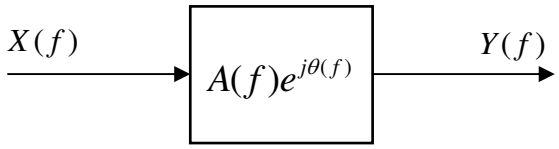
E6712: Homework 1 - Solution

Prepared by Anmo Kim and Emina Soljanin

1. (9.1) A channel is distortionless if the response $y(t)$ to an input $x(t)$ is $Kx(t-t_0)$, where K and t_0 are constants. Show that if the frequency response of the channel is $A(f)e^{j\theta(f)}$, where $A(f)$ and $\theta(f)$ are real, the necessary and sufficient conditions for distortionless transmission are $A(f) = K$ and $\theta(f) = 2\pi f t_0 \pm n\pi, n = 0, 1, 2, \ldots$

Solution



The impulse response function of above system is ditortionless iff,

$$\begin{aligned} Y(f) &= X(f) \cdot H(f) \\ &= X(f) \cdot A(f)e^{j\theta(f)} \\ &= \mathcal{F}\{Kx(t-t_0)\} \\ &= X(f) \cdot Ke^{-j2\pi f t_0} \end{aligned}$$

Thus, $A(f) = K$ and $\theta(f) = \pm 2\pi f t_0 \pm n\pi, n = 0, 1, 2, \ldots$ make the system distortionless.

* Notice that the positive sign of the first term and odd n are possible since K and t_0 can be positive or negative.

2. (9.3)

   (a) Show that (Poisson sum formula)

$$x(t) = \sum_{k=-\infty}^{\infty} g(t)h(t-kT) \Rightarrow X(f) = \sum_{n=-\infty}^{\infty} H(\frac{n}{T})G(f-\frac{n}{T}) \tag{1}$$

Solution

Let the Fourier series coefficient of impulse train, $\sum_{k=-\infty}^{\infty} \delta(t-kT)$, be a_n. We can get $a_n = 1/T$. Then,

$$\begin{aligned} \mathcal{F}\left\{\sum_{k=-\infty}^{\infty} h(t-kT)\right\} &= \mathcal{F}\left\{h(t) * \sum_{k=-\infty}^{\infty} \delta(t-kT)\right\} \\ &= H(f) \cdot \mathcal{F}\left\{\sum_{n=-\infty}^{\infty} a_n e^{j2\pi n\frac{t}{T}}\right\} \\ &= H(f) \cdot \sum_{n=-\infty}^{\infty} a_n \int_{-\infty}^{\infty} e^{j2\pi n\frac{t}{T}} e^{-j2\pi f t} dt \\ &= H(f) \cdot \frac{1}{T} \sum_{n=-\infty}^{\infty} \delta(f-\frac{n}{T}) \end{aligned}$$

From eqn. (1),

$$
\begin{aligned}
X(f) &= G(f) * \mathcal{F}\left\{\sum_{k=-\infty}^{\infty} h(t-kT)\right\} \\
&= G(f) * \left\{H(f)\cdot\frac{1}{T}\sum_{n=-\infty}^{\infty}\delta(f-\frac{n}{T})\right\} \\
&= \int_{-\infty}^{\infty}\left\{G(f-\omega)\cdot H(\omega)\frac{1}{T}\sum_{n=-\infty}^{\infty}\delta(\omega-\frac{n}{T})\right\}d\omega \\
&= \frac{1}{T}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\left\{G(f-\omega)H(\omega)\delta(\omega-\frac{n}{T})\right\}d\omega \\
&= \frac{1}{T}\sum_{n=-\infty}^{\infty}H(\frac{n}{T})G(f-\frac{n}{T})
\end{aligned}
$$

(b) Using the result in (a), verify the following versions of the Poisson sum:

$$\sum_{k=-\infty}^{\infty} h(kT) = \frac{1}{T}\sum_{n=-\infty}^{\infty} H(\frac{n}{T}) \tag{2}$$

$$\sum_{k=-\infty}^{\infty} h(t-kT) = \frac{1}{T}\sum_{n=-\infty}^{\infty} H(\frac{n}{T})exp\left(\frac{j2\pi nt}{T}\right) \tag{3}$$

$$\sum_{k=-\infty}^{\infty} h(kT)exp(-j2\pi kTf) = \frac{1}{T}\sum_{n=-\infty}^{\infty} H(f-\frac{n}{T}) \tag{4}$$

Solution

i. Let $g(t) = \delta(t)$.

$$
\begin{aligned}
\mathcal{F}\{x(t)\} &= \sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty}\delta(t)h(t-kT)e^{-j2\pi ft}dt \\
&= \sum_{k=-\infty}^{\infty} h(-kT) \\
&= \sum_{k=-\infty}^{\infty} h(kT)
\end{aligned}
$$

By inserting $G(f) = 1$ into eqn.(1), eqn.(2) can be proved.

ii. let $g(t) = \delta(t-\tau)$.

$$\mathcal{F}\{x(t)\} = \sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty}\delta(t-kT)\cdot h(t-\tau)e^{-j2\pi ft}dt \tag{5}$$

$$= \sum_{k=-\infty}^{\infty} h(\tau-kT)e^{-j2\pi f\tau} \tag{6}$$

Inserting $G(f) = e^{-j2\pi f\tau}$ into eqn.(1) and let $f = 0$,

$$
\begin{aligned}
X(0) &= \sum_{n=-\infty}^{\infty} H(\frac{n}{T})\exp(-j2\pi(0-\frac{n}{T})\tau) \\
&= \sum_{n=-\infty}^{\infty} H(\frac{n}{T})\exp(j2\pi\frac{n}{T}\tau)
\end{aligned}
$$

By letting f be 0 in eqn.(6), eqn.(3) will be proved.

iii. let $h(t) = \delta(t)$.

$$\mathcal{F}\{x(t)\} = \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} g(t) \cdot \delta(t-\tau) e^{-j2\pi ft} dt \tag{7}$$

$$= \sum_{k=-\infty}^{\infty} g(kT) \exp(-j2\pi kTf) \tag{8}$$

By applying $H(f) = 1$ into eqn.(1),

$$X(f) = \frac{1}{T} \sum_{n=-\infty}^{\infty} 1 \cdot G\left(f - \frac{n}{T}\right) \tag{9}$$

$$= \frac{1}{T} \sum_{n=-\infty}^{\infty} G\left(f - \frac{n}{T}\right) \tag{10}$$

Here, eqn.(8) must be same as eqn.(10).

$$\sum_{k=-\infty}^{\infty} g(kT) \exp(-j2\pi kTf) = \frac{1}{T} \sum_{n=-\infty}^{\infty} G\left(f - \frac{n}{T}\right) \tag{11}$$

It proves the eqn.(4).

(c) Derive the condition for no intersymbol interference (Nyquist criterion) by using the Poisson sum formula.

Solution

By the eqn.(9.2-12) in the textbook, the necessary and sufficient condition for x(t) having no intersymbol interference is:

$$\sum_{m=-\infty}^{\infty} X(f + m/T) = T \tag{12}$$

By the Poisson sum formula, eqn.(12) becomes:

$$T \cdot \sum_{k=-\infty}^{\infty} x(kT) \exp(-j2\pi kTf) = T \tag{13}$$

$$\sum_{k=-\infty}^{\infty} x(kT) \exp(-j2\pi kTf) = 1 \tag{14}$$

Eqn.(14) is the necessary and sufficient condition for no intersymbol interference.

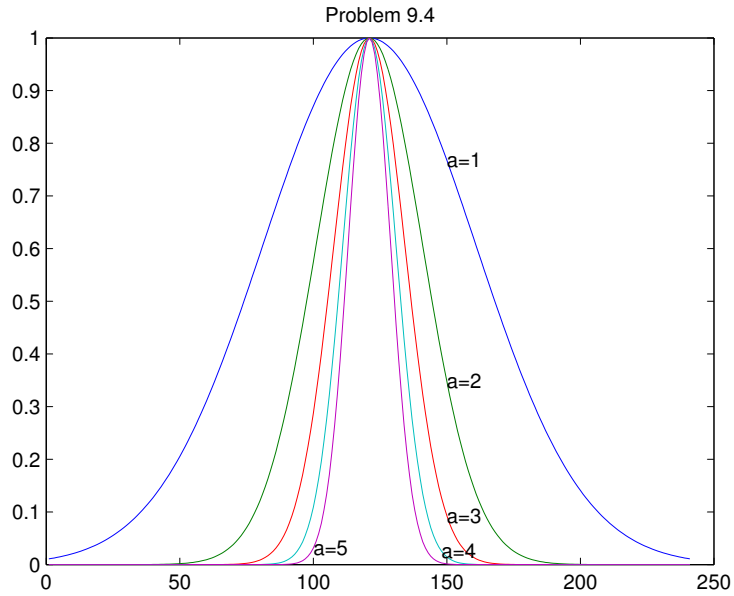
3. (9.4) Suppose a digital communication system employs Gaussian-shaped pulses of the form

$$x(t) = \exp(-\pi a^2 t^2)$$

To reduce the level of intersymbol interference to a relatively small amount, we impose the condition that $x(T) = 0.01$, where T is the symbol interval. The bandwidth W of the pulse $x(t)$ is defined as that value of W for which $X(W)/X(0) = 0.01$, where $X(f)$ is the Fourier transform of $x(t)$. Determine the value of W and compare this value to that of raised cosine spectrum with 100 percent rolloff.

Solution

Following figures shows the plot of $x(t)$:



From $x(T) = 0.01$,

$$x(T) = 0.01 \tag{15}$$

$$-\pi a^2 T^2 = \ln 0.01 \tag{16}$$

$$a^2 = \frac{\ln 100}{\pi T^2} \tag{17}$$

Fourier transform of $x(t)$ is:

$$\begin{aligned}
\mathcal{F}\{x(t)\} &= \int_{-\infty}^{\infty} e^{-\pi a^2 t^2} e^{-j2\pi ft} dt \\
&= \int_{-\infty}^{\infty} e^{-\pi(|a|t + j\frac{f}{|a|})^2} e^{-\pi\frac{f^2}{|a|^2}} dt \\
&= e^{-\pi\frac{f^2}{|a|^2}} \int_{-\infty}^{\infty} e^{-\pi(|a|t - j\frac{f}{|a|})^2} dt
\end{aligned}$$

Here, let $u\sqrt{(2)} = \sqrt{\pi}(|a|t - jf/|a|)$. Then, $dt = 1/(|a|\sqrt{2\pi})du$. By substituting u for t,

$$\begin{aligned}
X(f) &= e^{-\pi\frac{f^2}{|a|^2}} \int_{-\infty}^{\infty} e^{\frac{-u^2}{2}} du \\
&= \frac{1}{|a|} e^{-\pi\frac{f^2}{|a|^2}}
\end{aligned}$$

By the given condition, $X(W)/X(0) = 0.01$,

$$\frac{X(W)}{X(0)} = \frac{\frac{1}{|a|} e^{-\pi\frac{W^2}{|a|^2}}}{\frac{1}{|a|}} = 0.01$$

$$-\pi\frac{W^2}{|a|^2} = \ln 0.01$$

$$W^2 = |a|^2 \frac{\ln 100}{\pi}$$

By applying eqn.(17),

$$W^2 = \frac{\ln 100}{\pi T^2} \cdot \frac{\ln 100}{\pi}$$
$$W = \pm \frac{\ln 100}{\pi T}$$
$$W = 1.4659T$$

In case of the raised cosine filter with 100% rolloff factor,

$$W = \frac{1}{T}$$

Thus, the given Gaussian-shaped filter requires 1.4659 times of bandwidth compared with the raised cosine filter. (i.e. the raised cosine filter shows better performance.)

4. (9.10) A band-limited signal having bandwidth W can be represented as

$$x(t) = \sum_{n=-\infty}^{\infty} x_n \frac{\sin[2\pi W(t - n/2W)]}{2\pi W(t - n/2W)} \tag{18}$$

(a) Determine the spectrum $X(f)$ and plot $|X(f)|$ for the following cases:

$$x_0 = 2, \quad x_1 = 1, \quad x_2 = -1, \quad x_n = 0, n \neq 0, 1, 2$$
$$x_{-1} = -1, \quad x_0 = 2, \quad x_1 = -1, \quad x_n = 0, n \neq -1, 0, 1$$

Solution

We know that,

$$\mathcal{F}\left\{\frac{\sin 2\pi W t}{\pi t}\right\} = \begin{cases} 1 & , |f| < W \\ 0 & , \text{otherwise} \end{cases}$$

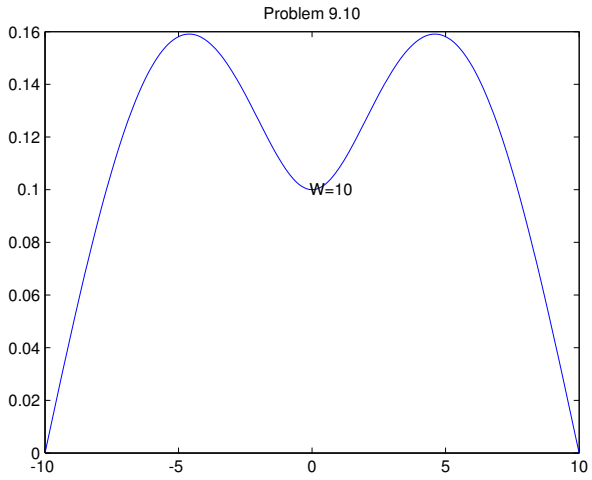
Thus,

$$\mathcal{F}\left\{\frac{\sin[2\pi W(t - n/2W)]}{2\pi W(t - n/2W)}\right\} = \begin{cases} \frac{1}{2W}\exp(-j2\pi f \frac{n}{2W}) & , |f| < W \\ 0 & , \text{otherwise} \end{cases}$$

For the first case, $x_0 = 2, x_1 = 1, x_2 = -1, x_n = 0, n \neq 0, 1, 2,$

$$X(f) = \begin{cases} \frac{1}{W} + \frac{1}{2W}\exp(-j2\pi f \frac{1}{2W}) - \frac{1}{2W}\exp(-j2\pi f \frac{2}{2W}) & , |f| < W \\ 0 & , \text{otherwise} \end{cases}$$
$$|X(f)| = \begin{cases} \frac{1}{2W}\left\{6 + 2\cos(2\pi f/2W) - 4\cos(2\pi f/W)\right\}^{1/2} & , |f| < W \\ 0 & , \text{otherwise} \end{cases}$$

The plot of above equation is as follows:

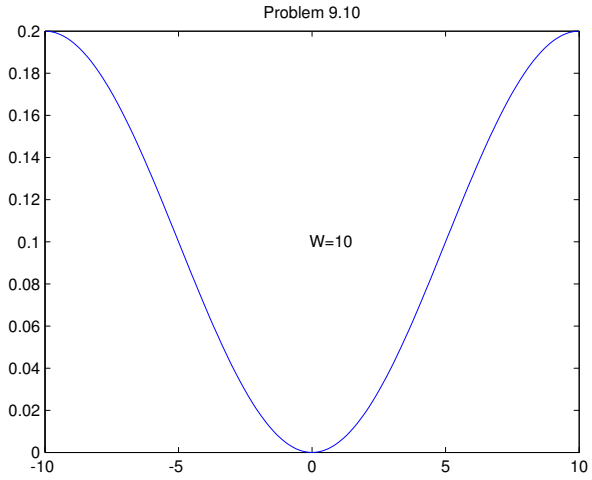


For the second case, $x_{-1} = -1, x_0 = 2, x_1 = -1, x_n = 0, n \neq -1, 0, 1,$,

$$X(f) = \begin{cases} \frac{1}{W} - \frac{1}{2W}\exp(j2\pi f \frac{1}{2W}) - \frac{1}{2W}\exp(-j2\pi f \frac{1}{2W}) & , |f| < W \\ 0 & , \text{otherwise} \end{cases}$$

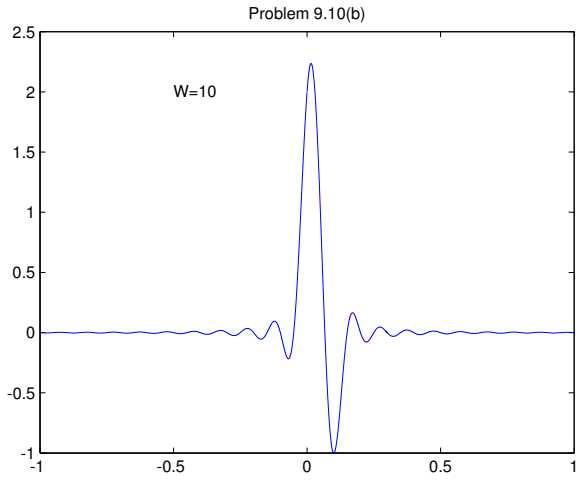
$$|X(f)| = X(f) = \begin{cases} \frac{1}{W}\{1 - \cos(\pi f/W)\} & , |f| < W \\ 0 & , \text{otherwise} \end{cases}$$

The plot of above equation is as follows:

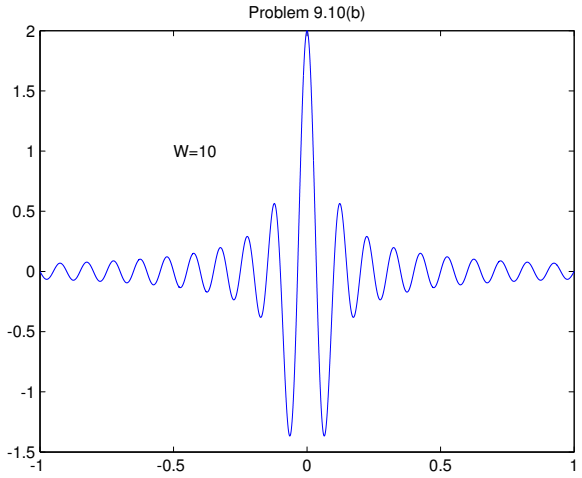


(b) Plot x(t) for these two cases.

For the first case, $x_0 = 2, x_1 = 1, x_2 = -1, x_n = 0, n \neq 0, 1, 2,$



For the second case, $x_{-1} = -1, x_0 = 2, x_1 = -1, x_n = 0, n \neq -1, 0, 1,$,



(c) If these signals are used for binary signal transmission, determine the number of received levels possible at the sampling instants $t = nT = n/2W$ and the probabilities of occurrence of the received levels. Assume that the binary digits at the transmitter are equally probable.

Solution

Because $t = nT = n/2W$,

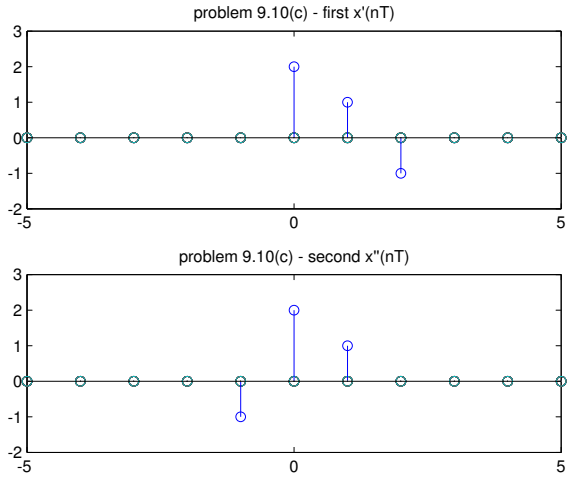
$$x(nT) = \sum_{k=-\infty}^{\infty} x_k \frac{\sin[\pi(n-k)]}{\pi(n-k)}$$

For the first case, $x_0 = 2, x_1 = 1, x_2 = -1, x_n = 0, n \neq 0, 1, 2,$

$$x(nT) = 2 \cdot \frac{\sin[\pi n]}{\pi n} + \frac{\sin[\pi(n-1)]}{\pi(n-1)} - \frac{\sin[\pi(n-2)]}{\pi(n-2)}$$

Then, we can get $x(0) = 2, x(1) = 1, x(2) = -1, x(n) = 0, n \neq 0, 1, 2.$
In the same way we can get $x(-1) = -1, x(0) = 2, x(1) = -1, x(n) = 0, n \neq -1, 0, 1,$ for the second case.

Let the first $x(t)$ be $x'(t)$ and the second $x(t)$ be $x''(t)$. The plot of those two signals are as follows:



Because each of those signals has the length of three, we can make all possible combinations by observing three consecutive signals. Let this A_0, A_1, A_2.
Then, in case of $x'(nT)$, $B = A_0(0) + A_1(1) + A_2(2)$ will have fair distribution of all possible values of the signal.

$A_2(2)$	-1·(-1)	-1·(-1)	-1·(-1)	-1·(-1)	1·(-1)	1·(-1)	1·(-1)	1·(-1)
$A_1(1)$	-1·1	-1·1	1·1	1·1	-1·1	-1·1	1·1	1·1
$A_0(0)$	-1·2	1·2	-1·2	1·2	-1·2	1·2	-1·2	1·2
B	-2	2	0	4	-4	0	-2	2

Then, the probability of each symbol is:

$$P(0) = \frac{1}{4}, P(2) = \frac{1}{4}, P(-2) = \frac{1}{4},$$
$$P(4) = \frac{1}{8}, P(-4) = \frac{1}{8}$$

In the same way, $x''(nT)$ can have:

$A_1''(1)$	-1·1	-1·1	-1·1	-1·1	1·1	1·1	1·1	1·1
$A_0''(0)$	-1·2	-1·2	1·2	1·2	-1·2	-1·2	1·2	1·2
$A''_{-1}(-1)$	-1·(-1)	1·(-1)	-1·(-1)	1·(-1)	-1·(-1)	1·(-1)	-1·(-1)	1·(-1)
B''	-2	-4	2	0	0	-2	4	2

Then, the probability of each symbol is same as $x'(t)$:

$$P(0) = \frac{1}{4}, P(2) = \frac{1}{4}, P(-2) = \frac{1}{4},$$
$$P(4) = \frac{1}{8}, P(-4) = \frac{1}{8}$$

5. (9.24) The binary sequence 10010110010 s the input to a precoder whose output is used to modulate a duobinary transmitting filter. Construct a table as in Table 9.2-1 showing the precoded sequence, the transmitted amplitude levels, the received signal levels, and the decoded sequence.

Solution

Data sequence D_n		1	0	0	1	0	1	1	0	0	1	0
Precoded sequence P_n	0	1	1	1	0	0	1	0	0	0	1	1
Transmitted sequence I_n	-1	1	1	1	-1	-1	1	-1	-1	-1	1	1
Received sequence B_n		0	2	2	0	-2	0	0	-2	-2	0	2
Decoded sequence D_n		1	0	0	1	0	1	1	0	0	1	0

6. (9.32) Determine the capacity of a (0,1) runlength-limited code. Compare its capacity with that of a $(1, \infty)$ code and explain the relationship.

Solution

For (0,1) runlength-limited code,

$$\mathbf{D} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

Then, the characteristic function and its solution is:

$$\det(\mathbf{D} - \lambda\mathbf{I}) = \det \begin{bmatrix} 1-\lambda & 1 \\ 1 & -\lambda \end{bmatrix} = \lambda^2 - \lambda - 1$$

$$\lambda = \frac{1 \pm \sqrt{5}}{2}$$

Thus, the capacity is:

$$C(0,1) = \log_2 \lambda_{max} = \log_2 \frac{1+\sqrt{5}}{2} = 0.6942$$

From the table 9.4-1, the capacity of $C(1, \infty)$ code is 0.6942.

The capacity of $(1, \infty)$ codes is the same as the one of $(0, 1)$ codes because there is a 1-to-1 mapping between the two systems. See the figure below, and notice that one system can be obtained from the other by flipping each symbol.

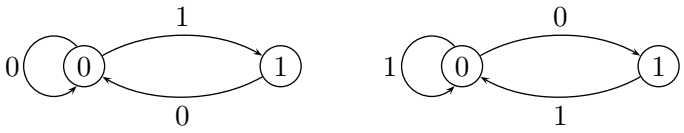


Figure 1: Graphs for the $(1, \infty)$ and $(0, 1)$ constraints.

7. (9.37) A simplified model of the telegraph code consists of two symbols. A dot consists of one time unit of line closure followed by one time unit of line open. A dash consists of three units of the line closure followed by one time unit of line open.

   (a) Viewing this code as a constrained code with symbols of equal duration, give the constraints.

   Solution

   Let the line closure be 1 and the line open be 0. Then allowed sequences consist of arbitrary concatenations of strings 10 and 111.

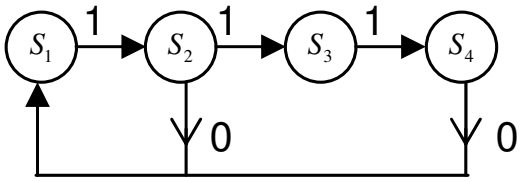
   constraints

   - each 0 is followed by a 1,
   - the number of 1's between two surrounding 0's must be one or three.

   (b) Determine the state transition matrix.

   Solution

   The finite state machine for this code is as follows:



Then, the state transition matrix is

$$\mathbf{D} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

(c) Determine the capacity.

Solution

The characteristic function and its solution is

$$\det(\mathbf{D} - \lambda\mathbf{I}) = \det \begin{bmatrix} -\lambda & 1 & 0 & 0 \\ 1 & -\lambda & 1 & 0 \\ 0 & 0 & -\lambda & 1 \\ 1 & 0 & 0 & -\lambda \end{bmatrix} = \lambda^4 - \lambda^2 - 1$$

$$\lambda = \pm\sqrt{\frac{1 \pm \sqrt{5}}{2}}$$

Then, the capacity is

$$\log_2 \lambda_{max} = \log_2 \sqrt{\frac{1+\sqrt{5}}{2}} = \log_2 1.272 = 0.3471$$

8. (10.16) In a magnetic recording channel, where the readback pulse resulting from a positive transition in the write current has the form

$$p(t) = \left[1 + \left(\frac{2t}{T_{50}}\right)^2\right]^{-1}$$

a linear equalizer is used to equalize the pulse to a partial response. The parameter T_{50} is defined as the width of the pulse at the 50 percent amplitude level. The bit rate is $1/T_b$ and the ration of $T_{50}/T_b = \Delta$ is the normalized density of the recording. Suppose the pulse is equalized to the partial-response values

$$x(nT) = \begin{cases} 1 & (n = -1, 1) \\ 2 & (n = 0) \\ 0 & (\text{otherwise}) \end{cases}$$

where $x(t)$ represents the equalized pulse shape.

(a) Determine the spectrum $X(f)$ of the band-limited equalized pulse

Solution

$$\begin{aligned} x(nT) &= x(t) * \delta(t - nT) = 2\delta(t) + \delta(t - T) + \delta(t + T) \\ \mathcal{F}\{x(nT)\} &= X(f)e^{-j2\pi fnT} = 2 + e^{-j2\pi fT} + e^{j2\pi fT} \\ \Rightarrow X(f) &= e^{j2\pi fnT}\left[2 + e^{-j2\pi fT} + e^{j2\pi fT}\right] \\ &= 2e^{j2\pi fnT}\left[1 + \cos(2\pi fT)\right] \end{aligned}$$

(b) Determine the possible output levels at the detector, assuming that successive transitions can occur at the rate $1/T_b$.

Solution

As the problem (9.10), we can make all possible combinations by observing three consecutive signals. Let this A_0, A_1, A_2. And the observed signal is defined as: $B = A_0(-1) + A_1(0) + A_2(1)$.

$A_2(1)$	-1·1	-1·1	-1·1	-1·1	1·1	1·1	1·1	1·1
$A_1(0)$	-1·2	-1·2	1·2	1·2	-1·2	-1·2	1·2	1·2
$A_0(-1)$	-1·1	1·1	-1·1	1·1	-1·1	1·1	-1·1	1·1
B	-4	-2	0	2	-2	0	2	4

The possible output levels are $-4, -2, 0, 2, 4$.

(c) Determine the error rate performance of the symbol-by-symbol detector for this signal, assuming that the additive noise is zero-mean Gaussian with variance σ^2.

Solution

The minimum distance between possible outputs is 2. Therefore, the error rate has lower bound

$$Q\left(\sqrt{\frac{2}{\mathcal{N}_0}}\right),$$

and the upper union bound

$$4 \cdot Q\left(\sqrt{\frac{2}{\mathcal{N}_0}}\right).$$

9. (10.22) A pulse and its (raised cosine) spectral characteristic are shown in Figure P10.22. This pulse is used for transmitting digital information over a band-limited channel at a rate 1/T symbols/s.

(a) What is the roll-off factor β?

Solution

3dB point of G(t) is $1/2T$.

$$R_s = \frac{1}{2T} = 900$$
$$T = \frac{1}{450}$$

Using the relationship between roll-off factor and bandwidth,

$$\frac{1+\beta}{2} 2R_s = W = 1200$$
$$\beta = \frac{1}{3}$$

(b) What is the pulse rate?

Solution

$$\frac{1}{T} = 1800\text{pulses/sec}$$

(c) The channel distorts the signal pulses. Suppose the sampled values of the filtered received pulse $x(t)$ are as shown in Figure P10.22c. It is obvious that there are five interfering signal components. Give the sequence of +1s and -1s that will cause the largest (destructive or constructive) interference and the corresponding value of the interference (the peak distortion).

Solution

If 1, -1, 1, 1, -1, 1 or -1, 1, -1, -1, 1, -1is given in that order, $\pm(0.1+0.3+0.6+1.0+0.4+0.2) = \pm 2.6$ can be generated. Then, the distortion is $2.6 - 1.0 = 1.6$.

(d) What is the probability of occurrence of the worst sequence obtained in (c), assuming that all binary digits are equally probable and independent?

Solution

Because the probability of symbol 1 and -1 are all 1/2,

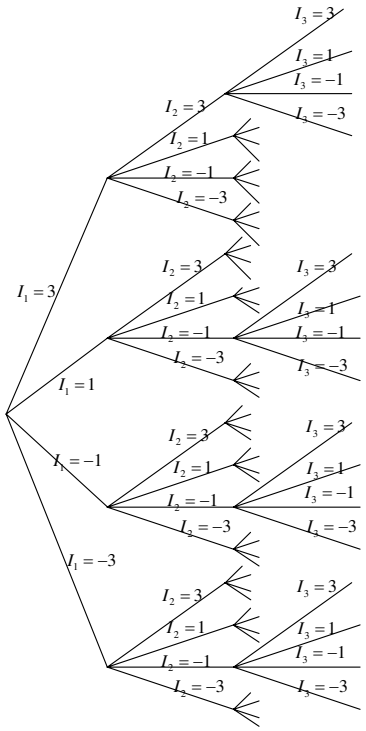
$$P\,(\text{distortion} = 1.6) = \left(\frac{1}{2}\right)^6 \cdot 2 = \left(\frac{1}{2}\right)^5.$$

10. (10.24) Consider a four-level PAM system with possible transmitted levels, 3, 1, -1, and -3. The channel through which the data is transmitted introduces intersymbol interference over two successive symbols. The equivalent discrete-time channel model is shown in Figure P10.24. $\{\eta_k\}$ is a sequence of real-valued independent zero-mean Gaussian noise variables with variance $\sigma^2 = N_0$. The received sequence is

$$\begin{aligned} y_1 &= 0.8I_1 + n_1 \\ y_2 &= 0.8I_2 - 0.6I_1 + n_2 \\ y_3 &= 0.8I_3 - 0.6I_2 + n_3 \\ &\vdots \\ y_k &= 0.8I_k - 0.6I_{k-1} + n_k \end{aligned}$$

(a) Sketch the tree structure, showing the possible signal sequences for the received signals y_1, y_2, and y_3.

Solution



Supposing that there is no noise, the possible received signals are:

I_1	3	3	3	3	3	3	3	3	3	3	3	3	3	...
I_2	3	3	3	3	1	1	1	1	-1	-1	-1	-1	-3	...
I_3	3	1	-1	-3	3	1	-1	-3	3	1	-1	-3	3	...
y_1	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.4	...
y_2	0.6	0.6	0.6	0.6	-1.2	-1.2	-1.2	-1.2	-2.6	-2.6	-2.6	-2.6	-4.2	...
y_3	0.6	-1.0	-2.6	-4.2	1.8	0.2	-1.4	-3.0	3.0	1.4	-0.2	1.8	4.2	...

(b) Suppose the Viterbi algorithm is used to detect the information sequence. How many probabilities must be computed at each stage of the algorithm?

Solution

There are four possible states and four possible transitions at each state. Therefore, 16 probabilities exist.

(c) How many surviving sequences are there in the Viterbi algorithm for the channel?

Solution

Because there are four possible states, the number of surviving sequences are also four.

(d) Suppose that the received signals are

$$y_1 = 0.5, y_2 = 2.0, y_3 = -1.0$$

Determine the surviving sequences through stage y_3 and the corresponding metrics.

Solution

The metrics for y_1 are:

$$\min_{I_1} (y_1 - 0.8I_1)^2$$

The metrics for $y_k, k \geq 2$ are:

$$\min_{I_k, I_{k-1}} (y_k - 0.8I_k + 0.6I_{k-1})^2$$

Surviving sequences are

$$\begin{matrix} 1 & 3 & 3 \\ 1 & 3 & 1 \\ -1 & -1 & -1 \\ -1 & -1 & -3 \end{matrix}$$

(e) Give a tight upper bound for the probability of error for four-level PAM transmitted over this channel.

Solution

Single symbol errors are most likely. Therefore

$$3 \cdot Q\left(\sqrt{\frac{d^2 \cdot (0.8^2 + 0.6^2)}{2\mathcal{N}_0}}\right),$$

where

$$d^2 = \frac{3}{M^2 - 1} E = 1$$