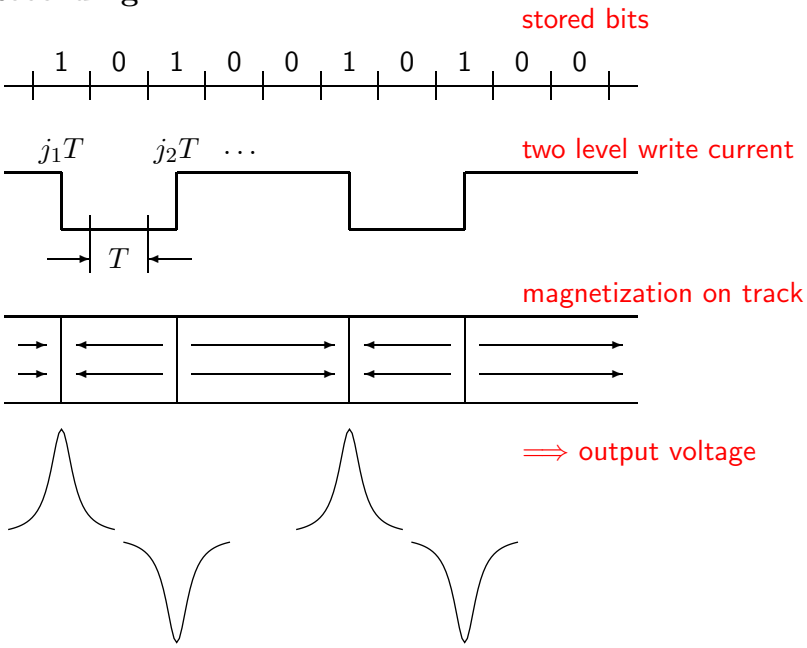
Feb. 25, 2004

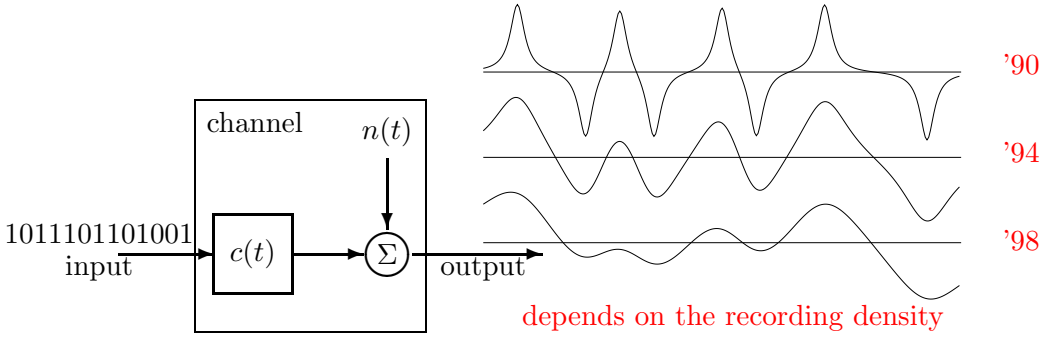
1 The Magnetic Recording Channel

1.1 Saturation Recording



$$g(t) = \frac{1}{1 + (2t/T_{50})^2}$$

1.2 The Continuous-Time Channel



2 Channels with ISI

2.1 Discrete Channel Model

The transmitted signal:

$$v(t) = \sum_n I_n g(t - nT),$$

where T is the symbol interval, $g(t)$ is the pulse shape, and $\{I_n\}$ is the information-bearing sequence of symbols.

The received signal:

$$r(t) = \sum_n I_n h(t - nT) + z(t), \text{ where } h(t) = \int g(\tau)c(t - \tau)d\tau$$

$z(t)$ is AWGN and $h(t)$ is the received pulse.

matched filter output $r(t) * h(t)$:

$$y(t) = \sum_n I_n x(t - nT) + \nu(t)$$

Sampling and discrete time equivalent:

$$y(kT + \tau_0) = \sum_n I_n x(kT - nT + \tau_0) + \nu(kT + \tau_0)$$

$$\therefore \boxed{y_k = \sum_n I_n x_{n-k} + \nu_k}$$

Set $x_0 = 0$ for convenience:

$$y_k = I_k + \underbrace{\sum_{n \neq k} I_n x_{n-k}}_{\text{ISI}} + \nu_k$$

2.2 The Nyquist Criterion for no ISI

Claim: The necessary and sufficient condition for $x(t)$ to satisfy

$$x_n = x(nT) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

is that its Fourier transform $X(f)$ satisfy

$$B(f) \triangleq \boxed{\sum_m X(f + m/T) = T}$$

Since $B(f)$ is periodic with period T, we check only the Nyquist period $[-\frac{1}{2T}, \frac{1}{2T}]$.

Case 1 $\frac{1}{2T} > W$, cannot have zero ISI.

Case 2 $\frac{1}{2T} = W$, use brick wall:

$$X(f) = \begin{cases} T, & |f| < W; \\ 0, & \text{otherwise.} \end{cases} \qquad \therefore \quad x(t) = \frac{\sin(\pi t/T)}{\pi t/T} = \text{sinc}\Big(\frac{\pi t}{T}\Big).$$

Case 3 $\frac{1}{2T} < W$, many solutions.

2.3 Pulse Shaping for no ISI

The raised cosine pulse with the *roll-off factor* β:

$$X_{\mathrm{rc}}(f) = \begin{cases} T, & 0 \le |f| \le \frac{1-\beta}{2T} \\ \frac{T}{2}\Big[1 + \cos\Big(\frac{\pi T}{\beta}(|f| - \frac{1-\beta}{2T})\Big)\Big], & \frac{1-\beta}{2T} \le |f| \le \frac{1+\beta}{2T} \\ 0, & |f| > \frac{1+\beta}{2T} \end{cases} \quad \therefore x_{\mathrm{rc}}(t) = \mathrm{sinc}\Big(\frac{\pi t}{T}\Big)\frac{\cos\Big(\frac{\pi\beta t}{T}\Big)}{1 - 4\beta^2 t^2/T^2}$$

Draw $x_{\mathrm{rc}}(t)$ and $X_{\mathrm{rc}}(f)$.

2.4 Controlled ISI - Partial Response

$$\sum_{n=0}^{K} \gamma_n D^n$$

duobinary $1 + D$

dicode $1 - D$

modified duobinary $1 - D^2$

Class 4 $(1 - D)(1 + D)^N$

Precoding for the duobinary channel:

Data sequence $\{D_n\}$

Precoded sequence $P_n = D_m + P_{n-1} \mod 2$

Transmitted sequence $I_n = 2P_n - 1$

Decoded sequence: $D_n = \frac{1}{2}B_n + 1 \mod 2$

Example: Table 9.2-1