E6712: Lecture 4

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1 PSK - Review

Last time we studied the following M-ary modulation scheme:

$$\sqrt{E_m}s_m(t) = \Re\{A_m \cdot g(t) \cdot e^{j\theta_m} \cdot e^{j2\pi f_c t}\}, \quad 1 \le m \le M.$$

PSK: $\theta_m = 2\pi(m-1)/M$ carries the information, $A_m = \sqrt{E}$; $d_{\min} = 2\sqrt{E}\sin(\pi/M)$.

Draw the constellation, mention the Gray code.

2 The Optimal Receiver and the Equivalent Discrete Time Channel

Draw the receiver.

Discrete-time equivalent:

$$r_c(k) = A(k)\cos(\theta(k)) + n_{ck} \quad \text{and} \quad r_s(k) = A(k)\sin(\theta(k)) + n_{sk}$$

or

$$r(k) = A(k)e^{j\theta(k)} + n_k$$

Have to estimate the phase of the carrier! (training symbols where θ is known)

Differential encoding.

3 Differentially Encoded PSK

3.1 The Idea

Does not require the estimation of the carrier phase.

Differential encoding:

original phases in PSK :	$\theta(0)$	$\theta(1)$	$\theta(2)$	$\ldots$
transmitted phases in DE-PSK :	$\bar{\theta}(0)$	$\bar{\theta}(1)$	$\bar{\theta}(2)$	$\ldots$

where $\bar{\theta}(k) = \bar{\theta}(k-1) + \theta(k) \mod 2\pi$ and $\theta(-1) = 0$.

Decoding: $\theta(k) = \bar{\theta}(k) - \bar{\theta}(k-1) \mod 2\pi$.

Example: BPSK

original phases:		0	π	π	π	0	π	0	π	π
transmitted phases:	0	0	π	0	π	π	0	0	π	0
received phases:	π	π	0	π	0	0	π	π	0	π
decoded phases:		0	π	π	π	0	π	0	π	π

Draw the receiver.

3.2 Performance

Coherently demodulate $\bar{\theta}$, then decode $\theta(k) = \bar{\theta}(k) - \bar{\theta}(k-1) \mod 2\pi$.

Probability of error is $P\{(\hat{\bar{\theta}}(k) \neq \bar{\theta}(k), \hat{\bar{\theta}}(k-1) = \bar{\theta}(k-1)) \bigcup (\hat{\bar{\theta}}(k) = \bar{\theta}(k), \hat{\bar{\theta}}(k-1) \neq \bar{\theta}(k-1))\}$

$P^e_{\text{DE-BPSK}} = 2P^e_{\text{BPSK}}(1 - P^e_{\text{BPSK}}) \approx 2P^e_{\text{BPSK}}$, where $P^e_{\text{BPSK}} = Q(\sqrt{2E_b/\mathcal{N}_0})$.

4 DPSK

DE-PSK also allows a form of demodulation that does not require the estimation of the carrier phase.

We have

$$\begin{aligned} r(k) &= \sqrt{E_s} e^{j(\theta(k)-\phi)} + n_k \\ r(k-1) &= \sqrt{E_s} e^{j(\theta(k-1)-\phi)} + n_{k-1} \end{aligned}$$

Consider $r(k) \cdot r^*(k-1)$:

$$\begin{aligned} r(k) \cdot r^*(k-1) = & E_s e^{j(\theta(k)-\theta(k-1))} + \\ & n_k \sqrt{E_s} e^{-j(\theta(k-1)-\phi)} + n_{k-1} \sqrt{E_s} e^{j(\theta(k)-\phi)} + \\ & n_k n_{k-1} \end{aligned}$$

Draw the receiver.

DE-PSK decoded in this manner is called differential PSK or DPSK.

Performance?

5 Noncoherent Demodulation - AWGN

Last time we studied the following M-ary modulation scheme:

$$\sqrt{E_m} s_m(t) = \Re\{A_m \cdot g(t) \cdot e^{j\theta_m} \cdot e^{j2\pi f_c t}\}, \quad 1 \leq m \leq M.$$

Suppose that the receiver knows (or can estimate) f_c but not the phase of the received signal.

In general

$$\begin{aligned} r(t) &= \Re\{\tilde{s}(t) e^{j(2\pi f_c t + \theta)}\} + n(t) \\ &= s(t;\theta) + n(t) \end{aligned}$$

where $s(t;\theta) \in \{s_1(t;\theta) \ldots, s_M(t;\theta)\}$, $\int_0^T |s_m(t;\theta)|^2 dt = E_m$, and θ is a RV with PDF $f(\theta)$.

Conditional likelihood:

$$\begin{aligned} \Lambda_m(\theta) &\triangleq p(r(t), t \in [0,T] | s_m(t), t \in [0,T], \theta) \\ &\propto \exp\Big(\frac{2}{\mathcal{N}_0} \int_0^T r(t) s_m(t;\theta) dt - \frac{E_m}{\mathcal{N}_0}\Big) \end{aligned}$$

Likelihood:

$$\begin{aligned}
\Lambda_m &= \triangleq p(r(t), t \in [0,T] | s_m(t), t \in [0,T]) \\
&= \int_{-\pi}^{\pi} \Lambda_m(\theta) f(\theta) d\theta \\
&\propto \int_{-\pi}^{\pi} \exp\Big(\frac{2}{\mathcal{N}_0} \int_0^T r(t) s_m(t;\theta) dt - \frac{E_m}{\mathcal{N}_0}\Big) f(\theta) d\theta \\
&= \exp\Big(-\frac{E_m}{\mathcal{N}_0}\Big) \cdot \int_{-\pi}^{\pi} \exp\Big(\frac{2}{\mathcal{N}_0} \int_0^T r(t) s_m(t;\theta) dt\Big) f(\theta) d\theta \\
&= \exp\Big(-\frac{E_m}{\mathcal{N}_0}\Big) \cdot \int_{-\pi}^{\pi} \exp\Big(\frac{2}{\mathcal{N}_0} \Re\Big\{ e^{j\theta} \underbrace{\int_0^T r(t) \tilde{s}_m(t) e^{j2\pi f_c t} dt}_{L_m = |L_m| e^{j\varphi_m}} \Big\}\Big) f(\theta) d\theta \\
&= \exp\Big(-\frac{E_m}{\mathcal{N}_0}\Big) \cdot \int_{-\pi}^{\pi} \exp\Big(\frac{2}{\mathcal{N}_0} |L_m| \cos(\varphi_m + \theta)\Big) f(\theta) d\theta
\end{aligned}$$

Special case – uniform phase $f(\theta) = 1/(2\pi)$ on $[-\pi, \pi]$:

$$\Lambda_m = \exp\Big(-\frac{E_m}{\mathcal{N}_0}\Big) \cdot \underbrace{\frac{1}{2\pi} \int_{-\pi}^{\pi} \exp\Big(\frac{2}{\mathcal{N}_0} |L_m| \cos(\phi)\Big) d\phi}_{I_0\left(\frac{2}{\mathcal{N}_0}|L_m|\right)}$$

where

$$I_0 \triangleq \frac{1}{\pi} \int_0^{\pi} \exp(x \cos\theta) d\theta.$$

$$\boxed{\Lambda_m = \exp\Big(-\frac{E_m}{\mathcal{N}_0}\Big) \cdot I_0\Big(\frac{2}{\mathcal{N}_0}|L_m|\Big)}$$

Example: Equal energy symbols

$$\hat{m} = \max_{1 \le m \le M} |L_m|^2$$

5.1 Computation of L_m

$$L_m \triangleq \int_0^T r(t) \underbrace{\tilde{s}_m(t) e^{j2\pi f_c t}}_{s_m(t) + j\hat{s}_m(t)} dt$$

$\Rightarrow$

$$|L_m|^2 = \Big(\int_0^T r(t) s_m(t) dt\Big)^2 + \Big(\int_0^T r(t) \hat{s}_m(t) dt\Big)^2$$

5.2 Performance

Examples with no phase error: BPSK, binary orthogonal signals.