A Combinatorial Technique for Constructing High-Rate MTR–RLL Codes

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***Abstract*—We present advanced combinatorial techniques for constructing maximum runlength-limited (RLL) block codes and maximum transition run (MTR) codes. These codes find widespread application in recording systems. The proposed techniques are used to construct a high-rate multipurpose modulation code for recording systems. The code, a rate 16/17, (0, 3, 2, 2) MTR code, that also fulfills (0, 15, 9, 9) RLL constraints is a high-rate distance-enhancing code with additional constraints for improving timing and gain control. The encoder and decoder have a particularly efficient architecture and allow an instantaneous translation of 16-bit source words into 17-bit codewords and vice versa. The code has been implemented in Lucent read-channel chips and has excellent performance.**

***Index Terms*—Constrained coding, intersymbol interference, magnetic recording, runlength codes.**

I. Introduction

A VARIETY of block codes that fulfill certain channel input constraints have been developed for recording channels during the last several decades (see [1]–[4] and references therein). One important category is the class of runlength-limited codes (RLL codes). The codewords of an RLL code are runlength-limited sequences of a fixed length n, also known as (d, k) sequences, that are characterized by the parameters d and k, which indicate the minimum and maximum number of "zeros" between consecutive "ones" in the binary sequence, respectively. Usually, a "one" and a "zero" correspond to a reversal and a nonreversal in magnetization, respectively. As a consequence, the parameter d controls the intersymbol interference and the parameter k determines the self-clocking properties of the sequence.

Current high-density recording systems use a detection technique that combines partial response equalization with maximum-likelihood sequence detection, often referred to as PRML. Intersymbol interference can be mitigated by reconstructing the recorded sequence from sample values of a suitably equalized readback channel. The current systems often use a partial response channel model with transfer function $h(D) = (1-D)(1+D)^N$, which is referred to as a $\mathrm{E}^N\mathrm{PR4}$ channel [5], [6]. Sequence detection can be enhanced for these channels by reducing the set of possible recording sequences in order to increase the minimum distance. In [6]–[8] and references therein, the pairs of sequences with low minimum distance have been identified for important classes of partial response channels, and codes based on constrained systems have been proposed. For the $\mathrm{E}^2\mathrm{PR4}$ channel, which is an appropriate model for the current linear recording densities, the constraints that have been specified can be realized by restricting the maximum runlength of consecutive "ones." The corresponding codes, known as maximum transition run codes (MTR codes), are equivalent to $(0, k)$ RLL codes with "ones" and "zeros" interchanged. It has been shown in [7]–[9] that a (0, 2) MTR code will be sufficient to increase the minimum distance. However, the upper bound on the rate of a (0, 2) code is only 0.8791 [1]. By relaxing the constraints, higher rates can be achieved by specifying constraints that do not remove all undesired pairs but instead significantly reduce the number of undesired pairs of sequences with minimum distance [8], [9]. In this way, a tradeoff is made between code rate, code complexity, and detection performance.

We consider the construction of $(0, k, k_l, k_r)$ block codes, where k_l and k_r denote the maximum number of leading and trailing "zeros" of a codeword. Generally, $k_l + k_r \leq k$, to allow the construction of long $(0, k)$ sequences by concatenating the codewords of a $(0, k, k_l, k_r)$ block code. As indicated, $(0, k, k_l, k_r)$ MTR codes are $(0, k, k_l, k_r)$ RLL codes with "ones" and "zeros" interchanged. Several $(0, k, k_l, k_r)$ codes that are employed in recording systems have been implemented with look-up tables or combinatorial circuitry [1], [3], [5], [10]–[13]. This imposes limitations on the block length and the achievable rate. The majority of codes have a rate 8/9, and only recently, codes with longer block lengths have been developed, either by using algorithms [4], [14], [15] or by interleaving a short block code with uncoded bits [15]–[17]. The latter methods increase the rate and limit error propagation, but the constraints of the resulting code are certainly not the best possible for the given rate. The combinatorial construction methodology that is proposed here is shown to be a very powerful tool for the design of high-rate constrained codes. Integration of the code into a device is feasible because of its highly parallelized architecture and its small delay and high throughput.

In Section II, we briefly describe some properties of $(0, k, k_l, k_r)$ sets. In Section III, we give an outline of the design methodology for the construction of high-rate runlength-limited codes. We illustrate this methodology in Section IV by detailing the construction of a rate 16/17, (0, 3, 2, 2) MTR code that satisfies (0, 15, 9, 9) RLL constraints. The (0, 3, 2, 2) MTR constraints are the smallest possible for the given code rate, and the (0, 15, 9, 9) RLL constraints are sufficient to provide timing and gain control.

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II. Definitions

Let $\mathcal{A}^m$ denote the set of binary sequences of length m. A sequence $X \in \mathcal{A}^m$ is represented as a string of symbols $x_1x_2\cdots x_m$ of length m. The concatenation of two sequences X and Y is denoted by XY, and $P\mathcal{S}^{(m)} = \{PX | X \in \mathcal{S}^{(m)}\}$ denotes a set of sequences of length $p+m$ with prefix $P \in \mathcal{A}^p$ and suffix $X \in \mathcal{S}^{(m)}$, where $\mathcal{S}^{(m)} \subseteq \mathcal{A}^m$. The null string Λ represents a string of length 0 for which $\Lambda X = X\Lambda = X$. A run of m consecutive symbols a is written as a^s. We use $*$ and $(*)^m$ to denote any element of $\mathcal{A}$ and $\mathcal{A}^m$, respectively.

The set $\mathcal{G}^{(n)}_{k,k_l,k_r}$ of length-n sequences that satisfy the $(0, k, k_l, k_r)$ constraints on the runs of consecutive ones and its size $G^{(n)}_{k,k_l,k_r}$ can be determined recursively. For $n \leq \min(k_l, k_r)$, we have $\mathcal{G}^{(n)}_{k,k_l,k_r} = \mathcal{A}^n$, and for $n > k_l$, we have

$$\mathcal{G}^{(n)}_{k,k_l,k_r} = \bigcup_{i=0}^{k_l} 1^i 0 \mathcal{G}^{(n-i-1)}_{k,k,k_r}. \tag{1}$$

The set $\mathcal{G}^{(t)}_{k,k,k_r}$ can be determined using (1) for $t \geq k+2$. For $t \leq k_r$, we have $\mathcal{G}^{(t)}_{k,k,k_r} = \mathcal{A}^t$, and for $t = k_r+1$, we have $\mathcal{G}^{(t)}_{k,k,k_r} = \mathcal{A}^t \backslash \{1^{k_r+1}\}$. For $k_r+2 \leq t \leq k+1$, it can be easily verified that $\mathcal{G}^{(t)}_{k,k,k_r} = 0\mathcal{G}^{(t-1)}_{k,k,k_r} \cup 1\mathcal{G}^{(t-1)}_{k,k,k_r}$. For $t \leq k+1$, we thus have

$$G^{(t)}_{k,k,k_r} = \begin{cases} 2^t, & \text{if } t \leq k_r \\ 2^{t-k_r-1}(2^{k_r+1}-1), & \text{if } k_r < t \leq k+1. \end{cases}$$

Because the sets in the union of (1) are mutually disjoint, we obtain the following recursion for $n \geq k+2$:

$$G^{(n)}_{k,k_l,k_r} = \sum_{i=0}^{k_l} G^{(n-i-1)}_{k,k,k_r}.$$

We define $\mathcal{F}^{(n)}_{k,k_l,k_r}$ to be the set of length-n sequences that satisfy the $(0, k, k_l, k_r)$ constraints on the runs of consecutive zeros and denote by $F^{(n)}_{k,k_l,k_r}$ the size of $\mathcal{F}^{(n)}_{k,k_l,k_r}$. By reversing the zeros and the ones in the expressions above, we obtain identical results for $\mathcal{F}^{(n)}_{k,k_l,k_r}$ and $F^{(n)}_{k,k_l,k_r}$. In particular, $F^{(n)}_{k,k_l,k_r} = G^{(n)}_{k,k_l,k_r}$. The above expressions will be frequently used to determine the number of (sub)sequences of a given length that satisfy given $(0, k, k_l, k_r)$ constraints.

III. Combinatorial Construction Methods

We present an efficient methodology for constructing $(0, k, k_l, k_r)$ codes by mapping $(n-1)$-bit source words onto n-bit codewords of a $(0, k, k_l, k_r)$ code. It improves on techniques presented in [15]. If the number of sequences of length n that satisfy the $(0, k, k_l, k_r)$ constraints exceeds 2^{n-1}, it is always possible to perform this mapping, although the complexity may be high. For several codeword lengths, it is possible to realize this mapping procedure using a small, digital combinatorial circuit. The codes that have been developed are characterized by a set of source word–dependent mapping rules. The encoder first checks whether the source word can be mapped onto a candidate codeword by inserting one extra fixed-valued bit. If this is the case, the encoder copies the source word and inserts one extra bit to indicate that the other bits of the codeword are a copy of the source word. Otherwise, the extra bit indicates that a mapping rule had to be applied to transform the source word into an $(n-1)$-bit coded sequence that satisfies the imposed $(0, k, k_l, k_r)$ constraints. The coded sequence, which uniquely represents the source word, consists of bits that identify the mapping used, as well as the rearranged bits of the source word. The mapping structure is often symmetric to simplify the design process and the structure of the digital combinatorial circuitry.

The construction methodology has the following characteristics.

- A mapping is defined for a set of source words rather than for one word. Based on the specific constraints, the set of source words and the set of codewords are partitioned into disjoint subsets. For every set, a mapping is specified. The words belonging to the same subset are transformed in the same fashion.
- Symmetry is used whenever possible; i.e., if $Y = \psi(X)$ represents the mapping of X onto Y, then X' and Y', representing the words X and Y in reversed order, satisfy whenever possible the relation $Y' = \psi(X')$. This significantly simplifies the design.
- The symbols of the source word are, whenever possible, directly mapped onto corresponding codeword positions. Whenever changes are required, the source symbols that cause the constraint to be violated are specified by identifying the type of violation, and the other source symbols that contain information are permuted.
- The translation of $(n-1)$ information bits into n-bit codewords and vice versa is done in parallel.
- The combinatorial circuitry that results from the methodology classifies the incoming source word and rearranges the bits of the source word based on the classification in order to satisfy the constraints.

This methodology is particularly useful for the construction of tightly constrained codes with a short and medium wordlength (4 up to 32 bits and possibly up to 64 bits with slightly looser constraints). The constructed codes can also be used as a basic code to obtain longer and higher rate codes with less tight constraints using interleaving with uncoded source symbols [17].

IV. Design of the Rate 16/17, (0, 3, 2, 2) MTR, (0, 15, 9, 9) RLL Code

In this section, we will describe the construction of the rate 16/17, (0, 3, 2, 2) MTR–RLL code in detail. The objective is to construct a code $\mathcal{C}$ that is a subset of $\mathcal{G}^{(17)}_{3,2,2}$ and has a cardinality 2^{16}. Because $G^{(17)}_{3,2,2} = 65\,753$, we have a surplus of 217 codewords. We will show at a later stage that this will allow us to simultaneously fulfill (0, 15, 9, 9) RLL constraints by excluding 199 words to obtain a code $\mathcal{C} \subset \mathcal{G}^{(17)}_{3,2,2} \cap \mathcal{F}^{(17)}_{15,9,9}$ with a surplus of 18 candidate words.

We will detail the technique of mapping source words onto codewords that fulfill both the (0, 3, 2, 2) MTR constraints and the (0, 15, 9, 9) RLL constraints. This method inherently specifies the structure of the encoder and decoder that map 16-bit source words in parallel onto 17-bit codewords and vice versa.

TABLE I
DISJOINT PARTITION OF THE SET $\mathcal{A}^8\backslash\mathcal{V}_0$

	string representation	characterization	size
$\mathcal{V}_1$	$\mathbf{1}\ \mathbf{1}\ \mathbf{1}\ x_4\ x_5\ x_6\ x_7\ x_8$	$x_4x_5x_6x_7x_8 \in \mathcal{A}^5$	32
$\mathcal{V}_2$	$\mathbf{0}\ \mathbf{1}\ \mathbf{1}\ \mathbf{1}\ \mathbf{1}\ x_6\ x_7\ x_8$	$x_6x_7x_8 \in \mathcal{A}^3$	8
$\mathcal{V}_3$	$x_1\ \mathbf{0}\ \mathbf{1}\ \mathbf{1}\ \mathbf{1}\ \mathbf{1}\ x_7\ x_8$	$x_1x_7x_8 \in \mathcal{A}^3$	8
$\mathcal{V}_4$	$x_1\ x_2\ \mathbf{0}\ \mathbf{1}\ \mathbf{1}\ \mathbf{1}\ \mathbf{1}\ x_8$	$x_1x_2x_8 \in \mathcal{A}^3$	8
$\mathcal{V}_5$	$x_1\ x_2\ x_3\ \mathbf{0}\ \mathbf{1}\ \mathbf{1}\ \mathbf{1}\ \mathbf{1}$	$x_1x_2x_3 \in \mathcal{A}^3\backslash\{111\}$	7

A. Partition of the Set of Data Words

We will develop a mapping technique in which the symmetry relative to the central position of the codeword plays an important role. Let $Y = y_1y_2\cdots y_{17}$ be an arbitrary element of $\mathcal{A}^{17}$, and denote $Y' = y_{17}\cdots y_2y_1$ to be the sequence Y in reversed order. The set $\mathcal{G}^{(17)}_{3,2,2} \cap \mathcal{F}^{(17)}_{15,9,9}$ has the property that $Y \in \mathcal{G}^{(17)}_{3,2,2} \cap \mathcal{F}^{(17)}_{15,9,9}$ if and only if $Y' \in \mathcal{G}^{(17)}_{3,2,2} \cap \mathcal{F}^{(17)}_{15,9,9}$. Let $Y = \psi(X)$ denote the mapping of a 16-bit source word onto a 17-bit codeword. Whenever possible, Y' will be the codeword associated with X', i.e., $Y' = \psi(X')$. This will help to reduce the complexity of the analysis and the design. The symmetry is taken into account by decomposing $X \in \mathcal{A}^{16}$ into two sequences $X^l = x_1^lx_2^l\cdots x_8^l$ and $X^r = x_1^rx_2^r\cdots x_8^r$, where $X^l \in \mathcal{A}^8$ and $X^r \in \mathcal{A}^8$. Let $X^{r'}$ be the sequence X^r in reversed order, i.e., $X^{r'} = x_8^rx_7^r\cdots x_1^r$, The source word X is given by

$$X = X^lX^{r'} = x_1^lx_2^lx_3^lx_4^lx_5^lx_6^lx_7^lx_8^lx_8^rx_7^rx_6^rx_5^rx_4^rx_3^rx_2^rx_1^r.$$

Let Y denote the codeword, where the symbols are indexed as

$$Y = y_1^ly_2^ly_3^ly_4^ly_5^ly_6^ly_7^ly_8^ly_cy_8^ry_7^ry_6^ry_5^ry_4^ry_3^ry_2^ry_1^r,$$

where y_c denotes the central position of the 17-bit codeword. The indexes reflect the symmetry relative to the central position of the codeword.

We now consider the 17-bit word $Y = X^l0X^{r'}$, i.e., $y_i^l = x_i^l$, $y_i^r = x_i^r$ for $1 \le i \le 8$, and $y_c = 0$. It can be verified that $Y \in \mathcal{G}^{(17)}_{3,2,2}$ if and only if $X^l \in \mathcal{G}^{(8)}_{3,2,3}$ and $X^{r'} \in \mathcal{G}^{(8)}_{3,3,2}$, i.e., $X^r \in \mathcal{G}^{(8)}_{3,2,3}$, and therefore, $G^{(8)}_{3,2,3} \times G^{(8)}_{3,3,2} = 193 \times 193 = 37\,249$ out of 65 536 words can be mapped in this way.

Let $\mathcal{V}_0 = \mathcal{G}^{(8)}_{3,2,3}$ denote the set of words of length 8 that fulfill the (0, 3, 2, 3) MTR constraints. The set $\mathcal{A}^8\backslash\mathcal{V}_0$ is partitioned into five disjoint subsets $\mathcal{V}_1, \ldots, \mathcal{V}_5$, which are specified in Table I. Every set $\mathcal{V}_i, 1 \le i \le 5$, consists of words that have in common a substring that violates the imposed (0, 3, 2, 3) MTR constraints. Define V_i to be the size of the set $\mathcal{V}_i$. The values of V_1 up to V_5 are specified in Table I. The size of $\mathcal{V}_0$ is given by $V_0 = G^{(8)}_{3,2,3} = 193$.

Because the six sets $\mathcal{V}_0, \ldots, \mathcal{V}_5$ are disjoint and their union equals $\mathcal{A}^8$, every source word $X \in \mathcal{A}^{16}$ can be represented by $X = X^lX^{r'}$, where $X^l \in \mathcal{V}_i$ and $X^r \in \mathcal{V}_j$. We now introduce the notation $\mathcal{W}_{i,j} = \mathcal{V}_i \times \mathcal{V}_j$. Because the sets $\mathcal{V}_0, \ldots, \mathcal{V}_5$ are disjoint, the sets $\mathcal{W}_{i,j}$ are consequently disjoint, and therefore, the set of source words $\mathcal{A}^{16}$ is uniquely partitioned. This implies that there are 36 different categories that have to be considered. Define $W_{i,j}$ to be the number of elements of $\mathcal{W}_{i,j}$. Table II lists the values of $W_{i,j} = V_i \cdot V_j$ for each of the 36 sets $\mathcal{W}_{i,j}$. It can be verified that the sum of the values $W_{i,j}$ in Table II is equal to 65 536, as expected. To determine to which set $\mathcal{V}_i$ a word $X \in \mathcal{A}^8$ belongs, we evaluate the binary equations

$$\begin{aligned}
v_1 &= x_1 \cdot x_2 \cdot x_3,\\
v_2 &= \overline{x}_1 \cdot x_2 \cdot x_3 \cdot x_4 \cdot x_5,\\
v_3 &= \overline{x}_2 \cdot x_3 \cdot x_4 \cdot x_5 \cdot x_6,\\
v_4 &= \overline{x}_3 \cdot x_4 \cdot x_5 \cdot x_6 \cdot x_7,\\
v_5 &= \overline{x}_4 \cdot x_5 \cdot x_6 \cdot x_7 \cdot x_8 \cdot \overline{v}_1,\\
v_0 &= \overline{v}_1 \cdot \overline{v}_2 \cdot \overline{v}_3 \cdot \overline{v}_4 \cdot \overline{v}_5,
\end{aligned}$$

where $\overline{x}_i$ denotes the inverse of the binary value x_i and the dot-operator denotes the binary AND operation. We can easily verify that $v_i = 1$ if $X \in \mathcal{V}_i$, and $v_i = 0$ otherwise.

In order to determine the set $\mathcal{W}_{i,j}$ a source word $X = X^lX^{r'}$ belongs to, this set of equations is evaluated for X^l and X^r.

TABLE II
NUMBER OF ELEMENTS OF THE SET $\mathcal{W}_{i,j}$

$W_{i,j}$	$j=0$	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
$i=0$	37249	6176	1544	1544	1544	1351
$i=1$	6176	1024	256	256	256	224
$i=2$	1544	256	64	64	64	56
$i=3$	1544	256	64	64	64	56
$i=4$	1544	256	64	64	64	56
$i=5$	1351	224	56	56	56	49

TABLE III
DISJOINT PARTITION OF THE SET $\mathcal{G}^{(17)}_{3,2,2}$

	format	set	size
$\mathcal{M}_0$	$\cdots\cdots\cdots 0 \cdots\cdots\cdots$	$\mathcal{G}^{(8)}_{3,2,3} \times \mathcal{G}^{(8)}_{3,3,2}$	$193 \times 193 = 37249$
$\mathcal{M}_1$	$\cdots\cdots\cdots 110 \cdots\cdots\cdots$	$\mathcal{G}^{(7)}_{3,2,1} \times \mathcal{G}^{(7)}_{3,3,2}$	$79 \times 100 = 7900$
$\mathcal{M}_2$	$\cdots\cdots\cdots 011 \cdots\cdots\cdots$	$\mathcal{G}^{(7)}_{3,2,3} \times \mathcal{G}^{(7)}_{3,1,2}$	$100 \times 79 = 7900$
$\mathcal{M}_3$	$\cdots\cdots 10100 \cdots\cdots$	$\mathcal{G}^{(6)}_{3,2,2} \times \mathcal{G}^{(6)}_{3,3,2}$	$48 \times 52 = 2496$
$\mathcal{M}_4$	$\cdots\cdots 00101 \cdots\cdots$	$\mathcal{G}^{(6)}_{3,2,3} \times \mathcal{G}^{(6)}_{3,2,2}$	$52 \times 48 = 2496$
$\mathcal{M}_5$	$\cdots\cdots 00100 \cdots\cdots$	$\mathcal{G}^{(6)}_{3,2,3} \times \mathcal{G}^{(6)}_{3,3,2}$	$52 \times 52 = 2704$
$\mathcal{M}_6$	$\cdots\cdots 01110 \cdots\cdots$	$\mathcal{G}^{(6)}_{3,2,3} \times \mathcal{G}^{(6)}_{3,3,2}$	$52 \times 52 = 2704$
$\mathcal{M}_7$	$\cdots\cdots 10101 \cdots\cdots$	$\mathcal{G}^{(6)}_{3,2,2} \times \mathcal{G}^{(6)}_{3,2,2}$	$48 \times 48 = 2304$

B. Partition of the Set of Codewords

The set $\mathcal{G}^{(17)}_{3,2,2}$ will be partitioned into eight disjoint subsets $\mathcal{M}_0, \ldots, \mathcal{M}_7$, which are specified in Table III. The dots in this table are placeholders for binary symbols of elements of the set specified in the third column of the table. It is easy to see that all possible combinations of the central positions are specified. The right-most column gives the number of codewords of $\mathcal{M}_i$.

TABLE IV
MAPPING OF THE ELEMENTS $X \in \mathcal{W}_{0,0}$

		y_1^l	y_2^l	y_3^l	y_4^l	y_5^l	y_6^l	y_7^l	y_8^l	y_c	y_8^r	y_7^r	y_6^r	y_5^r	y_4^r	y_3^r	y_2^r	y_1^r
$\mathcal{W}_{0,0}^{[1]}$	X	x_1^l	x_2^l	x_3^l	x_4^l	x_5^l	x_6^l	1	**0**		**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	Y	x_1^l	x_2^l	x_3^l	$\underline{0}$	$\underline{1}$	$\underline{1}$	$\underline{0}$	$\underline{0}$	$\underline{1}$	$\underline{0}$	$\underline{1}$	x_4^l	x_5^l	x_6^l	$\underline{1}$	$\underline{0}$	$\underline{1}$
$\mathcal{W}_{0,0}^{[2]}$	X	x_1^l	x_2^l	x_3^l	x_4^l	x_5^l	x_6^l	0	**0**		**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	Y	x_4^l	$\underline{0}$	$\underline{1}$	x_1^l	x_2^l	x_3^l	$\underline{1}$	$\underline{0}$	$\underline{1}$	$\underline{0}$	$\underline{0}$	$\underline{1}$	$\underline{1}$	$\underline{1}$	0	x_5^l	x_6^l
$\mathcal{W}_{0,0}^{[3]}$	X	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**		**0**	1	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r
	Y	$\underline{1}$	$\underline{0}$	$\underline{1}$	x_6^r	x_5^r	x_4^r	$\underline{1}$	$\underline{0}$	$\underline{1}$	$\underline{0}$	$\underline{0}$	$\underline{1}$	$\underline{1}$	$\underline{0}$	x_3^r	x_2^r	x_1^r
$\mathcal{W}_{0,0}^{[4]}$	X	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**		**0**	0	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r
	Y	x_6^r	x_5^r	0	$\underline{1}$	$\underline{1}$	$\underline{1}$	$\underline{0}$	$\underline{0}$	$\underline{1}$	$\underline{0}$	$\underline{1}$	x_3^r	x_2^r	x_1^r	$\underline{1}$	$\underline{0}$	x_4^r
$\mathcal{W}_{0,0}^{[6]}$	X	x_1^l	x_2^l	x_3^l	x_4^l	x_5^l	x_6^l	x_7^l	x_8^l		x_8^r	x_7^r	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r
	Y	x_1^l	x_2^l	x_3^l	x_4^l	x_5^l	x_6^l	x_7^l	x_8^l	$\underline{0}$	x_8^r	x_7^r	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r

The sum of the last column equals $G_{3,2,2}^{(17)} = 65\,753$. Because of the symmetry of the central positions, we have for $1 \le m \le 3$, $|\mathcal{M}_{2m-1}| = |\mathcal{M}_{2m}|$. The sets $\mathcal{M}_i$, where $1 \le i \le 7$, automatically fulfill the (0, 15, 9, 9) RLL constraints in addition to the (0, 3, 2, 2) MTR constraints.

C. Mapping Techniques

Having completed partitioning the set of source words and the set of candidate codewords, we will specify the assignment of codewords to source words in a compact format by mappings that inherently define the encoder and the decoder. It is often necessary to further partition the sets $\mathcal{W}_{i,j}$ into $P_{i,j}$ disjoint subsets, denoted by $\mathcal{W}_{i,j}^{[p]}$, where $1 \le p \le P_{i,j}$. These partitions have the property that

$$\mathcal{W}_{i,j}^{[p]} \subseteq \mathcal{W}_{i,j} \Big\backslash \bigcup_{s=1}^{p-1} \mathcal{W}_{i,j}^{[s]} \quad \text{and} \quad \mathcal{W}_{i,j} = \bigcup_{p=1}^{P_{i,j}} \mathcal{W}_{i,j}^{[p]}. \tag{2}$$

The mapping of a set of source words $X \in \mathcal{W}_{i,j}^{[p]}$ onto a set of codewords $Y \in \mathcal{M}$ is specified by the templates $\mathtt{X}_{i,j[p]}$ and $\mathtt{Y}_{i,j[p]}$. The first string $\mathtt{X}_{i,j[p]}$ specifies the set $\mathcal{W}_{i,j}^{[p]}$ by indicating the relative positions of zeros, ones, and source symbols. The second string $\mathtt{Y}_{i,j[p]}$ specifies the subset of $\mathcal{M}_s$, onto which the elements of $X \in \mathcal{W}_{i,j}^{[p]}$ are mapped.

We choose to let the central positions indicate at which side the violation occurred. The elements X of the sets $\mathcal{W}_{i,0}$, where $1 \le i \le 5$, will be mapped onto elements of the sets $\mathcal{M}_1$, $\mathcal{M}_3$, and $\mathcal{M}_5$. This makes it easy to use the symmetry and to use the central positions in reversed order to handle the violations $\mathcal{W}_{0,i}$. The mapping is "symmetric" and marked by S if an element $X \in \mathcal{W}_{i,j}^{[p]}$ is translated into an element $Y \in \mathcal{M}_{2m-1}$, $1 \le m \le 3$, and the element $X' \in \mathcal{W}_{j,i}^{[p]}$ is translated into $Y' \in \mathcal{M}_{2m}$. We will often specify one mapping for several sets $\mathcal{W}_{i,j}$.

We first consider the mapping of elements $X \in \mathcal{W}_{0,0}$, specified in a compact format in Table IV. The elements $X \in \mathcal{W}_{0,0}$ are mapped onto the elements of $\mathcal{M}_0$, unless the (0, 15, 9, 9) RLL constraints are violated. Table IV specifies the subsets $\mathcal{W}_{0,0}^{[1]}$ up to $\mathcal{W}_{0,0}^{[4]}$ for which the (0, 15, 9, 9) RLL constraints would be violated, and the remaining subset $\mathcal{W}_{0,0}^{[5]}$, which satisfies the (0, 15, 9, 9) RLL constraints. The subsequences that violate the constraints are boldfaced to emphasize the kind of violation that is under consideration. The underlined zeros and ones in the template Y indicate which positions uniquely specify the mapping. The decoder checks these positions, and if there is a match, the mapping will be reversed to reconstruct the source word.

We now consider the mapping of the elements $X \in \mathcal{W}_{0,0}^{[1]}$ in more detail. The template X specifies the additional constraints that are imposed. Because $X^l \in \mathcal{V}_0$, we have $x_1^l x_2^l \cdots x_6^l \in \mathcal{G}_{3,2,2}^{(6)}$. In other words, X specifies $G_{3,2,2}^{(6)} = 48$ different source words. The template Y specifies the corresponding codewords. Because $x_1^l x_2^l \cdots x_6^l \in \mathcal{G}_{3,2,2}^{(6)}$, we have $x_1^l x_2^l x_3^l \ne 111$ and $x_4^l x_5^l x_6^l \ne 111$. This guarantees that the words corresponding to template Y satisfy the imposed constraints.

The five mappings in Table IV specify the transformation of $W_{0,0} = 37\,249$ source words into codewords. The sets $\mathcal{W}_{0,0}^{[1]}, \ldots, \mathcal{W}_{0,0}^{[4]}$ are mapped onto subsets of $\mathcal{M}_3$ and $\mathcal{M}_4$, and the set $\mathcal{W}_{0,0}^{[5]}$ is mapped onto $\mathcal{M}_0$.

Let $z^l = \overline{x}_1^l \cdot \overline{x}_2^l \cdot \overline{x}_3^l \cdot \overline{x}_4^l \cdot \overline{x}_5^l \cdot \overline{x}_6^l \cdot \overline{x}_7^l \cdot \overline{x}_8^l \cdot \overline{x}_8^r$ and $z^r = \overline{x}_1^r \cdot \overline{x}_2^r \cdot \overline{x}_3^r \cdot \overline{x}_4^r \cdot \overline{x}_5^r \cdot \overline{x}_6^r \cdot \overline{x}_7^r \cdot \overline{x}_8^r \cdot \overline{x}_8^l$. To determine to which set $\mathcal{W}_{0,0}^{[i]}$ a word $X \in \mathcal{W}_{0,0}$ belongs, we can evaluate the following set of equations, which were directly obtained from Table IV:

$$\begin{aligned}
w_{0,0[1]} &= v_0^l \cdot v_0^r \cdot x_7^l \cdot z^r \\
w_{0,0[2]} &= v_0^l \cdot v_0^r \cdot \overline{x}_7^l \cdot z^r \\
w_{0,0[3]} &= v_0^l \cdot v_0^r \cdot x_7^r \cdot z^l \\
w_{0,0[4]} &= v_0^l \cdot v_0^r \cdot \overline{x}_7^l \cdot z^l \cdot \overline{w}_{0,0[1]} \\
w_{0,0[5]} &= v_0^l \cdot v_0^r \cdot \overline{w}_{0,0[1]} \cdot \overline{w}_{0,0[2]} \cdot \overline{w}_{0,0[3]} \cdot \overline{w}_{0,0[4]}.
\end{aligned}$$

It can be verified that exactly one out of the five binary values $w_{0,0[1]}, \ldots, w_{0,0[5]}$ will be nonzero.

Table V specifies the mapping for the source words $X \in \mathcal{W}_{i,\{0,\ldots\}}$, $1 \le i \le 5$, where $\mathcal{W}_{i,\{0,\ldots\}}$ is the shorthand notation of $\mathcal{W}_{i,\{0,a,b\}} = \mathcal{W}_{i,0} \cup \mathcal{W}_{i,a} \cup \mathcal{W}_{i,b}$. The character S in the right-most column marks the sets $\mathcal{W}_{i,j}^{[p]}$ that have the property that by reversing the order of X and Y the mapping of the

TABLE V
MAPPING OF THE WORDS $X \in \mathcal{W}_{i,\{0,\dots\}}$, WHERE $1 \le i \le 5$

		y_1^l	y_2^l	y_3^l	y_4^l	y_5^l	y_6^l	y_7^l	y_8^l	y_c	y_8^r	y_7^r	y_6^r	y_5^r	y_4^r	y_3^r	y_2^r	y_1^r	
$\mathcal{W}^{[1]}_{1,\{0,5\}}$	X	**1**	**1**	**1**	x_4^l	x_5^l	**1**	**1**	x_8^l		x_8^r	x_7^r	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r	
	Y	x_8^l	0	1	x_4^l	x_5^l	0	x_8^r	1	1	0	x_7^r	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r	$\mathcal{S}$
$\mathcal{W}^{[2]}_{1,\{0,5\}}$	X	**1**	**1**	**1**	x_4^l	x_5^l	x_6^l	x_7^l	x_8^l		x_8^r	x_7^r	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r	
	Y	x_8^l	x_8^r	0	x_4^l	x_5^l	x_6^l	x_7^l	1	1	0	x_7^r	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r	$\mathcal{S}$
$\mathcal{W}^{[1]}_{2,\{0,4,5\}}$	X	**0**	**1**	**1**	**1**	**1**	1	1	1		x_8^r	x_7^r	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r	
	Y	0	1	1	0	x_8^r	x_7^r	1	0	1	0	0	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r	$\mathcal{S}$
$\mathcal{W}^{[2]}_{2,\{0,4,5\}}$	X	**0**	**1**	**1**	**1**	**1**	x_6^l	x_7^l	x_8^l		x_8^r	x_7^r	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r	
	Y	x_7^r	x_8^r	0	x_8^l	x_7^l	x_6^l	1	0	1	0	0	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r	$\mathcal{S}$
$\mathcal{W}_{3,\{0,4,5\}}$	X	x_1^l	**0**	**1**	**1**	**1**	**1**	x_7^l	x_8^l		x_8^r	x_7^r	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r	
	Y	x_1^l	x_8^l	0	x_7^l	x_8^r	x_7^r	0	0	1	0	0	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r	$\mathcal{S}'$
$\mathcal{W}^{[1]}_{4,\{0,5\}}$	X	x_1^l	1	**0**	**1**	**1**	**1**	**1**	1		x_8^r	x_7^r	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r	
	Y	x_1^l	0	1	0	x_8^r	1	0	1	1	0	x_7^r	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r	$\mathcal{S}$
$\mathcal{W}^{[2]}_{4,\{0,5\}}$	X	1	1	**0**	**1**	**1**	**1**	**1**	0		1	x_7^r	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r	
	Y	0	1	1	1	0	x_7^r	1	0	1	0	0	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r	$\mathcal{S}$
$\mathcal{W}^{[3]}_{4,\{0,5\}}$	X	1	0	**0**	**1**	**1**	**1**	**1**	x_8^l		1	x_7^r	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r	
	Y	x_8^l	0	1	1	0	1	0	1	1	0	x_7^r	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r	$\mathcal{S}$
$\mathcal{W}^{[4]}_{4,\{0,5\}}$	X	x_1^l	x_2^l	**0**	**1**	**1**	**1**	**1**	x_8^l		x_8^r	x_7^r	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r	
	Y	0	1	1	x_1^l	x_8^r	x_8^l	x_2^l	1	1	0	x_7^r	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r	$\mathcal{S}$
$\mathcal{W}^{[1]}_{5,0}$	X	x_1^l	x_2^l	x_3^l	**0**	**1**	**1**	**1**	**1**		x_8^r	1	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r	
	Y	x_8^r	0	1	x_1^l	x_2^l	x_3^l	1	0	1	0	0	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r	$\mathcal{S}$
$\mathcal{W}^{[2]}_{5,0}$	X	x_1^l	x_2^l	x_3^l	**0**	**1**	**1**	**1**	**1**		x_8^r	0	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r	
	Y	x_8^r	0	1	x_1^l	x_2^l	x_3^l	0	0	1	0	0	x_6^r	x_5^r	x_4^r	x_3^r	x_2^r	x_1^r	$\mathcal{S}'$

set $\mathcal{W}^{[p]}_{j,i}$ is specified. Sets having the same property with the exception that the central indicator 00100 is replaced by 01110 and vice versa, are marked by $\mathcal{S}'$.

We observe that in several cases, the indicator in the middle of the codeword causes the right-hand side of the codeword to satisfy the (0, 3, 3, 2) constraints. There are in fact 12 out of the 25 subsets $\mathcal{W}_{i,j}$, with $1 \le i, j \le 5$, for which the mapping is specified in Table V. The mapping for the 13 remaining sets will be specified in the next tables.

We first consider the sets $\mathcal{W}_{i,j}$, where $1 \le i \le 3$ and $1 \le j \le 3$. The mapping, specified in Table VI is of the form $Y = \psi(X) = \psi_l(X^l)1\psi_r(X^r) \in \mathcal{M}_7$; i.e., the left part of the codeword Y is specified by a mapping $\psi_l(X^l)$ that depends on X^l only. The same holds for the right part of the codeword, which is given by $\psi_r(X^r)$, depending on X^r only. As an example, consider the source word $X = X^l X^{r'} = 11101001\,11111110$. It is easy to determine that $X^l \in \mathcal{V}_1^{[2]}$ and $X^r \in \mathcal{V}_2^{[1]}$. Table VI prescribes that in this situation, X^l is mapped onto $Y^l = 10001010$ and X^r is mapped onto $Y^r = 10111010$. We therefore obtain $Y = Y^l 1 Y^{r'} = 10001010\,1\,01011101$.

It can be verified that $|\mathcal{V}_1^{[1]}| = 4$, $|\mathcal{V}_1^{[2]}| = 28$, $|\mathcal{V}_2^{[1]}| = 2$, $|\mathcal{V}_2^{[2]}| = 6$, and $|\mathcal{V}_3| = 8$. Because the mapping above applies to both the left-hand side and the right-hand side of an element

TABLE VI
SINGLE-SIDED MAPPING FOR $\mathcal{W}_{i,j}$, WHERE $1 \le i \le 3$ AND $1 \le j \le 3$

		1	2	3	4	5	6	7	8
$\mathcal{V}_1{}^{[1]}$	X	**1**	**1**	**1**	1	1	1	x_7	x_8
	Y	x_8	0	1	1	0	x_7	1	0
$\mathcal{V}_1{}^{[2]}$	X	**1**	**1**	**1**	x_4	x_5	x_6	x_7	x_8
	Y	x_8	x_7	0	x_4	x_5	x_6	1	0
$\mathcal{V}_2{}^{[1]}$	X	**0**	**1**	**1**	**1**	**1**	1	1	x_8
	Y	x_8	0	1	1	1	0	1	0
$\mathcal{V}_2{}^{[2]}$	X	**0**	**1**	**1**	**1**	**1**	x_6	x_7	x_8
	Y	0	1	1	x_6	x_7	x_8	1	0
$\mathcal{V}_3$	X	x_1	**0**	**1**	**1**	**1**	**1**	x_7	x_8
	Y	x_1	0	1	0	x_7	x_8	1	0

$X \in \mathcal{W}_{i,j}$, where $1 \le i \le 3$ and $1 \le j \le 3$, there are in total $48 \cdot 48 = 2304$ source words that fall into this category and that are transformed to conform to these rules. Note that the set $\mathcal{M}_7$ with central indicator 10101 is used exclusively to identify this category.

TABLE VII
MAPPING OF THE WORDS $X \in \mathcal{W}_{4,1}$

	y_1^l	y_2^l	y_3^l	y_4^l	y_5^l	y_6^l	y_7^l	y_8^l	y_c	y_8^r	y_7^r	y_6^r	y_5^r	y_4^r	y_3^r	y_2^r	y_1^r	
$\mathcal{W}_{4,1}^{[1]}$	1	1	**0**	1	1	1	1	x_8^l		x_8^r	x_7^r	1	1	1	1	1	1	
	0	1	1	0	x_8^r	x_8^l	0	0	1	0	0	x_7^r	0	1	1	1	0	S'
$\mathcal{W}_{4,1}^{[2]}$	1	1	**0**	1	1	1	1	x_8^l		x_8^r	x_7^r	x_6^r	x_5^r	x_4^r	1	1	1	
	0	1	1	0	x_8^r	x_8^l	0	0	1	0	0	x_6^r	x_5^r	x_4^r	1	0	x_7^r	S'
$\mathcal{W}_{4,1}^{[3]}$	x_1^l	x_2^l	**0**	1	1	1	1	x_8^l		x_8^r	x_7^r	x_6^r	x_5^r	x_4^r	1	1	1	
	0	1	1	x_1^l	x_2^l	x_8^l	0	0	1	0	0	x_6^r	x_5^r	x_4^r	0	x_8^r	x_7^r	S'

TABLE VIII
MAPPING OF THE WORDS $X \in \mathcal{W}_{4,4} \cup \mathcal{W}_{5,5}$

	y_1^l	y_2^l	y_3^l	y_4^l	y_5^l	y_6^l	y_7^l	y_8^l	y_c	y_8^r	y_7^r	y_6^r	y_5^r	y_4^r	y_3^r	y_2^r	y_1^r
$\mathcal{W}_{4,4}^{[1]}$	x_1^l	1	**0**	1	1	1	1	1		**0**	1	1	1	1	**0**	1	x_1^r
	0	1	1	1	0	x_1^l	0	0	1	0	0	x_1^r	0	1	1	1	0
$\mathcal{W}_{4,4}^{[2]}$	x_1^l	1	**0**	1	1	1	1	1		**1**	1	1	1	1	**0**	1	x_1^r
	0	1	1	1	0	x_1^l	0	1	1	1	0	x_1^r	0	1	1	1	0
$\mathcal{W}_{4,4}^{[3]}$	x_1^l	x_2^l	**0**	1	1	1	1	x_8^l		**1**	1	1	1	1	**0**	x_2^r	x_1^r
	x_1^l	0	1	x_2^l	x_8^l	x_2^r	0	1	1	1	0	x_1^r	0	1	1	1	0
$\mathcal{W}_{4,4}^{[4]}$	x_1^l	x_2^l	**0**	1	1	1	1	x_8^l		**0**	1	1	1	1	**0**	x_2^r	x_1^r
	0	1	1	1	0	x_1^l	0	0	1	0	0	x_2^l	x_2^r	x_8^l	1	0	x_1^r
$\mathcal{W}_{5,5}^{[1]}$	**1**	**1**	**0**	**0**	1	1	1	1		**1**	1	1	1	**0**	**0**	x_2^r	x_1^r
	0	1	1	1	0	1	1	0	1	0	0	1	1	1	0	x_2^r	x_1^r
$\mathcal{W}_{5,5}^{[2]}$	**1**	**1**	**0**	**0**	1	1	1	1		**1**	1	1	1	**0**	1	x_2^r	x_1^r
	0	1	1	0	1	1	0	1	1	1	0	x_2^r	x_1^r	0	1	1	0
$\mathcal{W}_{5,5}^{[3]}$	x_1^l	x_2^l	x_3^l	**0**	1	1	1	1		**1**	1	1	1	**0**	**0**	x_2^r	x_1^r
	0	1	1	x_1^l	x_2^l	x_3^l	0	0	1	0	0	x_2^r	x_1^r	0	1	1	0
$\mathcal{W}_{5,5}^{[4]}$	x_1^l	x_2^l	x_3^l	**0**	1	1	1	1		**1**	1	1	1	**0**	1	x_2^r	x_1^r
	0	1	1	0	x_1^r	x_2^r	0	1	1	1	0	x_3^l	x_2^l	x_1^l	1	1	0

The mapping of the words $X \in \mathcal{W}_{4,1}$ is specified in Table VII. The sets $\mathcal{W}_{4,4}$ and $\mathcal{W}_{5,5}$ are the only sets for which the mapping has not been specified yet. As there are only very few candidate words remaining, the sets have to be further partitioned in order to be able to map all of their elements. The resulting mapping is specified in Table VIII. At this stage, the specification is completed. There are 18 surplus candidate words, one or several of which might be used to replace codewords or as special recognition sequences for frame synchronization.

D. Derived Codes

The rate 16/17 (0, 3, 2, 2) code is suitable for the construction of higher rate codes because of its tight constraints. This is achieved by interspersing the codewords with uncoded source symbols, as described in [15]–[17]. Among others, we obtain a rate 32/33 (0, 7, 4, 4) code and a rate 48/49 (0, 11, 6, 6) code by inserting one and two arbitrary symbols between the bits of the codeword, respectively. Error propagation is limited to two bytes. To further suppress error propagation, we can use a strategy in which the interspersed bits are observed and the source symbols of the underlying constrained code are transformed only if the overall constraints are violated. Consider a rate $(n-1)/n$, $(0, k, k_l, k_r)$ RLL code that is obtained by interspersing a rate $(n'-1)/n'$, $(0, k', k'_l, k'_r)$ RLL code, as described in [15]–[17]. The mapping $X = X^l X^{r'} \in \mathcal{A}^{n'-1}$ onto $Y = X^l 1 X^{r'}$ is used as long as the resulting length-n sequence fulfills the $(0, k, k_l, k_r)$ constraints, even if Y does not fulfill the $(0, k', k'_l, k'_r)$ constraints. This strategy has the advantage that source symbols are transformed less often. In general, the probability that the translation of a source word $U = ST \in \mathcal{A}^s \times \mathcal{A}^t$ onto $Z = S1T$ satisfies the $(0, k, k_l, k_r)$ constraints is given by

$$\Pr\left[Z \in \mathcal{F}^{(n)}_{k,k_l,k_r}\right] = \frac{F^{(s)}_{k,k_l,k} \cdot F^{(t)}_{k,k,k_r}}{2^{s+t}}, \tag{3}$$

where $n = s+t+1$. For example, the probability that a source word for the rate 16/17 (0, 3, 2, 2) code needs to be transformed

is 0.432, whereas the probability that a source word of the rate 48/49 (0, 11, 6, 6) code needs to be transformed is only 0.0185 if the inserted bits surrounding the 17-bits of the 16/17 (0, 3, 2, 2) code are observed. As this resembles the "uncoded" situation, error propagation will consequently be further limited.

V. Conclusion

We have presented a methodology for efficiently converting user information into a constrained sequence in which restrictions are imposed on the runs of consecutive-like symbols. The techniques are specifically useful for the construction of MTR codes and RLL codes, which find widespread application in recording systems. The proposed techniques are exemplified by constructing a high-rate multipurpose modulation code that is of interest for novel recording systems. The constructed code, a rate 16/17, (0, 3, 2, 2) MTR code that also fulfills (0, 15, 9, 9) RLL constraints, has been described in detail. This particular code, which has been implemented in a Lucent read-channel chip, has an excellent performance. The MTR constraints and the RLL constraints are the best possible for the given rate 16/17. The code is a very efficient, multipurpose code and is expected to be very useful as a basic code for the construction of higher rate constrained codes by interleaving the codewords with uncoded bits.

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