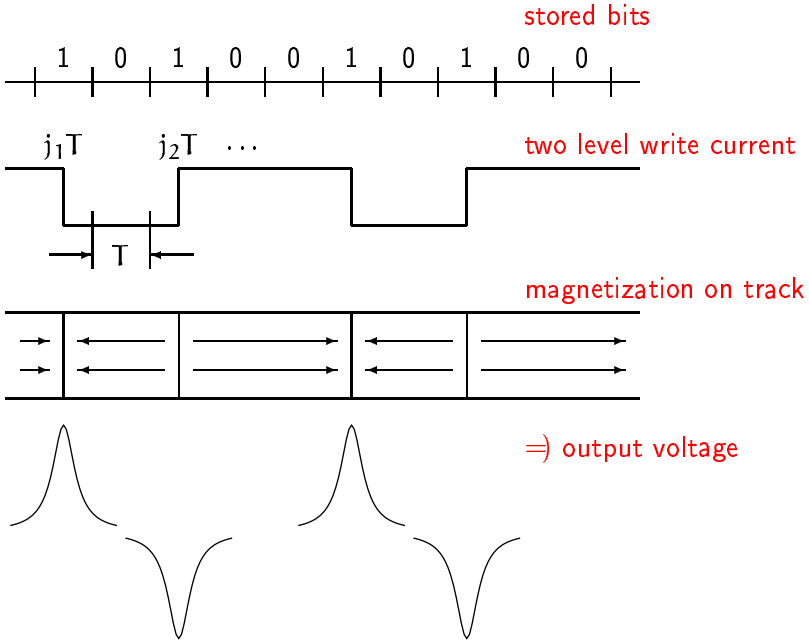
Lecture 6, E6712

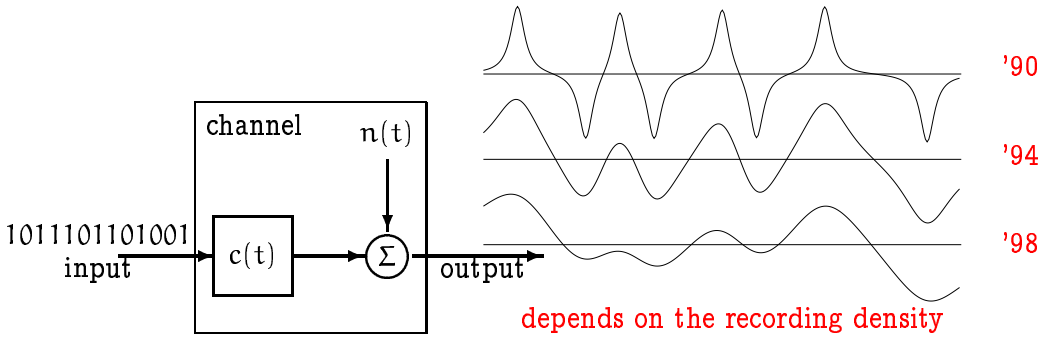
The Magnetic Recording Channel

Saturation Recording



$$g(t) = \frac{1}{1 + (2t/T_{50})^2}$$

The Continuous-Time Channel



Channels with ISI

Discrete Channel Model

The transmitted signal:

$$v(t) = \sum_n I_n g(t - nT),$$

where T is the symbol interval, $g(t)$ is the signal pulse waveform, and $\{I_n\}$ is the information-bearing sequence of symbols.

The received signal:

$$r(t) = \sum_n I_n h(t - nT) + z(t), \text{ where } h(t) = \int g(\tau)c(t - \tau)d\tau$$

$z(t)$ is AWGN and $h(t)$ is the received pulse.

matched filter output $r(t) * h(t)$:

$$y(t) = \sum_n I_n x(t - nT) + v(t)$$

Sampling and discrete time equivalent:

$$y(kT + \tau_0) = \sum_n I_n x(kT - nT + \tau_0) + v(kT + \tau_0)$$

$$\therefore \boxed{y_k = \sum_n I_n x_{n-k} + v_k}$$

Set $x_0 = 1$ for convenience:

$$y_k = I_k + \underbrace{\sum_{n \neq k} I_n x_{n-k}}_{\text{ISI}} + v_k$$

We may be able to design signals so that there is no ISI or design signals to "control" ISI. Note that $x(t) = r(t) * h(t) = r(t) * g(t) * c(t)$.

The Nyquist Criterion for no ISI

Claim: The necessary and sufficient condition for $x(t)$ to satisfy

$$x_n = x(nT) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

is that its Fourier transform $X(f)$ satisfy

$$B(f) \triangleq \boxed{\sum_m X(f + m/T) = T}$$

Proof: $x(t)$ is the inverse Fourier transform of $X(f)$:

$$x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}df \text{ and thus } x(nT) = \int_{-\infty}^{\infty} X(f)e^{j2\pi fnT}df$$

We further have

$$\begin{aligned} x(nT) &= \sum_{m=-\infty}^{\infty} \int_{(m-1)/2T}^{(m+1)/2T} X(f) e^{j2\pi f n T} df \\ &= \sum_{m=-\infty}^{\infty} \int_{-1/2T}^{1/2T} X(f + m/T) e^{j2\pi f n T} df \\ &= \int_{-1/2T}^{1/2T} \left[\sum_{m=-\infty}^{\infty} X(f + m/T) \right] e^{j2\pi f n T} df \end{aligned}$$

Since $B(f)$ is periodic with period T, we check only the Nyquist period $[-\frac{1}{2T}, \frac{1}{2T}]$.

Suppose that the channel has bandwidth W.

Case 1 $\frac{1}{2T} > W$, cannot have zero ISI.

Case 2 $\frac{1}{2T} = W$, use brick wall:

$$X(f) = \begin{cases} T, & |f| < W; \\ 0, & \text{otherwise.} \end{cases} \qquad \therefore \quad x(t) = \frac{\sin(\pi t/T)}{\pi t/T} = \text{sinc}\left(\frac{\pi t}{T}\right).$$

Case 3 $\frac{1}{2T} < W$, many solutions.

Pulse Shaping for no ISI

The raised cosine pulse with the *roll-off factor* β:

$$X_{rc}(f) = \begin{cases} T, & 0 \le |f| \le \frac{1-\beta}{2T} \\ \frac{T}{2}\left[1 + \cos\left(\frac{\pi T}{\beta}(|f| - \frac{1-\beta}{2T})\right)\right], & \frac{1-\beta}{2T} \le |f| \le \frac{1+\beta}{2T} \\ 0, & |f| > \frac{1+\beta}{2T} \end{cases} \quad \therefore x_{rc}(t) = \text{sinc}\left(\frac{\pi t}{T}\right) \frac{\cos\left(\frac{\pi \beta t}{T}\right)}{1 - 4\beta^2 t^2/T^2}$$

Draw $x_{rc}(t)$ and $X_{rc}(f)$.

Controlled ISI - Partial Response

$$\sum_{n=0}^{K} \gamma_n D^n$$

duobinary $1 + D$

dicode $1 - D$

modified duobinary $1 - D^2$

Class 4 $(1 - D)(1 + D)^N$

Precoding for the duobinary channel:

Data sequence $\{D_n\}$

Precoded sequence $P_n = D_m + P_{n-1} \mod 2$

Transmitted sequence $I_n = 2P_n - 1$

Received sequence $B_n = I_n + I_{n-1}$

Decoded sequence: $D_n = \frac{1}{2}B_n + 1 \mod 2$

Example: Table 9.2-1