A Coding Scheme for Generating Bipolar Dc–Free Sequences

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***Abstract*—A coding scheme for generating dc–free sequences over the bipolar alphabet $\{-1, 1\}$ is proposed. The scheme is based on dc–free coding over larger alphabets. The proposed technique is used to construct several new new dc–free codes with lower encoding and decoding complexity then the existing codes with comparable rates.**

I. Introduction

Magnetic recording systems often require that channel sequences have a spectral null at zero frequency. Such sequences are said to be *dc–free*. In partial response channels whose transfer function has a null at zero frequency, signaling with dc–free sequences gives a significant performance advantage. It is well known that the power density function of a sequence $\boldsymbol{x} = \{\ldots, x_{-1}, x_0, x_1, \ldots\}$ that can be obtained by a finite state machine vanishes at zero frequency if and only if its *running-digital-sum* (RDS), defined as $\mathrm{RDS}_i = \sum_{j=-\infty}^{i} x_j$, is bounded. See, for example, work by Yoshida and Yajima [1] and Pierobon [2].

Sequences of source symbols are translated into channel sequences with bounded RDS's by the means of dc–free modulation codes which can be finite-state codes [3] or block codes [4]. Several important implementation issues, such as quasi catastrophic error propagation and possibilities for suboptimal decoding, favor the use of block codes, where blocks of k source symbols, called information words, are mapped into blocks of n channel symbols called codewords. The simplest way to organize the mapping is to form a codebook of 2^k codewords and use a k-bit input word to specify (address) an n-bit codeword in the codebook. The ratio k/n defines the rate of the code.

To ensure that an arbitrary sequence of codewords has a bounded RDS, each codeword $\boldsymbol{w} = w_1, \ldots, w_n$ is required to have its *block-digital-sum* (BDS), defined as $\mathrm{BDS} = \sum_{j=1}^{n} w_j$, equal to zero. Codewords of bipolar symbols with BDS equal to zero are possible only if the codeword length n is even. The number of such codewords is then equal to $\binom{n}{n/2}$ but at most 2^k of them, where $k = \left\lfloor \log \binom{n}{n/2} \right\rfloor$, can be used to form a codebook for a k/n rate code. In magnetic recording applications codes of interest are expected to have rates higher than 3/4. This requires large block lengths and in turn large codebooks. For example, a code of rate $11/14 = .78\cdots$ requires a block length of 14 and a codebook of size $2^{11} = 2048$.

Large improvement can be achieved by using the technique of [5], as described in [6]. For example, a code of rate 0.775 can be organized in four codebooks with only 240 10-bit codewords combined. This approach, however, introduces additional complexity and the problem of large codebooks still remains. We propose a simple coding scheme which significantly reduces the codebook size for a given code rate, and reduces decoding complexity as well.

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II. Encoding Techniques

In the above method of dc–free block coding, each codeword was required to have a BDS equal to zero to ensure that an arbitrary sequence of codewords has a bounded RDS. Now the codewords are allowed to have different BDS's but the rules for their concatenation will be specified. We note that one of the rate 8/10 codes proposed recently by Fredrickson *et al.* in [7] is based on subsequence concatenation and is referred to as a code with *partitioned trellis structure*. The idea presented here is similar to the one used for construction of trellis codes for modems based on lattices and cosets [8].

Bipolar words of length l are grouped according to their block digital sums into codebooks $\mathcal{C}_i$, where $i \in \mathcal{A} \subseteq \{-l, -l+2, \cdots, l-2, l\}$ denotes the BDS of the words in $\mathcal{C}_i$. Note that these codebooks are of different sizes. Alphabet $\mathcal{A}$, which determines the number of codebooks, is chosen so that each codebook $\mathcal{C}_i$, $i \in \mathcal{A}$, contains at least 2^m words, for some positive integer m. The codebooks that contain more then 2^m words can be pruned by eliminating words that are *bad* for the channel, as for example words with long runs of same symbols for $1 - D$ channel.

Information words of length k are divided into two blocks of length p and $q \times m$, *i.e.*, $k = p + qm$, and translated into dc–free channel sequences as shown in Fig. 1. The first p bits are fed into a rate p/q dc–free code over alphabet $\mathcal{A}$, which serves to choose q codebooks $\mathcal{C}_{i_1}, \cdots, \mathcal{C}_{i_q}$. Each of the following q length m blocks of the information word is then used to select one of the 2^m bipolar words in $\mathcal{C}_{i_j}$, $j = 1, \cdots, q$. We see that there are two codes in this scheme: one is the *outer* rate p/q dc–free code, the other is the *inner* rate m/l look-up table code. The overall code rate is equal to $(p + qm)/(ql)$. If the rate p/q dc–free code is a block code, then the output is a sequence of dc–free words of length ql. We call an output word of length ql a

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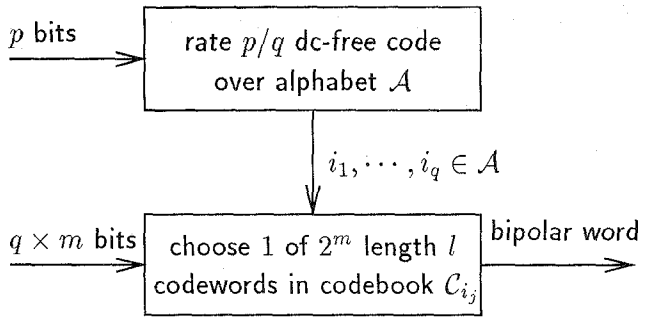


Fig. 1. Scheme for generating dc–free sequences at rate $(p+qm)/(ql)$

codeword, and its composing words of length l *subwords*.

This technique for generating dc–free sequences may at first seem complex, but we were able to design codes with much simpler encoders and decoders than the existing codes of comparable rates. The examples below illustrate the method and help appreciate reductions in complexity of encoders it allows. Reduced complexity decoders for codes designed based on the proposed technique are presented in [9] and [10].

III. Examples

A. An 11/14 Rate Code

We design a simple coding scheme of rate 11/14. Recall that a dc–free block code of this rate requires a codebook of 2048 14-bit codewords, and a dc–free code of slightly lower rate requires four codebooks with 240 10-bit codewords combined [6]. We choose for subwords the bipolar words of length $l = 7$. We form codebook $\mathcal{C}_1$ of 7-bit words with BDS equal to 1. There are $\binom{7}{3} = 35$ such codewords, and we choose $32 = 2^5$ of them for $\mathcal{C}_1$. Similarly we form codebook $\mathcal{C}_{-1}$ of 2^5 words with BDS equal to -1. Therefore $m = 5$ and $\mathcal{A} = \{-1, 1\}$. For the rate p/q dc–free code, we choose the rate 1/2 biphase code. Following the scheme shown in Fig. 1, we have an implementation shown in Fig. 2.

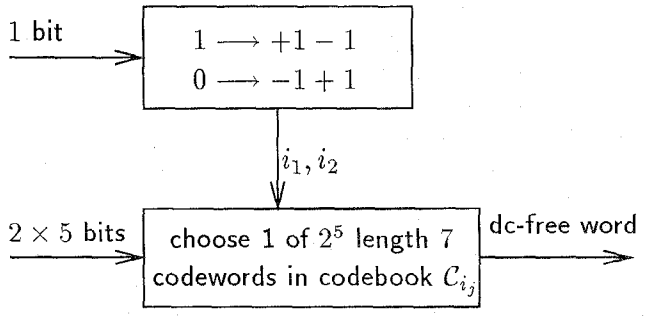


Fig. 2. Scheme for encoding $1+2\times5$ information bits into a dc–free codeword of 2×7 channel bits.

Implementation of the encoder may be simplified as follows. Note that, if the sign of each symbol of a codeword in $\mathcal{C}_1$ is inverted, the resulting *inverted* codeword has BDS equal to -1. Translation of an 11-bit input word into a 14-bit output dc–free block is done as shown in Fig. 3. We

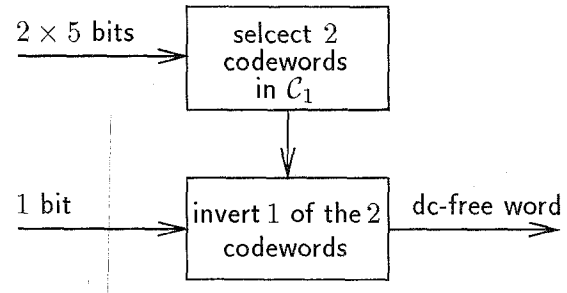


Fig. 3. Simplified scheme for encoding $1 + 2 \times 5$ information bits into a dc–free codeword of 2×7 channel bits.

divide the 11 bits into two 5-bit blocks and 1 additional bit. Each of the two 5-bit blocks serve to choose one 7-bit codeword in $\mathcal{C}_1$. The last bit in the input word specifies which of the two chosen codewords should be inverted.

B. A 46/56 Rate Code

As in the example above, we form two codebooks $\mathcal{C}_1$ and C_{-1} of 32 7-bit bipolar words. Therefore $m = 5$ and $\mathcal{A} = \{-1, 1\}$. For the rate p/q dc–free code, we choose a rate 6/8 dc–free code. Its encoder can be organized as a look-up table 8-bit bipolar words with BDS equal to 0. There are $\binom{8}{4} = 70$ such codewords, and we choose $64 = 2^6$ of them. Following the scheme shown in Fig. 1, we have an implementation shown in Fig. 4. Implementation

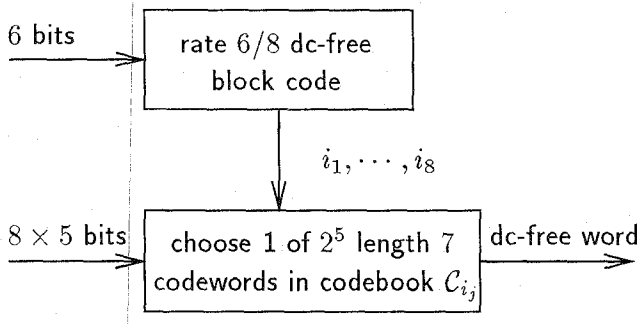


Fig. 4. Scheme for encoding $6+8\times5$ information bits into a dc–free codeword of 8×7 channel bits.

of the encoder may be further simplified as in the case of the 11/14 code above.

C. An 18/22 Rate Code

We choose for subwords the bipolar words of length $l = 11$. We form codebook $\mathcal{C}_1$ of 11-bit words with BDS equal to 1. There are $\binom{11}{5} = 462$ such words, and we choose $256 = 2^8$ of them for $\mathcal{C}_1$. Similarly we form codebook $\mathcal{C}_{-1}$ of 2^8 words with BDS equal to -1. In addition, we form codebook $\mathcal{C}_3$ of 11-bit words with BDS equal to 3. There are $\binom{11}{7} = 330$ such words, and we again choose $256 = 2^8$ of them for $\mathcal{C}_3$. Similarly we form codebook $\mathcal{C}_{-3}$ of 2^8 words with BDS equal to -3. Therefore $m = 8$ and $\mathcal{A} = \{-3, -1, 1, 3\}$. Following the scheme shown in Fig. 1, we have an implementation shown in Fig. 5. The rate p/q dc–free code is the 2/2 block code described in the figure.

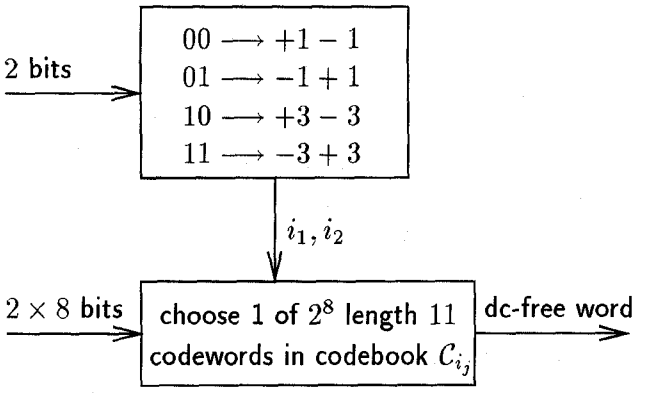


Fig. 5. Scheme for encoding $2+2\times 8$ information bits into a dc–free codeword of 2×11 channel bits.

Implementation of the encoder may be simplified as follows. We form only two codebooks of 11-bit words: $\mathcal{C}_1$ and $\mathcal{C}_3$. Translation of an 18-bit input word into a 22-bit output dc–free block is done as shown in Fig. 6. We

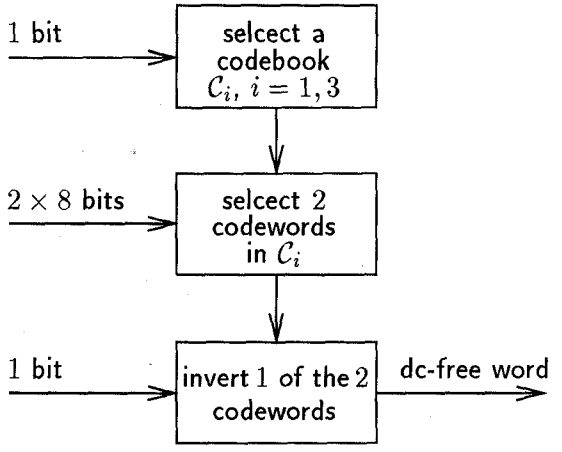


Fig. 6. Simplified scheme for encoding $1+1+2\times 8$ information bits into a dc–free codeword of 2×11 channel bits.

divide the 18 bits into two 8-bit blocks and 2 additional bits. Each of the two 8-bit blocks serves to choose one 11-bit subword in the same codebook $\mathcal{C}_i$, where $i \in \{1, 3\}$ is determined by the first of the last two bits in the input word. To form a 22-bit dc–free codeword, one of the two chosen codewords should be inverted, and which one is determined by the second of of the last two bits in the input word.

D. A 0.875 Rate Code

We briefly describe a rate 0.875 code. As in the example of rate 18/22 code of the previous subsection, we choose for subwords the bipolar words of length $l = 11$. We form codebook $\mathcal{C}_1$ of 11-bit words with BDS equal to 1. There are $\binom{11}{5} = 462$ such words, and this time we choose $432 = 2^8 + 2^7 + 2^5 + 2^4$ of them for $\mathcal{C}_1$. Similarly we form codebook $\mathcal{C}_{-1}$ of 432 words with BDS equal to -1. In addition, we form codebook $\mathcal{C}_3$ of 11-bit words with BDS equal to 3. There are $\binom{11}{7} = 330$ such codewords, and we choose $320 = 2^8 + 2^6$ of them for $\mathcal{C}_3$. Similarly we form codebook $\mathcal{C}_{-3}$ of 320 words with BDS equal to -3. The inner code can now be realized as described in [6]. With an outer rate 5/4 dc–free code over $\mathcal{A} = \{-3, -1, 1, 3\}$ we have a coding scheme of rate .875.

IV. Conclusions

We proposed a coding scheme for generating dc–free sequences that allowed construction of several new dc–free codes with lower encoding and decoding complexity then the existing codes with comparable rates. The codes we presented here to illustrate the proposed code construction technique are most suitable for magnetic recording applications in terms of the rate vs. complexity tradeoff. We constructed some other dc–free codes using the method presented here together with the method of [6] for addressing points in multidimensional constellations which in this case removes the restriction that the size of a codebook $\mathcal{C}$ be 2^m where m is an integer.

The encoding technique described here may be beneficial in other circumstances. For example some modulation schemes for optical communications may be looked at as block codes in which each codeword is required to have a given BDS not necessarily equal to zero. Such codewords can then be built by concatenating appropriate subwords as determined by an outer code.

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