]. \quad (10)
\end{aligned}
$$

From the extremal inequality introduced in [14, Theorem 1, Corollary 4], we have

$$
\begin{aligned}
&h\left(X_1^n+N_1^n\right)-\mu h\left(\sqrt{b}X_1^n+Z_2^n|N_2^n\right) \quad (11)\\
&\le\frac{n}{2}\log\left[2\pi e\left(P_1^*+\sigma_1^2\right)\right]-\frac{n\mu}{2}\log\left[2\pi e\left(bP_1^*+1-\rho_2^2\right)\right],
\end{aligned}
$$

and

$$
\begin{aligned}
&\mu h\left(X_2^n+N_2^n\right)-h\left(\sqrt{a}X_2^n+Z_1^n|N_1^n\right) \quad (12)\\
&\le\frac{n\mu}{2}\log\left[2\pi e\left(P_2^*+\sigma_2^2\right)\right]-\frac{\mu}{2}\log\left[2\pi e\left(aP_2^*+1-\rho_1^2\right)\right],
\end{aligned}
$$

where equalities hold when X_1^n and X_2^n are both Gaussian with covariance matrices $P_1^*\mathbf{I}$ and $P_2^*\mathbf{I}$ respectively. From (9)-(12) we obtain the outer bound (1).

The outer bound in (2) (resp. (3)) is obtained by letting the genie provide side information X_2 to receiver one (resp. X_1 to receiver two), and applying the extremely inequality, i.e.,

$$
\begin{aligned}
&n(R_1+\eta_1R_2)-n\epsilon\\
&\le I\left(X_1^n;Y_1^n,X_2^n\right)+\eta_1 I\left(X_2^n;Y_2^n\right)\\
&=h\left(X_1^n+Z_1^n\right)-\eta_1 h\left(\sqrt{b}X_1^n+Z_2^n\right)-h\left(Z_1^n\right)\\
&\quad+\eta_1 h\left(Y_2^n\right)\\
&\le\frac{n}{2}\log\left(\tilde{P}_1+1\right)-\frac{n\eta_1}{2}\log\left(b\tilde{P}_1+1\right)\\
&\quad+\frac{n\eta_1}{2}\log\left(1+bP_1+P_2\right), \quad (13)
\end{aligned}
$$

where $\tilde{P}_1=\frac{b\eta_1-1}{b-b\eta_1}$ for $\frac{1+bP_1}{b+bP_1}\le\eta_1\le\frac{1}{b}$. This is the bound in (2). Similarly, we obtain bound (3). ■

Remark 1: The bounds (1)-(3) are obtained by providing different genie-aided signals to the receivers. There is overlap of the range of μ, η_1, and η_2, and none of the bounds uniformly dominates the other two bounds.

Remark 2: Equations (2) and (3) are also outer bounds for the capacity region of a Z-IC, and a Z-IC is equivalent to a degraded IC [5]. For such channels, it can be shown that (2) and (3) are the same as the outer bounds in [15]. For $\eta_1=\frac{1+bP_1}{b+bP_1}$ and $\eta_2=\frac{a+aP_2}{1+aP_2}$, the bounds in (2) and (3) are tight for a Z-IC (or degraded IC) because $\tilde{P}_1=P_1,\tilde{P}_2=P_2$ in

(13), and there is no power sharing between the transmitters. Consequently, $\frac{1+bP_1}{b+bP_1}$ and $\frac{a+aP_2}{1+aP_2}$ are the negative slopes of the tangent lines for the capacity region at the corner points.

Remark 3: The bounds in (2)-(3) turn out to be the same as the bounds in [10, Theorem 2]. This can be shown by rewriting the bounds in [10, Theorem 2] in the form of a weighted sum rate.

Remark 4: The bounds in [10, Theorem 2] are obtained by getting rid of one of the interference links to reduce the IC into a Z-IC. In addition, the proof in [10] allowed the transmitters to share their power, which further reduces the Z-IC into a degraded broadcast channel. Then the capacity region of this degraded broadcast channel is an outer bound for the capacity region of the original IC. The bounds in (2) and (3) are also obtained by reducing the IC to a Z-IC. Although we do not explicitly allow the transmitters to share their power, it is interesting that these bounds are equivalent to the bounds in [10, Theorem 2] with power sharing. In fact, a careful examination of our derivation reveals that power sharing is implicitly assumed. For example, for the term $h\left(X_1^n+Z_1^n\right)-\eta_1 h\left(\sqrt{b}X_1^n+Z_2^n\right)$ of (13), user 1 uses power $\tilde{P}_1=\frac{b\eta_1-1}{b-b\eta_1}\leq P_1$, while for the term $\eta_1 h\left(Y_2^n\right)$ user 1 uses all the power P_1. This is equivalent to letting user 1 use the power $\tilde{P}_1$ for both terms, and letting user 2 use a power that exceeds P_2.

Remark 5: Theorem 1 improves [11, Theorem 3]. Specifically, the bound in (2) is tighter than the first sum-rate bound of [11, Theorem 3]. Similarly, the bound in (3) is tighter than the second sum-rate bound of [11, Theorem 3]. The third sum-rate bound in [11, Theorem 3] is a special case of (1).

Remark 6: Our outer bound is not always tighter than that of [11] for all rate points. The reason is that in [11, last two equations of (39)], different genie-aided signals are provided to the same receiver. Our outer bound can also be improved in a similar and more general way by providing different genie-aided signals to the receivers. Specifically the starting point of the bound can be modified to be

$$\begin{aligned} n\left(R_1+\mu R_2\right) &\leq \sum_{i=1}^{k}\lambda_i I\left(X_1^n;Y_1^n,U_i\right) \\ &\quad +\sum_{j=1}^{m}\mu_i I\left(X_2^n;Y_2^n,W_j\right)+n\epsilon, \end{aligned} \quad (14)$$

where $\sum_{i=1}^{k}\lambda_i=1, \sum_{j=1}^{m}\mu_j=\mu, \lambda_i>0, \mu_j>0$.

B. Sum-rate capacity for noisy interference

The outer bound in Theorem 1 is in the form of an optimization problem. Four parameters $\rho_1,\rho_2,\sigma_1^2,\sigma_2^2$ need to be optimized for different choices of the weight μ. When $\mu=1$, the bound (1) of Theorem 1 leads directly to the following sum-rate capacity result.

Theorem 2: For the IC(a,b,P_1,P_2) satisfying

$$\sqrt{a}(bP_1+1)+\sqrt{b}(aP_2+1)\leq 1, \quad (15)$$

the sum-rate capacity is

$$C=\frac{1}{2}\log\left(1+\frac{P_1}{1+aP_2}\right)+\frac{1}{2}\log\left(1+\frac{P_2}{1+bP_1}\right). \quad (16)$$

Proof: By choosing

$$\begin{aligned} \sigma_1^2 &= \frac{1}{2b}\Big\{b(aP_2+1)^2-a(bP_1+1)^2+1 \\ &\quad \pm\sqrt{\left[b(aP_2+1)^2-a(bP_1+1)^2+1\right]^2-4b(aP_2+1)^2}\Big\} \\ \sigma_2^2 &= \frac{1}{2a}\Big\{a(bP_1+1)^2-b(aP_2+1)^2+1 \\ &\quad \pm\sqrt{\left[a(bP_1+1)^2-b(aP_2+1)^2+1\right]^2-4a(bP_1+1)^2}\Big\} \end{aligned}$$

$$\rho_1=\sqrt{1-a\sigma_2^2} \quad (17)$$

$$\rho_2=\sqrt{1-b\sigma_1^2}, \quad (18)$$

the bound (1) with $\mu=1$ is

$$R_1+R_2\leq\frac{1}{2}\log\left(1+\frac{P_1}{1+aP_2}\right)+\frac{1}{2}\log\left(1+\frac{P_2}{1+bP_1}\right). \quad (19)$$

But one can achieve equality in (19) by treating the interference as noise at both receivers. In order that the choices of σ_i^2 and ρ_i^2 are feasible, (15) must be satisfied. ■

Remark 7: Consider the bound (1) with $\mu=1$, we further let

$$1-\rho_1^2\geq a\sigma_2^2, \quad 1-\rho_2^2\geq b\sigma_1^2. \quad (20)$$

From (5) and (6) we have $P_1^*=P_1, P_2^*=P_2$. Thus,

$$\begin{aligned} R_1 &\leq \frac{1}{2}\log\left(1+\frac{P_1}{\sigma_1^2}\right)-\frac{1}{2}\log(aP_2+1-\rho_1^2) \\ &\quad +\frac{1}{2}\log\left[1+aP_2+P_1-\frac{(P_1+\rho_1\sigma_1)^2}{P_1+\sigma_1^2}\right] \\ &= \frac{1}{2}\log\left[\frac{P_1(1+aP_2)}{1+aP_2-\rho_1^2}\left(\frac{1}{\sigma_1}-\frac{\rho_1}{1+aP_2}\right)^2+1+\frac{P_1}{1+aP_2}\right] \\ &\triangleq f(\rho_1,\sigma_1). \end{aligned} \quad (21)$$

Therefore, for any given ρ_1, when

$$\rho_1\sigma_1=1+aP_2, \quad (22)$$

then $f(\rho_1,\sigma_1)$ achieves its minimum which is user 1's single-user detection rate. Similarly, we have $\rho_2\sigma_2=1+bP_1$. Since the constraint in (20) must be satisfied, we have

$$\frac{1+aP_2}{\rho_1}\leq\sqrt{\frac{1-\rho_2^2}{b}}, \quad \frac{1+bP_1}{\rho_2}\leq\sqrt{\frac{1-\rho_1^2}{a}}. \quad (23)$$

As long as there exists a $\rho_i\in(0,1)$ such that (23) is satisfied, we can choose σ_i to satisfy (22) and hence the bound in (1) is tight. By cancelling ρ_1,ρ_2, we obtain (15).

Remark 8: The most special choices of ρ_1,ρ_2 are in (17) and (18), since (11) and (12) with $\mu=1$ become

$$\begin{aligned} h\left(X_1^n+N_1^n\right)-h\left(\sqrt{b}X_1^n+Z_2^n|N_2^n\right) &= -n\log\sqrt{b} \\ h\left(X_2^n+N_2^n\right)-h\left(\sqrt{a}X_2^n+Z_1^n|N_1^n\right) &= -n\log\sqrt{a}. \end{aligned}$$

Therefore, we do not need the extremal inequality [14] to prove Theorem 2.

Remark 9: The sum-rate capacity for a Z-IC with $a = 0$, $0 < b < 1$ is a special case of Theorem 2 since (15) is satisfied. The sum-rate capacity is therefore given by (16).

Remark 10: Theorem 2 follows directly from Theorem 1 with $\mu = 1$. It is remarkable that a genie-aided bound is tight if (15) is satisfied since the genie provides extra signals to the receivers without increasing the rates. This situation is reminiscent of the recent capacity results for vector Gaussian broadcast channels (see [16]). Furthermore, the sum-rate capacity (16) is achieved by treating the interference as noise. We therefore refer to channels satisfying (15) as ICs with *noisy interference*.

Remark 11: For a symmetric IC where $a = b, P_1 = P_2 = P$, the constraint (15) implies that

$$a \le \frac{1}{4}, \quad P \le \frac{\sqrt{a} - 2a}{2a^2}. \tag{24}$$

Noisy interference is therefore *weaker* than *weak interference* as defined in [5] and [17], namely $a \le \frac{\sqrt{1+2P}-1}{2P}$ or

$$a \le \frac{1}{2}, \quad P \le \frac{1-2a}{2a^2}. \tag{25}$$

Recall that [17] showed that for "weak interference" satisfying (25), treating interference as noise achieves larger sum rate than time- or frequency-division multiplexing (TDM/FDM), and [5] claimed that in "weak interference" the largest known achievable sum rate is achieved by treating the interference as noise.

III. Sum-rate Capacity for m-user IC with Noisy Interference

For an m-user IC, the receive signal at user i is defined as

$$Y_i = X_i + \sum_{j=1, j\neq i}^{m} \left(\sqrt{c_{ji}} X_j\right) + Z_i, \quad i = 1, 2, \ldots, m, \tag{26}$$

where c_{ji} is the channel gain from jth transmitter to ith receiver, Z_i is unit-variance Gaussian noise, and the transmit signals have the block power constraints $\sum_{l=1}^{n} \mathcal{E}(X_{il}^2) \le nP_i$. We have the following sum-rate capacity result.

Theorem 3: For an m-user IC defined in (26), if there exist $\rho_i \in (0,1), i = 1, \ldots, m$, such that the following conditions are satisfied

$$\sum_{j=1, j\neq i}^{m} \frac{c_{ji}(1+Q_j)^2}{\rho_j^2} \le 1 - \rho_i^2 \tag{27}$$

$$\sum_{j=1, j\neq i}^{m} \frac{c_{ij}}{1 + Q_j - \rho_j^2} \le \frac{1}{P_i + (1+Q_i)^2/\rho_i^2}, \tag{28}$$

where Q_i is the interference power at receiver i, defined as

$$Q_i = \sum_{j=1, j\neq i}^{m} c_{ji} P_j, \tag{29}$$

the sum-rate capacity is

$$C = \frac{1}{2} \sum_{i=1}^{m} \log\left(1 + \frac{P_i}{1+Q_i}\right). \tag{30}$$

Similar to the proof of bound (1) in Theorem 1, we assume a genie provides receiver i with $X_i + N_i$ where N_i is Gaussian noise with variance σ_i^2 that is correlated with Z_i with correlation coefficient ρ_i. Then from Fano's inequality and the entropy power inequality, it can be shown that the best upper bound on the sum rate is (30). Specifically, to show that Gaussian X_i are optimal, the following condition must be satisfied

$$\sum_{j=1, j\neq i}^{m} c_{ji} \sigma_j^2 \le 1 - \rho_i^2. \tag{31}$$

Furthermore, the following condition makes Gaussian X_i with variance P_i optimal

$$\sum_{j=1, j\neq i}^{m} \frac{c_{ij}}{1 + Q_j - \rho_j^2} \le \frac{1}{P_i + \sigma_i^2}. \tag{32}$$

Finally by setting the resulting upper bound to be equal to the the single user detection sum rate we have

$$\rho_i \sigma_i = 1 + Q_i. \tag{33}$$

By substituting (33) into (31) and (32) we obtain (27) and (28). Details of the proof are omitted due to space limitations.

Therefore, if there exist $\rho_1, \ldots, \rho_m$, such that (27) and (28) are satisfied for all $i = 1, \ldots, m$, the sum-rate capacity of an m-user IC can be achieved by treating interference as noise. It is clear that Theorem 2 is a special case of Theorem 3.

Consider a uniformly symmetric m-user IC where $c_{ji} = c$, for all $i, j = 1, \ldots, m, i \neq j$, and $P_i = P$. The bounds (27) and (28) with $\rho_i = \rho$ for all i reduce to

$$c \le \frac{1}{4(m-1)}, \quad P \le \frac{\sqrt{(m-1)c} - 2(m-1)c}{2(m-1)^2 c^2}. \tag{34}$$

IV. Numerical examples

A comparison of the outer bounds for a Gaussian IC is given in Fig. 1. Some part of the outer bound from Theorem 1 overlaps with Kramer's outer bound due to (2) and (3). Since this IC has noisy interference, the proposed outer bound coincides with the inner bound at the sum rate point.

The lower and upper bounds for the sum-rate capacity of the symmetric IC are shown in Fig 2 for high power level. The upper bound is tight up to point A. The bound in [11, Theorem 3] approaches the bound in Theorem 1 when the power is large, but there is still a gap. Fig. 2 also provides a definitive answer to a question from [17, Fig. 2]: whether the sum-rate capacity of symmetric Gaussian IC is a decreasing function of a, or there exists a bump like the lower bound when a varies from 0 to 1. In Fig. 2, our proposed upper bound and Sason's inner bound explicitly show that the sum capacity is not a monotone function of a (this result also follows by the bounds of [11]).

V. Conclusions, extensions and parallel work

We derived an outer bound for the capacity region of 2-user Gaussian ICs by a genie-aided method. From this outer bound, the sum-rate capacity for ICs that satisfy (15) is obtained. The sum-rate capacity for m-user Gaussian ICs that satisfy (27) and (28) are also obtained.

Finally, we wish to acknowledge parallel work. After submitting our 2-user bounds and capacity results on the arXiv.org e-Print archive [18], two other teams of researchers - Motahari and Khandani from the University of Waterloo, Annapureddy and Veeravalli from the University of Illinois at Urbana-Champaign - let us know that they derived the same 2-user sum-rate capacity results which are included in [19] and [20].

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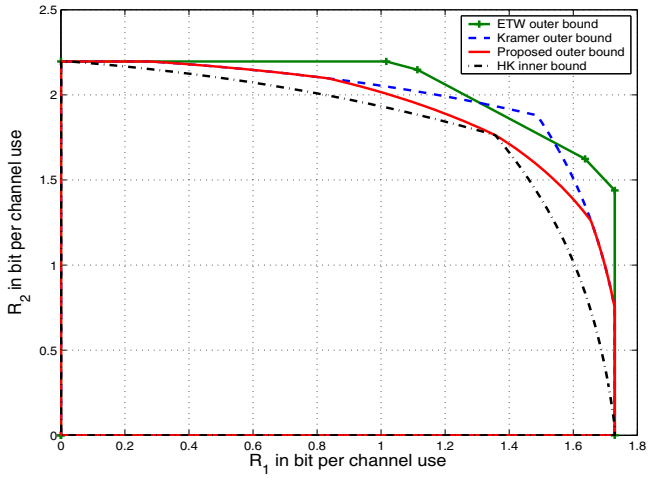


Fig. 1. Inner and outer bounds for the capacity region of Gaussian ICs with $a = 0.04, b = 0.09, P_1 = 10, P_2 = 20$. The ETW bound is by Etkin, Tse and Wang in [11, Theorem 3]; the Kramer bound is from [10, Theorem 2]; the HK inner bound is based on [4] by Han and Kobayashi.

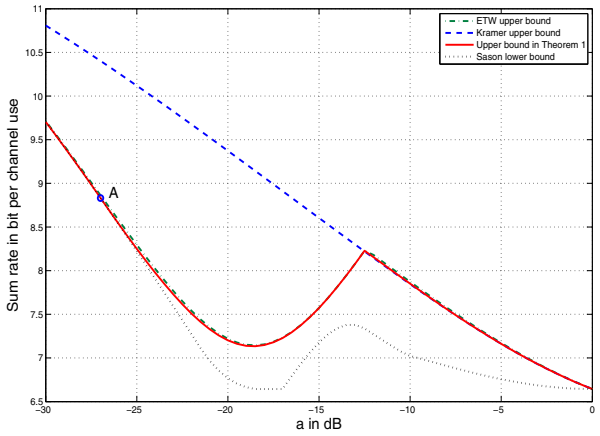


Fig. 2. Lower and upper bounds for the sum-rate capacity of symmetric Gaussian ICs with $a = b, P_1 = P_2 = 5000$. The channel gain at point A is $a = -26.99$dB. Sason's bound is an inner bound obtained from Han and Kobayashi's bound by a special time sharing scheme [17, Table I].

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