Lecture 7, E6712

Modulation Codes for Spectrum Shaping

Two Simple Rate $1/2$ Codes that Facilitate Timing Recovery

- original data

  $$10001101$$

  requires a piece of track of length $8 \times \Delta$

- FM coded sequence

  $$\underline{1}1\underline{1}0\underline{1}0\underline{1}0\underline{1}1\underline{1}1\underline{1}0\underline{1}1$$

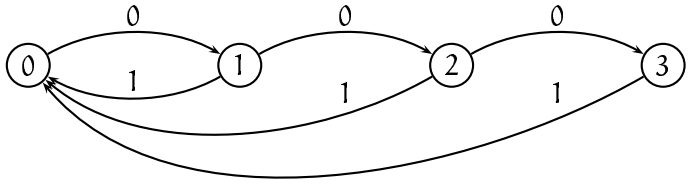
  requires a piece of track of length $2 \times 8 \times \Delta$

- MFM (or Miller) coded sequence

  $$\underline{0}1\underline{0}0\underline{1}0\underline{1}0\underline{0}1\underline{0}1\underline{0}0\underline{0}1$$

  requires a piece of track of length $2 \times 8 \times \Delta/2$

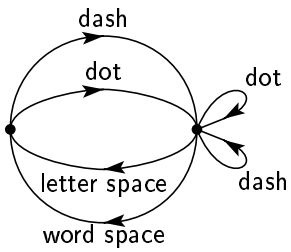
- $(1,3)$ run-length limited constraint



- (d,k) run-length limited constraint: between two 1's, there are at least d and at most k 0's.

Constrained Codes

Shannon's representation of the constraints on telegraph symbols:



Finite state transition diagram (FSTD).

Shannon capacity (topological entropy) for FSTD F:

$$C(F) = \lim_{n!1} \frac{\log_2(N(n;(F)))}{n} = \log_2 \lambda(A(F)),$$

where $A(F)$ is the constraint adjacency matrix, and $\lambda(A(F))$ is its maximum eigenvalue.

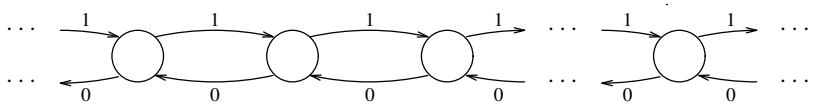
Code rate:

$$p/q \leq C(F).$$

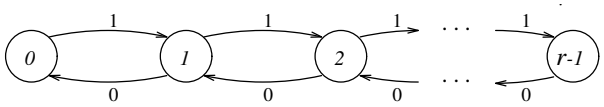
For the $(1,3)$ constraint:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad \lambda = 1.4656 \quad C(1,3) = 0.5525$$

Canonical diagram for sequences with spectral null at 0:



Finite case:

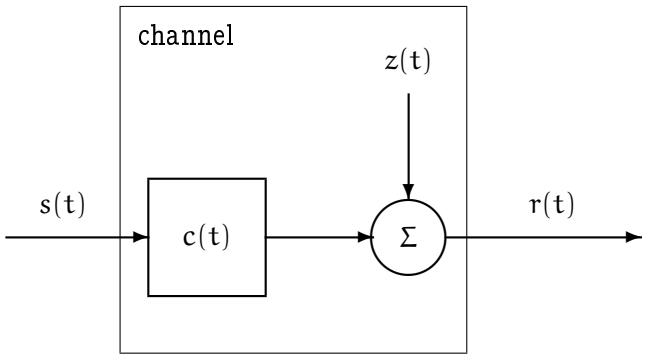


$$C(F_r) = \log_2 \left[2 \cos \frac{\pi}{r+1} \right].$$

Encoder/Decoder design: state splitting, sliding block decoders.

Channels with ISI and AWGN

Continuous-time channel model:



Sequence of symbols $\{I_n\}$ is transmitted or stored as a continuous time signal

$$s(t) = \sum_n I_n g(t - nT),$$

The received signal:

$$r(t) = \sum_n I_n h(t - nT) + z(t), \qquad z(t) \sim (0, \mathcal{N}_0/2).$$

The sampled MF output:

$$\begin{aligned} y_k &= \int_{-\infty}^{\infty} h(t - kT) r(t) dt \\ &= \sum_n I_n x_{k-n} + \zeta_k \\ &= (I * x)_k + \zeta_k, \end{aligned}$$

where

$$x_k = \int_{-\infty}^{\infty} h(t) h^*(t - kT) dt$$

and

$$\zeta_k = \int_{-\infty}^{\infty} z(t) h^*(t - kT) dt$$

is discrete-time noise with the autocorrelation of ζ_n and ζ_m given by $x_{n-m}\mathcal{N}_0/2$.

Note that $x^*_{-k} = \int_{-\infty}^{\infty} h(t)^* h(t + kT) dt = x_k$. Thus, $X(z) = X^*(1/z^*)$.

Therefore $X(z)$ can be factored as

$$\{x_n\} \overset{\mathcal{Z}}{\longleftrightarrow} X(z) = F(z) F^*(z^{-1}).$$

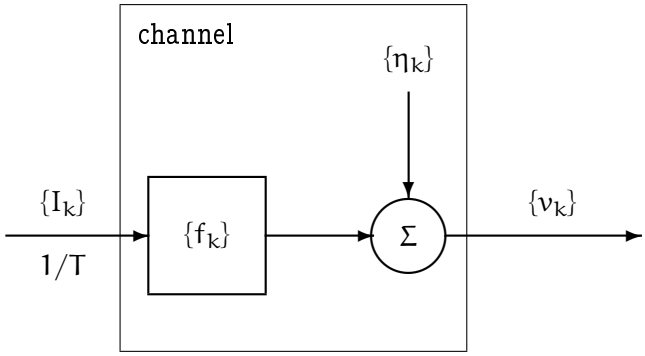
We will assume that $F(z)$ has unit norm. Choice of $F(z)$ is not unique. We will choose the *minimum-phase* $F(z)$.

After sequence $\{y_n\}$ passes through the *noise-whitening* filter with response $1/F^*(z^{-1})$, we get

$$v_k = \sum_n I_n f_{k-n} + \eta_k,$$

where $\{\eta_k\}$ is AWGN.

The equivalent discrete-time channel model:



The Optimal Viterbi Detection

Maximum likelihood sequence detection:

$$\max_{\{I_k\}\in\mathcal{C}} p(\{\nu_k\}|\{I_k\})$$

The log-likelihood function:

$$\Omega(\{I_k\}) = \sum_n [\nu_k - \sum_n I_n f_{k-n}]^2$$

Sequences $\{I_k\}$ are represented as paths through a trellis

We will use the $(1-D)$ channel as an example to explain principles, implementation issues, and performance of the Viterbi detection.