Punctured vs Rateless Codes for Hybrid ARQ

Emina Soljanin
Bell Labs, Lucent
Murray Hill NJ 07974, USA
Email: emina@lucent.com

Nedeljko Varnica
Marvell Semiconductor, Inc.
Santa Clara, CA 95054
Email: nvarnica@marvell.com

Philip Whiting
Bell Labs, Lucent
Murray Hill NJ 07974, USA
Email: pawhiting@lucent.com

***Abstract*— Two incremental redundancy hybrid ARQ (IR-HARQ) schemes are compared: one is based on LDPC code ensembles with random transmission assignments, the other is based on recently introduced Raptor codes. A number of important issues, such as rate and power control, and error rate performance after each transmission on time varying binary-input, symmetric-output channels are addressed by analyzing performance of LDPC and Raptor codes on parallel channels. The theoretical results obtained for random code ensembles are tested on several practical code examples by simulation. Both theoretical and simulation results show that both LDPC and Raptor codes are suitable for HARQ schemes. Which codes would make a better choice depends mainly on the width of the operating range of the HARQ scheme, prior knowledge of that range, and other design parameters and constraints dictated by standards.**

I. Introduction

In conventional *a*utomatic *r*epeat re*q*uest (ARQ) schemes, frame errors are examined at the receiving end by an error detecting (usually cyclic redundancy check (CRC)) code. If a frame passes the CRC, the receiving end sends an acknowledgement (ACK) of successful transmission to the receiver. If a frame does not pass the CRC, the receiving end sends a negative acknowledgement (NAK), requesting retransmission. User data and its CRC bits may be additionally protected by an error correcting code which increases the probability of successful transmission. Such ARQ schemes which combine the ARQ principle with error control coding are known as *h*ybrid ARQ (HARQ) schemes.

The standard measure of ARQ protocol efficiency is *throughput*, defined as the average number of user data bits accepted at the receiving end in the time required for transmission of a single bit. Therefore the level of redundancy of the error correcting code employed in an HARQ scheme has two opposing effects on the scheme efficiency, namely, with increased redundancy the probability of successful transmission increases but the percentage of user data in the frame decreases. Usually, a fixed rate code which is well suited to the channel characteristics and throughput requirements is selected.

In applications with fluctuating channel conditions within a range of signal-to-noise ratios (SNRs), such as mobile and satellite packet data transmission, the so called *incremental redundancy* (IR) HARQ schemes exhibit higher throughput efficiency by adapting their error correcting code redundancy to different channel conditions. At the transmitter, the information and CRC bits are encoded by a systematic "mother" code. Initially, only the systematic part of the codeword and a selected number of parity bits are transmitted. The selected parity bits together with the systematic bits form a codeword of a punctured mother code. Decoding of this code is performed at the receiving end. If a retransmission is requested, the transmitter sends additional parity bits possibly under different channel conditions or at different power. Decoding is again attempted at the receiving end, where the new parity bits are combined with those previously received. The procedure is repeated after each subsequent retransmission request until all the parity bits of the mother code are transmitted.

A historic overview of hybrid ARQ schemes up to 1998, can be found in [1]. Recent interest in the schemes comes from the quest for reliable and efficient transmission under fluctuating conditions in wireless networks. An information-theoretic analysis of some HARQ protocols, concerning throughput and average delay for block-fading, Gaussian, collision channels was reported in [2]. Another line of recent work on HARQ is concerned with the mother code and its puncturing since the throughput in these schemes is strongly affected by the power of the mother code used in the system and the family of codes obtained by puncturing [3], [4], [5], [6].

The HARQ scheme of the third generation wireless standards is based on powerful turbo codes. There are two crucial questions in this application of HARQ: 1) how to evaluate the error rate performance after each transmission (which is equivalent to evaluating performance of punctured codes on time varying channels [7]), and 2) how to choose the signal power and the number of bits for transmission m after a failed transmission $m-1$. In [8], both questions were successfully addressed by analyzing performance of IR-HARQ schemes averaged over certain ensembles of turbo codes and all possible transmission assignments (or puncturing patterns) of the mother code bits.

Throughout the paper we suppose that the channel is Binary Input Symmetric Channel (BISO). Input is taken from one of two discrete symbols and the channel is additive noise (discrete or continuous). Furthermore we assume that the channel is known only at the receiver and that the goal is to maximize the throughput. Consequently, we organize an IR-HARQ protocol as follows: Initially the transmitter sends only as many codeword symbols as necessary to ensure a high probability of successful maximum-likelihood (ML) decoding over a high SNR channel. If the decoding fails, the receiver sends a NACK and the channel information to the

transmitter. Taking into account the channel information of the past transmission(s), the transmitter sends only as many additional codeword symbols as necessary to insure a high probability of successful ML decoding assuming a high SNR channel during the current transmission. Note that, since the channel may not have a high SNR and the actual decoding is belief propagation, our approach is parsimonious, designed to maximize the throughput with possibly introducing delay.

In addition to LDPC codes, another family of codes seem to be a natural candidate for use in HARQ schemes. This is the class of *rateless* Fountain Codes, originally designed for reliable transmission of data over an erasure channel with unknown erasure probability. The first class of efficient Fountain Codes were LT-codes [9]. The codewords of an LT code are generated based on the k information symbols by the means of a probability distribution on the numbers $1, \ldots, k$. Each codeword symbol is obtained independently, by first sampling this distribution to obtain a number d, and then adding the values of d randomly chosen information symbols. Raptor codes are a modified version of LT codes in which the information sequence of k symbols is pre-coded by a high rate, block code, (*e.g.,* LDPC code), and then the n resulting symbols are used to generate the Raptor codeword symbols in the same manner as for LT codes, [10].

Both LT and Raptor codes were originally designed for erasure channels, but performance of Raptor codes on arbitrary symmetric channels have been studied in [11], where the following scenario was considered. The receiver collects outputs from the channel whose statistical characteristics do not change in time (*e.g.,* AWGN channel with a constant SNR), until the cumulative reliability of the collected bits reaches a certain value determining the overhead of the code, at which point the receiver attempts to recover the input bits. The main concern in [11] is to design Raptor Codes that achieve a reception overhead arbitrarily close to zero, while maintaining the reliability and efficiency of the decoding algorithm. Although this problem has been solved for the erasure channel [10], it is not yet fully understood for general symmetric channels. Nevertheless, significant progress has been made, and in particular, it has also been shown that there are no universal Raptor Codes whose performance comes arbitrarily close to the capacity regardless of the noise of the channel [11].

The ability of Raptor codes to produce, for a given set of information symbols, as many codeword symbols as needed for their successful decoding is what makes them *rateless* and of interest for use in HARQ schemes. In this paper, we first study the spectra of Raptor codes and the ML decoding error rates for HARQ schemes based on Raptor codes. As in the case of LDPC codes, we assume that the channel is known only at the receiver and the goal is to maximize the throughput. With that in mind, we organize an IR-HARQ scheme based on Raptor codes in a very similar fashion as in the HARQ scheme with LDPC codes. Taking into account the channel information of the past transmission(s), the transmitter generates and then sends only as many codeword symbols as necessary to insure a high probability of successful ML decoding assuming a high SNR channel during the current transmission.

Current communications systems (*e.g.,* 3G wireless) employ HARQ schemes that are based on punctured codes (*e.g.,* turbo and LDPC). In this paper, we compare the IR-HARQ schemes based on LDPC codes with those based on Raptor codes. We in addition discuss a scenario in which not the entire LDPC code based scheme is replaced by a Raptor code based one, but only the puncturing device is replaced by a device performing the LT transform, since that may be a likely evolution of the practical systems. The results and conclusions easily generalize to schemes which use other codes as mother codes or precodes instead of LDPC codes. Therefore, we can in general compare IR-HARQ schemes based on punctured codes with those based on rateless codes.

This paper is organized as follows. In Sec. II, we begin with a weight enumerator analysis of the LDPC code ensembles which we use to model both the mother code and the precode, depending on the scheme. In Sec. III, we present and analyze an IR-HARQ protocol based on LDPC codes. In Sec. IV, we present and analyze an IR-HARQ protocol based on Raptor codes. Throughout these sections, we compare the theoretical results for the LDPC-based schemes with their counterparts for the Raptor-based schemes. In Sec. V, we compare simulation results of several HARQ schemes based on LDPC codes with the corresponding HARQ schemes based on Raptor codes.

II. The Precode and the Mother Code

We will use regular LDPC code ensembles to model the precode and the mother code in our HARQ schemes, but generalization to other classes of codes with similar weight enumerator properties is straightforward. These classes of codes include other LDPC code ensembles [14], turbo code ensembles [15], and all "good" code sequences [16].

A. Weight Enumerator Properties

To analyze the performance of HARQ schemes based on punctured LDPC codes and Raptor codes which use them as precodes, we need to establish certain properties of LDPC code ensembles' weight enumerators, namely, the asymptotic behavior of the left tail and the quantity known as the *noise threshold* [15]. We will consider ensembles of regular binary LDPC codes whose $k \times n$ parity check matrices have constant row and column sums. The weight enumerators of these ensembles were studied in, *e.g.,* [14], and the properties of interest here (summarized below) in [12], [13].

For a binary linear code ensemble $[\mathcal{C}](n)$, we denote the number of codewords of weight h (weight enumerator) averaged over the ensemble by $\overline{A}_h^{[\mathcal{C}](n)}$. The following bound on the left tail of the weight enumerator was derived in [13]:

$$\sum_{h=1}^{\lceil n\theta_0 \rceil} \overline{A}_h^{[\mathcal{C}](n)} \leq O(n^{-\frac{1}{2}}). \tag{1}$$

The noise threshold of the ensemble is denoted by $c_0^{[\mathcal{C}]}$ and defined as

$$c_0^{[\mathcal{C}]} \triangleq \limsup_{n\to\infty} \max_{\lceil n\theta_0\rceil < h \le n} \frac{\log \overline{A}_h^{[\mathcal{C}](n)}}{h}.$$

or, equivalently,

$$c_0^{[\mathcal{C}]} \triangleq \limsup_{n\to\infty} \sup_{\theta_0 < \theta \le 1} \frac{\mathfrak{a}_\theta^{[\mathcal{C}](n)}}{\theta}, \tag{2}$$

where

$$\mathfrak{a}_\theta^{[\mathcal{C}](n)} \triangleq \frac{\log \overline{A}_{n\theta}^{[\mathcal{C}](n)}}{n}. \tag{3}$$

It was shown in [13] that $c_0^{[\mathcal{C}]}$ is finite as well as how to numerically obtain an upper bound on $c_0^{[\mathcal{C}]}$. From the definition of the noise threshold, we have

$$\mathfrak{a}_\theta^{[\mathcal{C}](n)} \le_n \theta c_0^{[\mathcal{C}]}, \quad \forall \theta,\ \theta_0 < \theta \le 1. \tag{4}$$

for sufficiently large n.

B. The Maximum Likelihood Decoding

Consider a binary input memoryless channel with output alphabet Ω and transition probabilities $W(y|0)$ and $W(y|1)$, $y \in \mathcal{Y}$. When a codeword $\boldsymbol{x}$ has been transmitted, the probability that the ML detector finds codeword $\boldsymbol{x'}$ at Hamming distance h from $\boldsymbol{x}$ more likely can be bounded as follows:

$$P_e(\boldsymbol{x}, \boldsymbol{x'}) \le \gamma^h, \tag{5}$$

where γ is the *Bhattacharyya noise parameter* defined as

$$\gamma = \sum_{y\in\mathcal{Y}} \sqrt{W(y|x=0)W(y|x=1)} \tag{6}$$

if $\mathcal{Y}$ is discrete and as

$$\gamma = \int_{\mathcal{Y}} \sqrt{W(y|x=0)W(y|x=1)}\ dy$$

if $\mathcal{Y}$ is a measurable subset of $\mathbb{R}$. It follows from the definition that $0 \le \gamma \le 1$. For a given channel model and the noise level, γ is determined by the signal power, and it increases when the signal power decreases.

The well known *union-Bhattacharyya bound* on the ML decoder word error probability averaged over the ensemble with weight enumerator $\overline{A}_h^{[\mathcal{C}](n)}$ is given by

$$\overline{P}_W^{[\mathcal{C}](n)} \le \sum_{h=1}^{n} \overline{A}_h^{[\mathcal{C}](n)} \gamma^h. \tag{7}$$

III. An IR HARQ Scheme Based on LDPC Codes

We consider an IR HARQ scheme with at most M transmissions where a bit is assigned to transmission j with probability α_j. Transmission m takes place if transmission $m-1$ fails. The rates α_m may be predetermined (*e.g.*, specified by a standard) or determined based on current network conditions. In both cases, we are interested in evaluating performance after m transmissions for all $1 \le m \le M$. In the latter case, we are interested in determining α_m to achieve some required performance. The random transmission assignment for an IR HARQ protocol can be implemented by an "on-the-fly" dynamic version of the algorithm described in [8].

Let $\mathcal{I} = \{1, \ldots, n\}$ denote the set indexing the bit positions in a codeword. For the M transmissions, set $\mathcal{I}$ is partitioned in M subsets $\mathcal{I}(j)$, for $1 \le j \le M$. During the j-th transmission, only bits at positions where $i \in \mathcal{I}(j)$ are transmitted. We assume that the channel is slowly time-varying, namely that $W(y|0)$ and $W(y|1)$ remain constant for all bits at positions i taking part in the same transmission. Consequently, the Bhattacharyya noise parameter for transmission j, denoted by γ_j, remains constant during the transmission.

The performance analysis for a particular transmission assignments of the bits of the mother codes is difficult (see [8] for details). However, a union-type bound to the performance after m, $1 \le m \le M$ transmissions, averaged over all possible transmission assignments where a bit of a mother code is assigned to transmission j with probability α_j, $\alpha_j > 0$, $\sum_j^M \alpha_j = 1$ was derived in [8]:

$$\overline{P}_W^{[\mathcal{C}](n)} \le \sum_{h=1}^{n} \overline{A}_h^{[\mathcal{C}](n)} \overline{\gamma}_m^h, \tag{8}$$

where $\overline{\gamma}_m$ is the *average* Bhattacharyya noise parameter seen by the mother code after the m-th transmission. To compute this parameter, we assume that the bits which are not yet transmitted are transmitted over a really bad channel, *i.e.*, a channel with the noise parameter equal to 1, and obtain

$$\overline{\gamma}_m = \Big(1 - \sum_{j=1}^{m} \alpha_j\Big) \cdot 1 + \sum_{j=1}^{m} \gamma_j \alpha_j. \tag{9}$$

Starting with (8), based on the results in Sec. II, we have

$$\begin{aligned} \overline{P}_W^{[\mathcal{C}](n)} &\le \sum_{h=1}^{\lceil n\theta_0\rceil} \overline{A}_h^{[\mathcal{C}](n)} + \sum_{h=\lceil n\theta_0\rceil+1}^{n} \overline{A}_h^{[\mathcal{C}](n)} \overline{\gamma}^h \\ &\le O(n^{-\frac{1}{2}}) + \sum_{h=\lceil n\theta_0\rceil+1}^{n} \exp[h(c_0^{[\mathcal{C}]} + \log \overline{\gamma}_m]. \end{aligned} \tag{10}$$

Assume that the decoding after transmission $m-1$ for some $1 \le m \le M$ failed, and we need to choose α_m or γ_m or both to guarantee that $\lim_{n\to\infty} \overline{P}_W^{[\mathcal{C}](n)} = 0$. Form (10), we see that

$$\overline{\gamma}_m < \exp(-c_0^{[\mathcal{C}]})$$

or, equivalently,

$$1 - \exp(-c_0^{[\mathcal{C}]}) < \sum_{i=1}^{m} \alpha_i (1 - \gamma_i) \tag{11}$$

implies $\overline{P}_W^{[\mathcal{C}](n)} \le O(n^{-\frac{1}{2}})$. Condition (11) can be written (using (9)) in a form which clearly shows the tradeoff between expected number of bits sent in the m-th transmission (*i.e.*, $n\alpha_m$) and the signal power (*i.e.*, γ_m):

$$\alpha_m(1-\gamma_m) > 1 - \exp(-c_0^{[\mathcal{C}]}) - \sum_{i=1}^{m-1} \alpha_i (1-\gamma_i). \tag{12}$$

To satisfy the above lower bound on the product of α_m and $1-\gamma_m$, the transmitter can either increase the code redundancy α_m or increase the signal power which results in a decrease of γ_m and increase of $1-\gamma_m$. An increase in redundancy results in the lower throughput of the user while an increase in the power results in a higher interference level experienced by other users in the network. Since γ_m is positive, there is a minimum redundancy requirement:

$$\alpha_m > 1 - \exp(-c_0^{[\mathcal{C}]}) - \sum_{i=1}^{j-1} \alpha_i(1-\gamma_i). \tag{13}$$

In the case of predetermined α_j, the required signal power is specified by

$$\gamma_m < \frac{\exp\left(-c_0^{[\mathcal{C}]}\right) - (1-\alpha_m) - \sum_{i=1}^{m-1} \alpha_i(1-\gamma_i)}{\alpha_m}. \tag{14}$$

IV. An HARQ Scheme Based on Raptor Codes

To obtain a Raptor codeword, the information sequence of k bits is pre-coded by an (n,k) binary linear code. The Raptor codeword bits are then obtained based on the n precode bits by the means of a probability distribution Ω on the numbers $1,\ldots,n$. The probability generating function of this distribution is

$$\Omega(x) = \sum_{d=1}^{n} \Omega_d x^d. \tag{15}$$

Each Raptor codeword bit is obtained independently, by first sampling this distribution to obtain a number d, and then adding the values of d randomly chosen precode bits. Note that, because of the probabilistic nature of the encoding, we are dealing here with an ensemble rather than a particular code.

Suppose now that $m-1$ transmissions have taken place and were unsuccessful. We are to determine the minimum number of Raptor symbols which should be generated at the m-th transmission (*i.e.,* R_{lm}), and the minimum power at which they should be transmitted (*i.e.,* γ_m), given the values of these parameters and the channel conditions during the previous unsuccessful transmissions to guarantee $\lim_{n\to\infty} \overline{P}_W^{\mathcal{R}} = 0$ after the m-the transmission. Based on the results in [13], we know that when a codeword $\boldsymbol{x}$ of a length-$(N_1+N_2+\cdots+N_m)$ Raptor code has been transmitted over the channel with the Bhattacharyya noise parameter γ_1 during the first N_1 symbol intervals, the channel with the Bhattacharyya noise parameter γ_2 during the following N_2 symbol intervals, and so on, the channel with the Bhattacharyya noise parameter γ_m during the last N_m symbol intervals, then the ML decoder word error probability can be bounded as

$$\begin{aligned}\overline{P}_W^{\mathcal{R}} &\le \sum_{h=1}^{n} \overline{A}_h^{[\mathcal{C}](n)} \prod_{i=1}^{m} \left[1-p(\theta)(1-\gamma_i)\right]^{N_i} \\ &= \sum_{\theta\in\mathcal{T}} \exp(n\mathfrak{a}_\theta^{[\mathcal{C}](n)}) \prod_{i=1}^{m} \left[1-p(\theta)(1-\gamma_i)\right]^{N_i}, \end{aligned} \tag{16}$$

where $\mathfrak{a}_\theta^{[\mathcal{C}](n)}$ is defined by (3), $\mathcal{T} = \{\frac{1}{n}, \cdots, \frac{n-1}{n}, 1\}$, and $p(\theta)$ denotes the probability of 1 in the Raptor codeword when the input LDPC codeword has weight $n\theta$. For a Raptor code with the degree distribution Ω, we have

$$\begin{aligned} p(\theta) &= \sum_d \Omega_d \sum_{j=1,\ j \text{ odd}}^{d} \binom{d}{j} \theta^j (1-\theta)^{d-j} \\ &= \frac{1}{2} - \frac{1}{2} \sum_d \Omega_d (1-2\theta)^d \end{aligned}$$

and $1-p(\theta) = \frac{1}{2} + \frac{1}{2}\sum_d \Omega_d (1-2\theta)^d$. Note that the sum in (16) goes over the weights of the LDPC precode rather than the Raptor code. It is interesting to compare the expression (16) with the corresponding expression (7) for the precode without the LT coding, which can be obtained from (16) merely by substituting γ^h in the place of $\prod_{i=1}^m \left[1-p(\theta)(1-\gamma_i)\right]^{N_i}$. Since $\gamma < \left[1-p(\theta)(1-\gamma)\right]$, the LT code has the effect of making the channel in a way appear noisier to the precode.

Starting with (16), by using the inequality $(1-x) \le e^{-x}$ to bound $\left[1-p(\theta)(1-\gamma)\right]^{N_i}$ and the LDPC code ensemble weight enumerator bound (4), we obtain

$$\overline{P}_W^{\mathcal{R}} \le O(n^{-\frac{1}{2}}) + \exp\Big\{n\theta\Big[c_0^{[\mathcal{C}]} - \frac{p(\theta)}{\theta} \sum_{i=1}^{m} \frac{1-\gamma_i}{R_{li}}\Big]\Big\},$$

where $R_{li} = n/N_i$ and

$$\pi_\Omega \triangleq \min_\theta \frac{p(\theta)}{\theta}$$

Therefore, when

$$\frac{c_0^{[\mathcal{C}]}}{\pi_\Omega} < \sum_{i=1}^{m} \frac{1-\gamma_i}{R_{li}}, \tag{17}$$

we have $\overline{P}_W^{\mathcal{R}} \le O(n^{-\frac{1}{2}})$.

Condition (17) can be written in a form which clearly shows the tradeoff between the rate of the m-th transmission code and the signal power:

$$R_{lm}^{-1}(1-\gamma_m) > \frac{c_0^{[\mathcal{C}]}}{\pi_\Omega} - \sum_{i=1}^{m-1} R_{li}^{-1}(1-\gamma_i). \tag{18}$$

To satisfy the above lower bound on the product of R_{lj}^{-1} and $1-\gamma_m$, the transmitter can either decrease the m-th code rate R_{lm} or increase the signal power which results in a decrease of γ_m and increase of $1-\gamma_m$. A decrease in code rate results in the lower throughput of the user while an increase in the power results in a higher interference level experienced by other users in the network. Since γ_m is positive, there is a maximum rate constraint:

$$R_{lm} < \Big[\frac{c_0^{[\mathcal{C}]}}{\pi_\Omega} - \sum_{i=1}^{m-1} R_{li}^{-1}(1-\gamma_i)\Big]^{-1}. \tag{19}$$

Note that this condition ensures that the expected error-rate of the ML decoding is bounded by $O(n^{-1/2})$ for high SNR. In the case of predetermined R_{lm} (as it is sometimes done in the practice), the required signal power is specified by

$$\gamma_m < \frac{-\frac{c_0^{[\mathcal{C}]}}{\pi_\Omega} + R_{lm}^{-1} + \sum_{i=1}^{m-1} R_{li}^{-1}(1-\gamma_i)}{R_{lj}^{-1}}. \tag{20}$$

Equations (17), (18), (19), and (20) constitute the m-th transmission's power and rate assignment rules after transmission $m-1$ fails. It is interesting to compare them with their counterparts for the LDPC-based scheme given by equations (11), (12), (13), and (14). It is easy to see that the rules are identical with $c_0^{[\mathcal{C}]}/\pi_\Omega$ in the Raptor-based scheme playing the role of $1-\exp(-c_0^{[\mathcal{C}]})$ in the LDPC-based scheme.

V. A Comparison of HARQ Schemes

We next compare the simulation results for IR-HARQ schemes based on several LDPC and Raptor codes. The LDPC codes used in the schemes include three different regular codes with $\lambda(x)=x^2$ and rates $R=0.4$, 0.8 and 0.9, as well as an irregular code designed based on the optimized LDPC graph degree polynomials $\lambda(x)=\sum_i \lambda_i x^{i-1} = 0.21991x+0.23328x^2+0.02058x^3+0.08543x^5+0.06540x^6+0.04767x^7+0.01912x^8+0.08064x^{18}+0.22798x^{19}$ and $\rho(x)=\sum_i \rho_i x^{i-1} = 0.64854x^7+0.34747x^8+0.00399x^9$ from [17]. The Raptor codes are based on the same three regular LDPC codes now used as precodes. The LT degree distribution polynomial $\Omega(x)$ in Raptor codes was chosen to be $\Omega(x)=0.05x+0.5x^2+0.05x^3+0.25x^4+0.05x^6+0.1x^8$. In Fig. 1 we plot the throughput performance of these schemes together

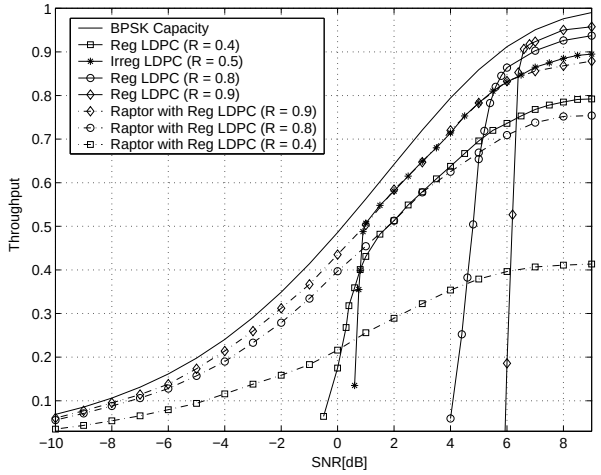


Fig. 1. Average throughput for IR-HARQ schemes with LDPC and Raptor codes.

with the BPSK channel capacity. We can observe that Raptor codes have an obvious advantage at low SNRs. In fact, Raptor codes can be used for signalling at extremely low SNRs and still perform very near the capacity, which is generally difficult to achieve with standard punctured LDPC and turbo codes. For example, when the precode is the regular rate 0.9 LDPC code, the Raptor code simulation shown in Fig. 1 demonstrates that the throughput of 0.1 is achieved at SNR about 0.65dB away from the Shannon limit. Moreover, on the range of throughputs from 0 to 0.75, the gap to the capacity for this Raptor code is less than 1dB. Therefore, Raptor codes provide a much broader dynamic range of rates (or SNRs) for HARQ than punctured LDPC codes, and in this sense are more robust than LDPC codes. On the other hand, using LT coding instead of puncturing LDPC codes results in a performance loss in the high-SNR region. This also can be seen in Fig. 1. In particular, we can observe that if the operating SNRs are guaranteed to be very high (say the SNR is always greater than 7dB), than it is more beneficial to use punctured LDPC codes with high rate mother codes than Raptor codes. Theoretical justification of this claim can be found in [13]. However, if the information about the range of SNR is not available or not reliable, it is more advisable to use Raptor coding schemes in IR-HARQ.

Our final comment is on the difference in the encoding and decoding complexities of these schemes, which are in general higher for the schemes based on Raptor codes. However, when Raptor codes are used, we only need to encode as many parity bits as we need to send in the initial transmission or in subsequent re-transmission(s). On the other hand, for punctured codes we need to encode all parity bits, even though we may need to send only a small fraction of them.

References

[1] D. J. Costello, J. H. Jr., H. Imai, and S. B. Wicker, "Applications of error-control coding," *IEEE Trans. Inform. Theory*, vol. 44, pp. 2531–2560, 1998.

[2] G. Caire and D. Tuninetti, "The throughput of Hybird-ARQ protocols for the Gaussion collision channel," *IEEE Trans. Inform. Theory*, vol. 47, pp. 1971–1988, July 2001.

[3] D. N. Rowitch and L. B. Milstein, "On the performance of hybrid FEC/ARQ systems using rate compatible punctured turbo (RCPT) codes," *IEEE Trans. Commun.*, vol. 48, pp. 948–959, June 2000.

[4] J. Li and K. Narayanan, "Rate-compatible low-density parity-check codes for capacity-approaching ARQ scheme in packet data communications," in *Int. Conf. on Comm., Internet, and Info. Tech. (CIIT)*, Nov. 2002.

[5] J. Ha, J. Kim, and S. McLaughlin, "Rate-compatible puncturing of low-density parity-check codes," *IEEE Trans. Inform. Theory*, vol. 50, pp. 2824–2836, Nov. 2004.

[6] M. Yazdani and A. Banihashemi, "On construction of rate-compatible low-density parity-check codes," *IEEE Commun. Lett.*, vol. 8, pp. 159–161, Mar. 2004.

[7] R. Liu, P. Spasojević, and E. Soljanin, "Reliable channel regions for good binary codes transmitted over parallel channels," *IEEE Trans. Inform. Theory*, to appear in 2006.

[8] E. Soljanin, R. Liu, and P. Spasojević, "Hybrid ARQ with random transmission assignments," in *Advances in Network Information Theory*, P. Gupta, G. Kramer, and A. Wijngarden, Eds. DIMACS Series in Discrete Mathematics and Thepretical Computer Science, American Mathematical Society, Aug. 2004, vol. 66, pp. 321–334.

[9] M. Luby, "LT codes," in *Proc. of the 43rd Annual IEEE Symp. on the Foundations of Comp. Science (STOC)*, 2002.

[10] A. Shokrollahi, "Raptor codes," *preprint*, 2002.

[11] O. Etesami and A. Shokrollahi, "Raptor codes on symmetric channels," *preprint*, 2005.

[12] N. Varnica, E. Soljanin, and P. Whiting, "LDPC codes for HARQ," in *Proc. IEEE Internat. Symp. Inform. Theory*, Adelaide, Australia, 4-9 Sept. 2005.

[13] E. Soljanin, N. Varnica, and P. Whiting, "Incremental redundancy hybrid ARQ with LDPC and raptor codes," *IEEE Trans. Inform. Theory*, submitted for publication, Sept. 2005.

[14] S. Litsyn and V. Shevelev, "On Ensmebles of Low Density Parity-Check Codes," *IEEE Trans. Inform. Theory*, vol. 48, pp. 887–908, Apr. 2002.

[15] H. Jin and R. J. McEliece, "Coding theorems for turbo code ensembles," *IEEE Trans. Inform. Theory*, vol. 48, pp. 1451–1461, June 2002.

[16] R. Liu, P. Spasojević, and E. Soljanin, "On the weight spectrum of good linear binary codes," *IEEE Trans. Inform. Theory*, vol. 51, pp. 4369–4373, Dec. 2005.

[17] T. Richardson, A. Shokrollahi, and R. Urbanke, "Design of capacity-approaching low-density parity-check codes," *IEEE Trans. Inform. Theory*, vol. 47, pp. 619–637, February 2001.