Adapting a Downlink Array from Uplink Measurements

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Abstract

In a frequency division duplex system, a transmitter adaptive antenna array can potentially improve the performance of a wireless downlink. But because the uplink and downlink channels have different wavelengths, and therefore different responses, direct downlink adaptation based on channel estimates of the uplink is generally not feasible. Instead, there has been some recent interest in adaptations that require only the second-order statistics of the uplink and downlink to be similar. These algorithms derive adaptive weights from the covariance of the received signal to apply to a downlink transmitter array. We make two contributions to this area. First, we introduce an array configuration employing $M + 1$ elements with log-periodic spacing, that comprises two overlapping subarrays, each with M elements, that are scaled versions of each other, with the scale factor equal to the ratio of the uplink wavelength to the downlink wavelength. This array has identical beampatterns at the two wavelengths, thus helping to fulfill the requirement that the uplink and downlink second-order statistics be the same. Second, we demonstrate that obtaining a good estimate of the uplink covariance matrix is not essential for the successful operation of the adaptive scheme: even when the mobile is at rest and the uplink information comprises only a single snapshot from the receiver array, an adaptive scheme can improve the signal-to-noise ratio.

Index Terms—Adaptive antenna array, log-periodic array, TDMA

1 Introduction

Transmitter-adaptive antenna arrays are inherently more difficult to utilize than receiver-adaptive antenna arrays. For a receiver array, training or pilot signals, along with the internal structure of the message-bearing signals, allow the receiver to boost its signal-to-noise and signal-to-interference ratios. For a transmitter array, one usually needs either *a priori* or auxiliary information concerning the propagation of signals from the array to the desired (in-cell) and undesired (out-of-cell) receivers.

In the case of IS-54/136 operating in flat-fading, considerable progress has been made on the use of receiver arrays for the uplink [4, 5]. Here the 14-symbol sync-sequence contained in each 162-symbol uplink slot is used as training to estimate the propagation coefficients associated with the desired mobile, and to estimate the covariance matrix for the interference. Both classical beamforming improvements as well as diversity gains have been demonstrated with this approach.

Comparatively little progress has been made for the downlink, where it is generally considered impractical to incorporate a receiver array within the mobile handset. If, instead, a transmitter array at the base station is used, how are the adaptive weights to be chosen? Conceivably one could send training signals during a portion of the downlink slot, and the mobile could estimate the downlink propagation coefficients and transmit them to the base station during the next uplink slot. However such a scheme would require considerable changes in standards, and would probably be infeasible in fast fading, where the propagation coefficients change quickly.

Recently some algorithms were proposed for adapting the downlink array [6, 7, 1], that rely on the hypothesis that, under certain conditions, the *spatial covariance matrices* associated with the uplink and the downlink propagation coefficients are nearly equal. It is tacitly assumed that the uplink and the downlink propagation coefficients are statistically independent because they operate at different frequencies, so no attempt is made to infer the downlink propagation coefficients themselves. Rather, one estimates spatial covariances associated with the uplink signals and then derives the weights from the uplink covariance estimates for use on the downlink transmitter array. The ostensible purpose is to increase the power that is transmitted to the desired mobile while minimizing the power that is transmitted to the undesired mobiles that occupy the same time/frequency slot.

The assumption that the spatial covariances of the propagation coefficients associated with the uplink and the downlink arrays are nearly equal is critical and must survive the change in wavelength from uplink to downlink. However the spatial structure of a plane wave front, for example, changes with wavelength.

To solve this problem, it is proposed in [6] (the *matched array*) to use two different arrays for the uplink and the downlink that are in close proximity and that are scaled versions of each other in proportion to the ratio of the two wavelengths. This approach suffers from the drawbacks that it is expensive, may provoke unwanted mutual coupling if the arrays are close together, or may be mismatched for near-field radiation if the arrays are far apart. Another approach suggested in [6] (the *duplex array*) uses a single array for both the uplink and the downlink, with the array artfully designed to have nearly the same response at the two wavelengths, but no details are given. A particular circular duplex array is described in [1] that permits the uplink and downlink covariances to be related mathematically.

Another major issue for adapting the downlink is estimating the spatial covariance matrix. In general, if the array has M elements, then a minimum of M statistically independent realizations (*snapshots*) of the array response are required to obtain a meaningful estimate for the $M \times M$ covariance matrix. Yet it is apparent that this condition is difficult to realize. For example, with a carrier frequency of 900 MHz, a symbol interval of 41 μs, and a vehicle speed of 25 m/s (approximately 55 miles-per-hour), the mobile travels less than one-half wavelength during a standard 162 symbol slot, which provides typically only one or two independent realizations of the array response to the propagation environment. To facilitate a more accurate estimate of the spatial covariance matrix when the mobile is nearly stationary, frequency-hopping is advocated in [6]. However, frequency hopping requires drastic changes to the existing standards.

Our research addresses the antenna array design and the covariance estimation problems. We propose a design, either linear or planar, in which the spacings between elements are log-periodic; the ratio between successive inter-element spacings is constant and equal to the ratio of the uplink and downlink wavelengths. In a linear array, for example, there are a total of $M+1$ elements, the first M of which are used on the uplink, and the last M of which are used on the downlink. The two overlapping subarrays have virtually identical responses at their respective wavelengths, and they have a relative displacement of only one element spacing.

We also show, perhaps remarkably, that the up/down adaptation scheme *does not require a good covariance estimate to be successful*. In particular we prove, both analytically and by simulation, that when the mobile is stationary and essentially only one snapshot of the uplink array response is available, the adaptive scheme will typically still provide improvements in signal-to-noise ratio on the downlink.

2 Signal Model

We assume that the system being analyzed is TDMA frequency-division duplex. On the uplink, a mobile sends a signal through a single transmitter antenna to an M-element receiver antenna array located at a base station. It is possible that one or more undesired signals, at the same time and frequency, may also arrive at the base station from other cells. On the downlink, the base station sends a signal through an M-element transmitter array, weighted by a complex scalar for each antenna, to a mobile that has a single receiver antenna. Inadvertently, some of the power may also be sent to out-of-cell mobiles that have the same time and frequency allocation. The goal is to choose the weights for the downlink transmitter array simultaneously to maximize the power that is sent to the desired mobile while minimizing the power that is sent to the undesired mobiles.

2.1 Uplink model

For an M-element receiver array, the baseband signal model is

$$\underline{x}_t^{\mathrm{u}} = \sqrt{\rho_0} s_{0t}^{\mathrm{u}} \underline{h}_{0t}^{\mathrm{u}} + \sqrt{\rho_1} s_{1t}^{\mathrm{u}} \underline{h}_{1t}^{\mathrm{u}} + \underline{\nu}_t^{\mathrm{u}}, \qquad t = 1, \cdots, T, \tag{1}$$

where t denotes symbol index, $\underline{x}_t^{\mathrm{u}}$ is the complex M-component received signal vector, and $\underline{\nu}_t^{\mathrm{u}}$ is additive receiver noise. The first term in (1), subscripted with 0, represents the signal arriving from the desired mobile (in-cell), while the second term, subscripted with 1, represents the signal arriving from an undesired mobile (out-of-cell). The modulated signals (typically QPSK) are complex scalars denoted by s_{0t}^{u} and s_{1t}^{u}. The complex time-varying propagation coefficients between the mobile transmitter antennas and the base station receiver array are denoted by the M-component vectors $\underline{h}_{0t}^{\mathrm{u}}$ and $\underline{h}_{1t}^{\mathrm{u}}$. The slot length is denoted by T (which is 162 symbols in IS-54/136). We shall model the receiver noise as zero-mean uncorrelated white complex Gaussian. We model the propagation coefficients as zero-mean complex Gaussian random vectors, the desired mobile's being independent of the undesired mobile's.

We suppose that the expected power of the modulated signals, the variance of the additive noise, and the average variance of the fading coefficients are all one; that is,

$$\mathrm{E}\{|s_{jt}^{\mathrm{u}}|^2\} = 1, \quad \mathrm{E}\{\underline{\nu}_t^{\mathrm{u}} \underline{\nu}_t^{\mathrm{u}\dagger}\} = I_M, \quad \frac{1}{M}\mathrm{E}\{\|\underline{h}_{jt}^{\mathrm{u}}\|^2\} = 1, \tag{2}$$

where the superscript † denotes conjugate-transpose, and

$$\|\underline{h}_{jt}^{\mathrm{u}}\|^2 \stackrel{\mathrm{def}}{=} \underline{h}_{jt}^{\mathrm{u}\dagger}\underline{h}_{jt}^{\mathrm{u}}.$$

Thus, ρ_0 and ρ_1 represent the expected signal-to-noise ratio (SNR), averaged over the receiver antennas, for the desired and the undesired mobile, respectively. We distinguish between the expected SNR, which involves a ratio of the average fading over the average additive noise, and the (instantaneous) SNR, which involves a ratio of the instantaneous fading over the average noise.

The problem of processing the uplink array data to maximize the desired signal-to-interference-plus-noise ratio has been tackled with considerable success [4, 5]. This type of adaptive algorithm produces a decoded message from which the modulated signal s_{0t}^{u} may be reconstructed, as well as an estimate for the propagation coefficients for the desired mobile, $\underline{h}_{0t}^{\mathrm{u}}$.

2.2 Downlink model

On the downlink, the M transmitter antenna elements are fed with a modulated signal s_{0t}^{d} that is weighted at the mth element by w_m^*. Different scalar signals are received by the desired mobile and possibly by mobiles in other cells that have the same time/frequency allocation,

$$x_{jt}^{\mathrm{d}} = \sqrt{\rho_j}\, s_{0t}^{\mathrm{d}}\underline{w}^\dagger\underline{h}_{jt}^{\mathrm{d}} + \nu_{jt}^{\mathrm{d}}, \qquad j = 0, 1, \;\; t = 1, \cdots, T. \tag{3}$$

Again, the subscript 0 denotes the desired mobile while the subscript 1 denotes the undesired mobile, the receiver noises are zero-mean uncorrelated white complex Gaussian, and the propagation coefficients for the different mobiles are statistically independent. The downlink expected SNR's are assumed to be equal to their uplink counterparts, and the downlink quantities are normalized in the same way as the uplink quantities,

$$\mathrm{E}\{|s_{0t}^{\mathrm{d}}|^2\} = 1, \;\; \mathrm{E}\{|\nu_{jt}^{\mathrm{d}}|^2\} = 1, \;\; \frac{1}{M}\mathrm{E}\{\|\underline{h}_{jt}^{\mathrm{d}}\|^2\} = 1. \tag{4}$$

The weight vector has unit energy,

$$\|\underline{w}\| = 1. \tag{5}$$

We say that the transmitter array use is *nonadaptive* if one antenna is picked at random (equally likely from among the M antennas) and fed all of the signal energy. Under the normalization (4), a nonadaptive use

results on average in a downlink expected SNR of ρ_j at the mobiles. To see this, let $\underline{w} = \underline{e}_m$ be the weight vector whose mth component is one and whose other components are zero. Then the expected nonadaptive SNR at the jth mobile is

$$\begin{aligned}
\frac{1}{M}\sum_{m=1}^{M}\frac{\mathrm{E}\left\{\rho_j|s_{0t}^{\mathrm{d}}|^2|\underline{e}_m^{\dagger}\underline{h}_{jt}^{\mathrm{d}}|^2\right\}}{\mathrm{E}\left\{|\nu_{jt}^{\mathrm{d}}|^2\right\}} &= \frac{1}{M}\sum_{m=1}^{M}\rho_j\underline{e}_m^{\dagger}\mathrm{E}\left\{\underline{h}_{jt}^{\mathrm{d}}\underline{h}_{jt}^{\mathrm{d}\dagger}\right\}\underline{e}_m \\
&= \frac{\rho_j}{M}\mathrm{tr}\left(\mathrm{E}\left\{\underline{h}_{jt}^{\mathrm{d}}\underline{h}_{jt}^{\mathrm{d}\dagger}\right\}\sum_{m=1}^{M}\underline{e}_m\underline{e}_m^{\dagger}\right) \\
&= \frac{\rho_j}{M}\mathrm{tr}\left(\mathrm{E}\left\{\underline{h}_{jt}^{\mathrm{d}}\underline{h}_{jt}^{\mathrm{d}\dagger}\right\}\right) \\
&= \frac{\rho_j}{M}\mathrm{E}\left\{\|\underline{h}_{jt}^{\mathrm{d}}\|^2\right\} \\
&= \rho_j.
\end{aligned}$$

Similarly, it can be shown that if the weight vector is chosen randomly (with $\|\underline{w}\| = 1$) then the expected SNR at the jth mobile is ρ_j.

2.3 Covariance model

We model the propagation coefficients $\{\underline{h}_{jt}^{\mathrm{u}},\ \underline{h}_{jt}^{\mathrm{d}},\ \ j = 0, 1\}$ as zero mean, stationary, complex Gaussian vector-valued random processes, with $M \times M$ covariance matrices

$$\mathrm{E}\{\underline{h}_{jt}^{\mathrm{u}}\underline{h}_{jt}^{\mathrm{u}\dagger}\} = K_j^{\mathrm{u}}, \quad \mathrm{E}\{\underline{h}_{jt}^{\mathrm{d}}\underline{h}_{jt}^{\mathrm{d}\dagger}\} = K_j^{\mathrm{d}}, \qquad j = 0, 1. \tag{6}$$

The normalizations (2) and (4) imply that

$$\mathrm{tr}\, K_j^{\mathrm{u}} = \mathrm{tr}\, K_j^{\mathrm{d}} = M, \qquad j = 0, 1. \tag{7}$$

We do not make any assumptions here about the K matrices. In general, the matrices tend to have low rank in a sparse scattering environment with closely-spaced antenna array elements, and tend to have full rank in a rich scattering environment with widely-spaced elements. We do not concern ourselves with the temporal correlation structure of the propagation coefficients.

Signals propagating to and from a mobile in another cell generally propagate through a different set of paths than signals to and from the desired mobile, making it reasonable to assume that $\underline{h}_{0t}^{\mathrm{u}}$ and $\underline{h}_{1t}^{\mathrm{u}}$ are statistically independent, and that $\underline{h}_{0t}^{\mathrm{d}}$ and $\underline{h}_{1t}^{\mathrm{d}}$ are statistically independent.

Given the different carrier frequencies used on the uplink and downlink, we may assume that the downlink propagation coefficients are statistically independent of the uplink propagation coefficients. Consequently, the adaptive algorithm should make no attempt to infer the actual values of the downlink propagation coefficients from the uplink measurements.

The crucial assumption for the adaptive array algorithm is that, for either mobile, the covariance matrix for the downlink propagation coefficients is equal to the covariance matrix for the uplink propagation coefficients, despite the different carrier frequencies employed,

$$K_0^{\mathrm{d}} = K_0^{\mathrm{u}} \stackrel{\mathrm{def}}{=} K_0, \qquad K_1^{\mathrm{d}} = K_1^{\mathrm{u}} \stackrel{\mathrm{def}}{=} K_1. \tag{8}$$

Although this assumption must ultimately be validated experimentally, there is some physical justification for believing that it might be true [6]. Specifically, suppose that there are L_0 distinct paths, created for example by L_0 scattering centers between the desired mobile and the base station array. Then, on the uplink, the signal arriving at the receiver array is equal to a sum of L_0 signals, and on the downlink the signal arriving at the mobile is a sum of L_0 signals. A similar argument applies to the undesired mobile (with L_0 replaced by L_1). Hence, we have

$$\underline{h}_{jt}^{\mathrm{u}} = A_j^{\mathrm{u}} \underline{\alpha}_{jt}^{\mathrm{u}}, \quad \underline{h}_{jt}^{\mathrm{d}} = A_j^{\mathrm{d}} \underline{\alpha}_{jt}^{\mathrm{d}}, \qquad j = 0, 1, \tag{9}$$

where A_j^{u} and A_j^{d} are $M \times L_j$ deterministic matrices, and $\underline{\alpha}_{jt}^{\mathrm{u}}$ and $\underline{\alpha}_{jt}^{\mathrm{d}}$ are L_j-dimensional random processes. The A-matrices embody the scatterer geometry while the $\underline{\alpha}$-vectors represent the complex scattering amplitudes. The ℓth column of A_0^{u} is, for example, the steering vector for the ℓth scattering center from the desired mobile to the base, $\ell = 1, \ldots, L_0$. For example, if the energy from the ℓth scattering center arrives at a linear array as a plane wave then $[A_0^{\mathrm{u}}]_{ml} = \exp\{i 2\pi u_\ell z_m / \lambda^{\mathrm{u}}\}$ where u_ℓ is the sine of the angle-of-arrival, λ^{u} is the uplink carrier wavelength, and z_m is the relative position of the mth array element. It is reasonable to assume that, if the effects due to the different carrier frequencies can be compensated for in the design of the transmitter and receiver arrays (see, for example, Section 3), then

$$A_j^{\mathrm{u}} = A_j^{\mathrm{d}} \stackrel{\mathrm{def}}{=} A_j, \qquad j = 0, 1. \tag{10}$$

Likewise, if each scattering center comprises a number of closely spaced subscatterers, then the L_j components of each $\underline{\alpha}_{jt}^{\mathrm{u,d}}$-vector should be independent of one another and be zero-mean complex Gaussian.

Furthermore, the reflectivity of each subscatterer should undergo a random phase shift between the uplink and the downlink, so that $\underline{\alpha}_{jt}^{\mathrm{u}}$ and $\underline{\alpha}_{jt}^{\mathrm{d}}$ should be statistically independent. On average, however, the power reflected from each of the L_j scatterers should be the same on the uplink and downlink, or $\mathrm{E}\left\{|[\underline{\alpha}_{jt}^{\mathrm{u}}]_\ell|^2\right\} = \mathrm{E}\left\{|[\underline{\alpha}_{jt}^{\mathrm{d}}]_\ell|^2\right\}$ for $\ell = 1, \ldots, L_j$. These properties, combined with (10), imply that the uplink and downlink covariance matrices are equal,

$$K_j^{\mathrm{u}} = A_j^{\mathrm{u}} \mathrm{E}\left\{\underline{\alpha}_{jt}^{\mathrm{u}} \underline{\alpha}_{jt}^{\mathrm{u}\dagger}\right\} A_j^{\mathrm{u}\dagger} = A_j^{\mathrm{d}} \mathrm{E}\left\{\underline{\alpha}_{jt}^{\mathrm{d}} \underline{\alpha}_{jt}^{\mathrm{d}\dagger}\right\} A_j^{\mathrm{d}\dagger} = K_j^{\mathrm{d}} \stackrel{\mathrm{def}}{=} K_j, \qquad j = 0, 1.$$

3 Log-Periodic Array

This section describes the geometry of a simple antenna array that has identical beampatterns at the uplink and downlink carrier frequencies. The array is slightly more complicated than an equivalent array designed for only one frequency, but it avoids the cumbersome and expensive alternative of designing two distinct possibly interfering or possibly non-colocated arrays for the two different frequencies. With the array, we can satisfy the requirement (10).

We rely on two principles to design the array. The first is that the antenna array response to an incoming plane wave from a particular direction is determined completely by the spacing between its elements (in one, two or three dimensions). The second is that an array operating at carrier wavelength λ^{d} has identical transmitter and receiver plane wave responses to another colocated array operating at wavelength λ^{u} that is configured identically except that its element spacing is scaled by the factor $\lambda^{\mathrm{u}}/\lambda^{\mathrm{d}}$.

The design can easily be described with the help of an example. Figure 1 displays the arrangement of a linear log-periodic array with nine elements. We assume that $\lambda^{\mathrm{u}} > \lambda^{\mathrm{d}}$. Let the spacing between the mth and $(m+1)$th elements be d_m, such that $d_m = (\lambda^{\mathrm{u}}/\lambda^{\mathrm{d}}) d_{m-1}$ for $m = 2, \ldots, 8$, with, for example, $d_1 = \lambda^{\mathrm{d}}/2$. We use the eight-element sub-array 1–8 on the downlink (operating at the shorter wavelength λ^{d}) and the eight-element subarray 2–9 on the uplink (operating at the longer wavelength λ^{u}). The name *log-periodic* refers to the fact that $\log(d_m/d_{m-1})$ is the same for all m, and is taken from a single wideband antenna (not a phased array) of the same name that has elements whose lengths and spacings follow a similar rule [2]. We see that the eight-element uplink array is identical to the eight-element downlink array except that it is scaled by $\lambda^{\mathrm{u}}/\lambda^{\mathrm{d}}$ and displaced by one element spacing.

We therefore conclude that the uplink array and downlink array beampatterns are identical, and the array responses to signals arriving from the same direction are approximately identical, the approximation needed

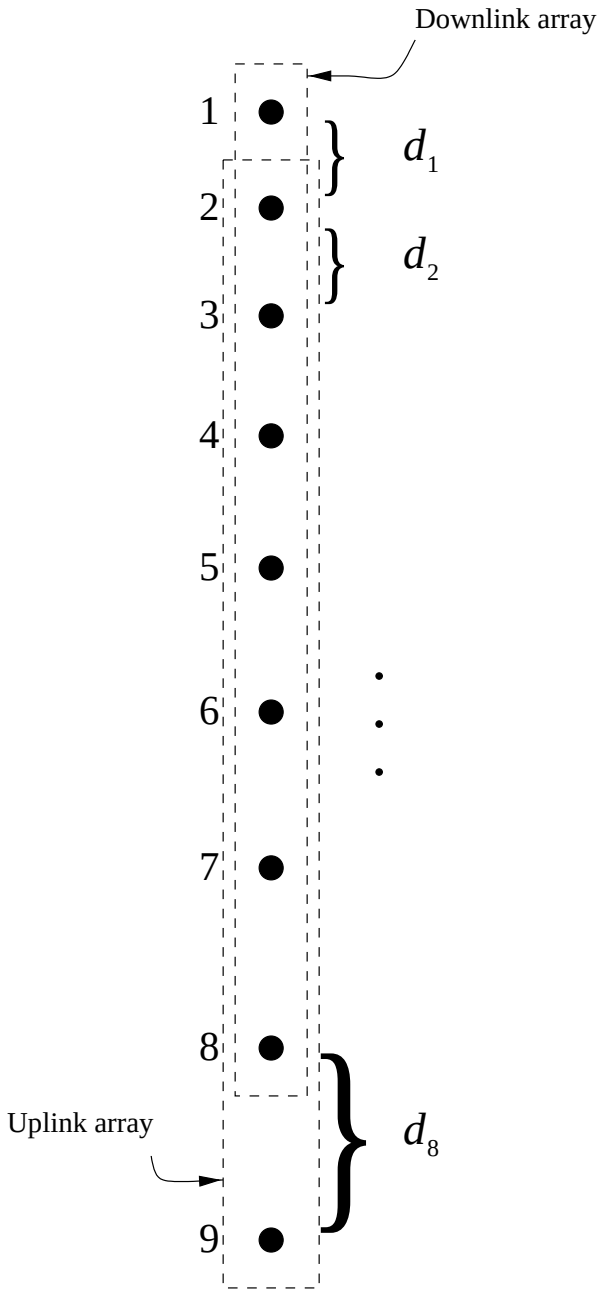


Figure 1: Linear log-periodic array with nine elements, where $d_1 = \lambda^{\mathrm{d}}/2$ and $d_m/d_{m-1} = \lambda^{\mathrm{u}}/\lambda^{\mathrm{d}}$ for $m = 2, \ldots, 8$. Elements 1–8 are used on the downlink and elements 2–9 are used on the uplink. It is assumed that $\lambda^{\mathrm{u}} > \lambda^{\mathrm{d}}$.

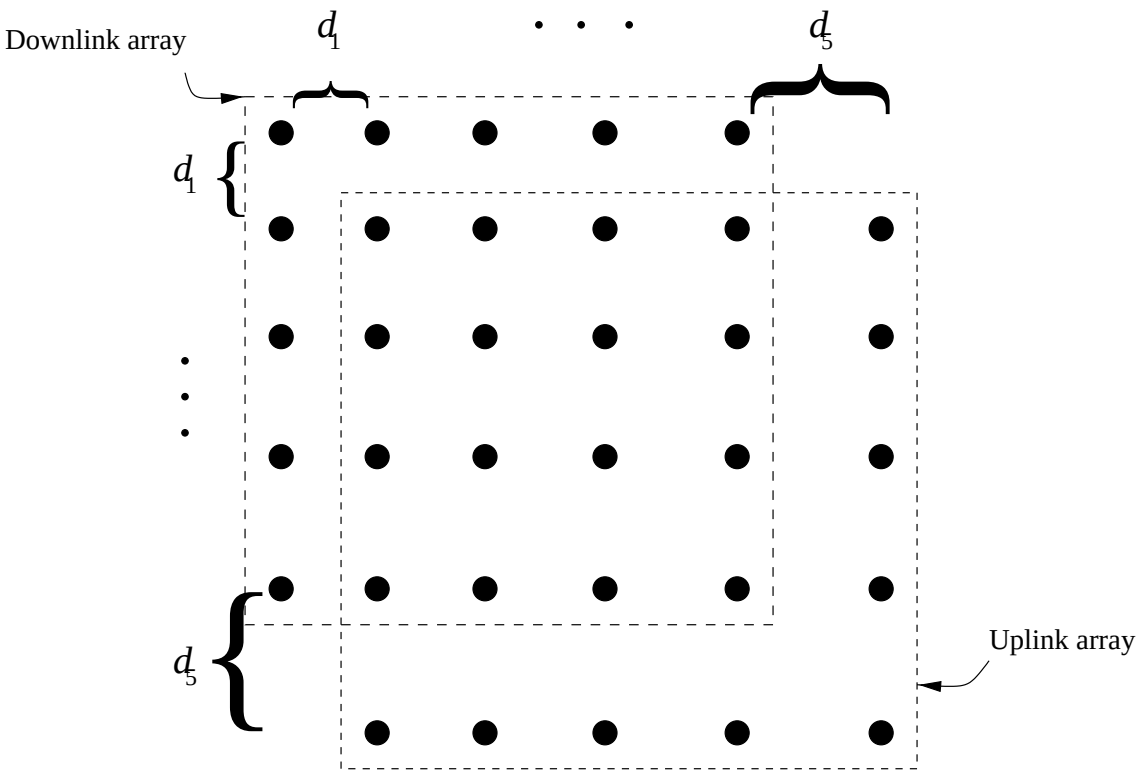


Figure 2: Two-dimensional log-periodic array with thirty-four elements, where $d_1 = \lambda^{\mathrm{d}}/2$ and $d_m/d_{m-1} = \lambda^{\mathrm{u}}/\lambda^{\mathrm{d}}$ for $m = 2, \ldots, 5$. The upper left group of sixteen elements (within the dashed square) are used on the downlink and lower right group on the uplink. It is assumed that $\lambda^{\mathrm{u}} > \lambda^{\mathrm{d}}$.

because the two arrays are not exactly colocated. Some mismatch may also occur for near field arrivals, but this mismatch should be very small. Observe that the log-periodic array requires only one more element than a linear array designed to operate at only one frequency. The choice of the spacing between the first two elements $d_1 = \lambda^{\mathrm{d}}/2$ can be changed to suit designer specifications on beamwidths and grating lobes.

Figure 2 displays a two-dimensional log-periodic array with a total of thirty-four elements, of which the upper-left group of twenty-five is used for the downlink and the lower-right group of twenty-five on the uplink. We see from the figure that the uplink array is exactly a scaled version of the downlink array, and therefore the two arrays have identical beampatterns at their respective frequencies.

This design technique of forming two overlapping arrays may also be extended to three dimensions. We omit a detailed description.

4 Adaptive Weight Selection

This section describes four distinct algorithms for choosing the adaptive weight vector $\underline{w}$ from the uplink measurements. The algorithms make different assumptions about the signal structure, and they have different objectives. Algorithms I and II seek to maximize the expected SNR at the desired mobile, irrespective of the SNR at the undesired mobile. Algorithms III and IV attempt to control both SNR's.

4.1 Algorithm I: Maximize SNR at desired mobile, assuming no undesired mobile present

This algorithm assumes that there are no undesired mobiles, so within (1) we have $\rho_1 = 0$. The measurements are used to obtain a covariance matrix estimate directly, without performing any demodulation,

$$\hat{K}_0 = \frac{1}{T} \sum_{t=1}^{T} \underline{x}_t^{\mathrm{u}} \underline{x}_t^{\mathrm{u}\dagger}. \tag{11}$$

The mean of this estimate is

$$\mathrm{E}\{\hat{K}_0\} = \rho_0 K_0 + I,$$

where I is the $M \times M$ identity matrix that is associated with the additive receiver noise. Although this estimate is biased, it does not require the receiver to know ρ_0, and it may be used in place of an unbiased estimate of K_0 because the eigenvectors of $\rho_0 K_0 + I$ are the same as the eigenvectors of K_0. The weight vector is chosen to be the eigenvector of $\hat{K}_0$ having the largest eigenvalue,

$$\hat{K}_0 \underline{w} = \lambda_{\max} \underline{w}, \tag{12}$$

with the implicit normalization $\|\underline{w}\|^2 = 1$. Finding the eigenvector with the largest eigenvalue is equivalent to maximizing a ratio of quadratic forms

$$\underline{w} = \arg \max_{\underline{y}} \frac{\underline{y}^\dagger \hat{K}_0 \underline{y}}{\underline{y}^\dagger \underline{y}}. \tag{13}$$

As shown later (see Section 5), $\underline{w}^\dagger K_0 \underline{w}$ is the conditional expected SNR at the desired mobile.

The principal drawback of this algorithm is that, in the presence of an interferer, the covariance estimate may be badly biased such that power intended for the desired mobile instead is sent uselessly and to the detriment of the interferer.

4.2 Algorithm II: Maximize SNR at desired mobile, assuming undesired mobile present

This algorithm accounts for the possible presence of one or more interfering mobiles, but still its criterion is to maximize the expected SNR at the desired mobile. The algorithm uses well-known adaptive receiver array techniques on the uplink [4, 5] to obtain estimates for the message and the fading associated with the desired mobile, $\hat{s}^{\mathrm{u}}_{0t}$, and $\widehat{\rho_0^{1/2}\underline{h}^{\mathrm{u}}_{0t}}$. The covariance estimate is obtained from the estimated fading process,

$$\hat{K}_0 = \frac{1}{T}\sum_{t=1}^{T}\left(\widehat{\rho_0^{1/2}\underline{h}^{\mathrm{u}}_{0t}}\right)\left(\widehat{\rho_0^{1/2}\underline{h}^{\mathrm{u}}_{0t}}\right)^{\dagger}. \tag{14}$$

The weight vector is again obtained by solving the eigenvector-eigenvalue problem (12), (13).

4.3 Algorithm III: Maximize ratio of SNR at desired mobile to SNR at undesired mobile

The goal of algorithms I and II is to maximize the expected SNR at the desired mobile, and there is some risk that excess power may be transmitted to one or more undesired mobiles in other cells. It is mutually beneficial to the various cells if such cross-cell transmissions are reduced. Algorithm III begins like Algorithm II by estimating the message and the fading associated with the desired mobile, and then forming the covariance matrix estimate $\hat{K}_0$ in (14). Then a second covariance matrix estimate $\hat{K}_1$ is obtained from the difference of the measured signal and the estimated signal for the desired mobile,

$$\hat{K}_1 = \frac{1}{T}\sum_{t=1}^{T}\left(\underline{x}^{\mathrm{u}}_t - \widehat{\rho_0^{1/2}\underline{h}^{\mathrm{u}}_{0t}}\right)\left(\underline{x}^{\mathrm{u}}_t - \widehat{\rho_0^{1/2}\underline{h}^{\mathrm{u}}_{0t}}\right)^{\dagger}. \tag{15}$$

The residuals that result from the subtraction contain both receiver noise and interferers, but no assumption is made regarding the number of interferers or their intensities. The weight vector is obtained by maximizing a ratio of quadratic forms,

$$\underline{w} = \arg\max_{\underline{y}} \frac{\underline{y}^{\dagger}\hat{K}_0\underline{y}}{\underline{y}^{\dagger}\hat{K}_1\underline{y}}, \tag{16}$$

which is equivalent to solving a generalized eigenvector-eigenvalue problem,

$$\hat{K}_0\underline{w} = \lambda_{\max}\hat{K}_1\underline{w}. \tag{17}$$

If the covariance estimates were equal to the actual covariances, this procedure could be interpreted as maximizing the ratio of the expected SNR at the desired mobile to the expected SNR at the undesired

mobile.

4.4 Algorithm IV: Maintain specified SNR at desired mobile while minimizing SNR at undesired mobile

Algorithm III tends to maximize the signal-to-interference ratio. In some situations however, this criterion may lead to inadequate power being transmitted to the desired mobile. Algorithm IV is a generalization of both Algorithms II and III which gives the user more direct control over this power. It differs from Algorithm III only with respect to how it obtains the weights from $\hat{K}_0$ and $\hat{K}_1$.

The weight vector is the solution to the following constrained optimization problem, where one seeks to minimize the power transmitted to the interferer(s) subject to the normalization constraint on the weights (in this case an inequality for greater generality), and subject to the constraint that the expected SNR at the desired mobile be greater than or equal to a specified value $\rho_{0\mathrm{d}}$,

$$\underline{w} = \arg\min_{\underline{y}} \underline{y}^\dagger \hat{K}_1 \underline{y}, \quad \text{subject to} \quad \underline{y}^\dagger \underline{y} \leq 1 \quad \text{and} \quad \underline{y}^\dagger \hat{K}_0 \underline{y} \geq \rho_{0\mathrm{d}}. \tag{18}$$

A weight vector that satisfies the two constraints exists if and only if $\rho_{0\mathrm{d}} \leq \lambda_{\max}(\hat{K}_0)$. Usually the two constraints in (18) will be satisfied with equality. To formulate the solution to this optimization problem we use the method of Lagrange multipliers and the Kuhn-Tucker conditions to rewrite (18) as

$$\underline{w} = \min_{\underline{y}} \left[\underline{y}^\dagger \hat{K}_1 \underline{y} - \mu_0 \left(\underline{y}^\dagger \underline{y} + r_0^2 - 1 \right) - \mu_1 \left(\underline{y}^\dagger \hat{K}_0 \underline{y} - r_1^2 - \rho_{0\mathrm{d}} \right) \right], \tag{19}$$

where μ_0 and μ_1 are Lagrange multipliers, and r_0 and r_1 are auxiliary variables whose squares are, respectively, added and subtracted to convert the constraint inequalities into equalities. If one increased the right-hand side of the first constraint in (18) (loosening the constraint that the energy of the weights be less than one), the optimized cost function would decrease. If one increased the right-hand side of the second constraint (resulting in a tighter constraint on the energy to the desired mobile) the optimized cost function would increase. Since the Lagrange multipliers are equal to the derivatives of the optimized cost with respect to the right-hand side of the constraints, we have

$$\mu_0 \leq 0 \leq \mu_1.$$

We now minimize the above cost function (19) with respect to all variables. Setting derivatives of the cost

function

$$\underline{y}^\dagger \hat{K}_1 \underline{y} - \mu_0 \left(\underline{y}^\dagger \underline{y} + r_0^2 - 1 \right) - \mu_1 \left(\underline{y}^\dagger \hat{K}_0 \underline{y} - r_1^2 - \rho_{0\mathrm{d}} \right) \tag{20}$$

with respect to μ_0 and μ_1 equal to zero yields the two equality constraints previously mentioned. The Kuhn-Tucker conditions are obtained as follows. Setting $\partial/\partial r_0$ of (20) equal to zero gives $\mu_0 r_0 = 0$, so if $\mu_0 < 0$ then $r_0 = 0$, implying that $\underline{y}^\dagger \underline{y} = 1$. Conversely if $\underline{y}^\dagger \underline{y} < 1$ then $r_0 \neq 0$ implying that $\mu_0 = 0$. Similarly we have that $\mu_1 r_1 = 0$, so if $\mu_1 > 0$ then $\underline{y}^\dagger \hat{K}_0 \underline{y} = \rho_{0\mathrm{d}}$, and if $\underline{y}^\dagger \hat{K}_0 \underline{y} > \rho_{0\mathrm{d}}$ then $\mu_1 = 0$. Finally after setting the gradient of the cost with respect to $\underline{y}$ equal to zero, and simplifying, we obtain

$$\left(\hat{K}_1 - \mu_0 I \right) \underline{w} = \mu_1 \hat{K}_0 \underline{w}, \tag{21}$$

which again is a generalized eigenvector-eigenvalue problem.

If $\mu_0 \to -\infty$ we obtain Algorithm II, and if $\mu_0 = 0$ we obtain Algorithm III. There is no direct way to solve the generalized eigenvector-eigenvalue problem (21) in conjunction with the Kuhn-Tucker conditions. One way to proceed is to solve (21) for a multiplicity of values of μ_0 and then evaluate $\underline{w}^\dagger \hat{K}_0 \underline{w}$ and $\underline{w}^\dagger \hat{K}_1 \underline{w}$ for each eigenvector.

5 Analysis of Performance

In this section, we evaluate the performances of three classes of algorithms for selecting the weights: a) maximum ratio weighting, which represents the optimum selection of weights, but which requires that the transmitter know the instantaneous values of the downlink fading coefficients, b) nonadaptive use of the transmit array, and c) the adaptive algorithms described in Section 4.

Our performance measures are the downlink SNR's at the desired and undesired mobiles, which in turn depend to a great extent on the eigenvector/eigenvalue structure of the covariance matrices,

$$K_0 = \Phi_0 \Lambda_0 \Phi_0^\dagger, \qquad K_1 = \Phi_1 \Lambda_1 \Phi_1^\dagger \tag{22}$$

where the rank of K_j (i.e., the number of nonzero eigenvalues) is $L_j \leq M$, Φ_j is an $M \times L_j$ complex matrix whose columns are orthonormal, $\Phi_j^\dagger \Phi_j = I$, and Λ_j is an $L_j \times L_j$ real diagonal matrix of positive

eigenvalues, ordered as $\lambda_{j1} \geq \cdots \geq \lambda_{jL_j} > 0$. The trace condition (7) implies that

$$\mathrm{tr}\,\{K_0\} = \sum_{m=1}^{L_0} \lambda_{0m} = \mathrm{tr}\,\{K_1\} = \sum_{m=1}^{L_1} \lambda_{1m} = M. \tag{23}$$

The eigenvector/eigenvalue decomposition allows the propagation coefficients to be represented as

$$\underline{h}_{0t}^{\mathrm{u,d}} = \Phi_0 \Lambda_0^{1/2} \underline{z}_{0t}^{\mathrm{u,d}} = \sum_{m=1}^{L_0} \sqrt{\lambda_{0m}}\, \underline{\phi}_{0m} [\underline{z}_{0t}^{\mathrm{u,d}}]_m \tag{24}$$

$$\underline{h}_{1t}^{\mathrm{u,d}} = \Phi_1 \Lambda_1^{1/2} \underline{z}_{1t}^{\mathrm{u,d}} = \sum_{m=1}^{L_1} \sqrt{\lambda_{1m}}\, \underline{\phi}_{1m} [\underline{z}_{1t}^{\mathrm{u,d}}]_m \tag{25}$$

where $\underline{\phi}_{jm}$ is the mth eigenvector of K_j (i.e., the mth column of Φ_j), λ_{jm} is the corresponding eigenvalue, and $[\underline{z}_{jt}^{\mathrm{u,d}}]_m$ are zero-mean random processes in t, uncorrelated with respect to m, with variance equal to one. This purely mathematical representation has a physical interpretation when there are L_j independent scattering centers that are perfectly resolvable by the array—in this case the columns of Φ_j are the associated steering vectors and we may use the notation of Section 2.3, with

$$\begin{aligned} A_0 &= \Phi_0, & A_1 &= \Phi_1, \\ \underline{\alpha}_{0t}^{\mathrm{u,d}} &= \Lambda_0^{1/2} \underline{z}_{0t}^{\mathrm{u,d}}, & \underline{\alpha}_{1t}^{\mathrm{u,d}} &= \Lambda_1^{1/2} \underline{z}_{1t}^{\mathrm{u,d}}. \end{aligned}$$

To simplify the analysis, we will often assume that the L_j nonzero eigenvalues are all equal. Equation (23) then implies that they must all be equal to M/L_j. When $L_j = M$, the covariance matrix is equal to the identity matrix, $K_j = I$, the M fading coefficients are independent and identically distributed and we have maximum diversity. Conversely, when $L_j = 1$ the M fading coefficients are perfectly correlated and we have minimum diversity.

For the adaptive algorithms we will generally consider two extreme cases. In one case, it is assumed that K_0 and K_1 are estimated perfectly on the uplink; this is the approximate outcome with high-speed mobiles that pass through many independent fades in one TDMA time-slot. In the second case, it is assumed that we obtain only rank-one estimates; this is consistent with quasi-stationary mobiles that do not pass through a fade. In either case, we assume that the uplink SNR's are high enough so that the effects of receiver noise may be ignored in our estimates of the covariance matrices. For the special case where the rank of the actual covariance matrix is one, a stationary mobile can obtain an essentially perfect estimate for the covariance

matrix when the SNR is high.

5.1 Maximum Ratio Weighting

If the downlink fading coefficients were known at each instant of time, the optimum weight vector would be

$$\underline{w} = \frac{\underline{h}_{0t}^{\mathrm{d}}}{\|\underline{h}_{0t}^{\mathrm{d}}\|}, \tag{26}$$

and the resulting instantaneous SNR at the desired mobile would be

$$\mathrm{SNR}_0 = \rho_0 \frac{|\underline{h}_{0t}^{\mathrm{d}\dagger}\underline{h}_{0t}^{\mathrm{d}}|^2}{\underline{h}_{0t}^{\mathrm{d}\dagger}\underline{h}_{0t}^{\mathrm{d}}} = \rho_0 \underline{h}_{0t}^{\mathrm{d}\dagger}\underline{h}_{0t}^{\mathrm{d}}. \tag{27}$$

No other selection of weights can give a higher SNR. The expected SNR is

$$\mathrm{E}\left\{\mathrm{SNR}_0\right\} = \rho_0 \mathrm{tr}\left(\mathrm{E}\left\{\underline{h}_{0t}^{\mathrm{d}}\underline{h}_{0t}^{\mathrm{d}\dagger}\right\}\right) = \rho_0 \mathrm{tr} K_0 = M\rho_0, \tag{28}$$

independently of K_0, but the probability density of the SNR depends critically on the eigenvalues of K_0. To see this we substitute the representation (24) into (27) to obtain

$$\mathrm{SNR}_0 = \rho_0 \underline{z}_{0t}^{\mathrm{d}\dagger} \Lambda_0^{1/2} \Phi_0^\dagger \Phi_0 \Lambda_0^{1/2} \underline{z}_{0t}^{\mathrm{d}} = \rho_0 \underline{z}_{0t}^{\mathrm{d}\dagger} \Lambda_0 \underline{z}_{0t}^{\mathrm{d}}, \tag{29}$$

so the SNR is equal to a weighted sum of squared-magnitudes of L_0 independent unit-variance complex Gaussian random variables. When the eigenvalues are different there is no simple expression for the probability density. For tractability we assume that the L_0 eigenvalues are equal, implying that the instantaneous SNR is proportional to a chi-square random variable with $2L_0$ degrees of freedom, $\mathrm{SNR}_0 \propto \chi^2_{2L_0}$. Although the *expected* SNR is the same irrespective of L_0, the maximum diversity case of $L_0 = M$ is far more favorable than the minimum diversity case of $L_0 = 1$ because the probability of an outage due to insufficient SNR is significantly lower.

The SNR at the undesired mobile is

$$\begin{aligned} \mathrm{SNR}_1 &= \rho_1 \frac{|\underline{h}_{0t}^{\mathrm{d}\dagger}\underline{h}_{1t}^{\mathrm{d}}|^2}{\underline{h}_{0t}^{\mathrm{d}\dagger}\underline{h}_{0t}^{\mathrm{d}}} \\ &= \rho_1 \frac{|\underline{z}_{0t}^{\mathrm{d}\dagger}\Lambda_0^{1/2}\Phi_0^\dagger\Phi_1\Lambda_1^{1/2}\underline{z}_{1t}^{\mathrm{d}}|^2}{\underline{z}_{0t}^{\mathrm{d}\dagger}\Lambda_0\underline{z}_{0t}^{\mathrm{d}}} \end{aligned} \tag{30}$$

We now assume that the eigenvalues for each covariance matrix are equal, and furthermore that the two covariance matrices have $L_{\rm c}$ eigenvectors in common (corresponding to nonzero eigenvalues), and that their remaining eigenvectors are mutually orthogonal,

$$\Lambda_j = (M/L_j) I_{L_j},\ j = 0, 1 \quad \Phi_0^\dagger \Phi_1 = \begin{bmatrix} I_{L_{\rm c}} & 0 \\ 0 & 0 \end{bmatrix}. \tag{31}$$

Of necessity $(L_0 + L_1 - M) \leq L_{\rm c} \leq \min(L_0, L_1)$. The substitution of (31) into (30) gives

$$\mathrm{SNR}_1 = \rho_1 (M/L_1) \left| \sum_{m=1}^{L_{\rm c}} [\underline{\theta}]_m^* [\underline{z}_{1t}^{\rm d}]_m \right|^2, \tag{32}$$

where $\underline{\theta} = \underline{z}_{0t}^{\rm d} / |\underline{z}_{0t}^{\rm d}|$. Thus $\underline{\theta}$ is a random unit vector that is independent of $\underline{z}_{1t}^{\rm d}$, and which may be shown to be *isotropically distributed* in the sense that its probability density is unchanged if the vector is multiplied by a deterministic unitary matrix. It has probability density [3]

$$p(\underline{\theta}) = \frac{\Gamma(L_0)}{\pi^{L_0}} \delta\left(\underline{\theta}^\dagger \underline{\theta} - 1\right), \tag{33}$$

where $\delta(\cdot)$ is the Dirac delta function. The expected SNR is

$$\mathrm{E}\left\{\mathrm{SNR}_1\right\} = \rho_1 (M/L_1) \sum_{m=1}^{L_{\rm c}} \mathrm{E}\left\{|[\underline{\theta}]_m|^2\right\} \mathrm{E}\left\{|[\underline{z}_{1t}^{\rm d}]_m|^2\right\} = \rho_1 (M/L_1) \sum_{m=1}^{L_{\rm c}} (1/L_0) = \frac{\rho_1 M L_{\rm c}}{L_0 L_1}. \tag{34}$$

A rearrangement of (32) gives

$$\mathrm{SNR}_1 = \rho_1 (M/L_1) \left(\sum_{m=1}^{L_{\rm c}} |[\underline{\theta}]_m|^2 \right) \left(\frac{\left| \sum_{m=1}^{L_{\rm c}} [\underline{\theta}]_m^* [\underline{z}_{1t}^{\rm d}]_m \right|^2}{\sum_{m=1}^{L_{\rm c}} |[\underline{\theta}]_m|^2} \right). \tag{35}$$

We claim that the two factors are independent. The second term is equal to the magnitude squared of a weighted combination of $L_{\rm c}$ $\mathrm{CN}(0,1)$ independent random variables, with the weight comprising a unit vector formed from $L_{\rm c}$ entries of $\underline{\theta}$. Conditioned on $\underline{\theta}$ this combination is $\mathrm{CN}(0,1)$, so this conditional density does not depend on $\underline{\theta}$. Therefore the two factors of (35) are independent, with the second term $\propto \chi_2^2$. The first factor is equal to the energy contained in $L_{\rm c}$ entries of the L_0-component isotropically distributed unit vector $\underline{\theta}$. We do not concern ourselves with its density, merely observing that the product of the two

independent random variables must be somewhat more "long-tailed" than a χ_2^2 random variable.

It is also possible to analyze what happens when both covariance matrices have rank one, but with arbitrary eigenvectors. In this case Φ_0 and Φ_1 are M-component steering vectors and $\underline{z}_{0t}^{\mathrm{u,d}}$ and $\underline{z}_{1t}^{\mathrm{u,d}}$ are scalars. The SNR expression (30) becomes

$$\mathrm{SNR}_1 = \rho_1 M \frac{\left| z_{0t}^{\mathrm{d}*} \Phi_0^\dagger \Phi_1 z_{1t}^{\mathrm{d}} \right|^2}{\left| z_{0t}^{\mathrm{d}} \right|^2} = \rho_1 M \left| \Phi_0^\dagger \Phi_1 \right|^2 \left| z_{1t}^{\mathrm{d}} \right|^2, \tag{36}$$

so $\mathrm{SNR}_1 \propto \chi_2^2$, and

$$\mathrm{E}\{\mathrm{SNR}_1\} = \rho_1 M \left| \Phi_0^\dagger \Phi_1 \right|^2. \tag{37}$$

For plane wavefronts the magnitude-squared of the inner product between the eigenvectors is equal to the cosine-squared of the difference of the angles of arrival.

We see from (34) that if $ML_{\mathrm{c}}/(L_0 L_1) > 1$ then the expected SNR at the undesired mobile can be larger than ρ_1, the nonadaptive SNR. We see from (37) that if the desired and undesired mobile have small angular separation, the expected SNR at the undesired mobile can be again be larger than ρ_1. Since the downlink signal intended for the desired mobile constitutes interference for the undesired mobile, a high SNR at the undesired mobile can be a very unwelcome side effect of maximum ratio weighting.

To summarize: Maximum ratio weighting gives a greater SNR at the desired mobile than any other possible scheme, but it requires knowledge of the instantaneous downlink fading coefficients. It always gives a factor of M improvement in expected SNR_0 compared with nonadaptive weighting. The proportionate probability density of SNR_0 ranges from χ_2^2 in the case of maximally correlated fading to χ_{2M}^2 in the case of uncorrelated fading. The expected SNR at an undesired mobile depends on the overlap of the subspaces spanned by the eigenvectors of K_0 and K_1. In any case SNR_1 has a long-tailed density, at best being χ_2^2.

If the instantaneous value of the downlink fading coefficients for the undesired mobile were also known, one would have direct control over the instantaneous SNR at the undesired mobile, and could implement schemes analogous to Algorithm III and IV involving trade-offs between SNR_1 and SNR_0.

5.2 Nonadaptive Weighting

With nonadaptive weighting (see Section 2.2), the SNR at either mobile is $\mathrm{SNR}_j = \rho_j |\underline{e}_m^\dagger \underline{h}_{jt}^{\mathrm{d}}|^2$. The weighted combination of the downlink fading coefficients is zero-mean complex Gaussian, so

$$\mathrm{SNR}_0 \propto \chi_2^2, \qquad \mathrm{SNR}_1 \propto \chi_2^2, \tag{38}$$

and

$$\mathrm{E}\{\mathrm{SNR}_0\} = \rho_0, \qquad \mathrm{E}\{\mathrm{SNR}_1\} = \rho_1. \tag{39}$$

5.3 Adaptive Weighting

For any of the four adaptive algorithms, the weights are derived from uplink measurements, which in turn are statistically independent of the downlink fading. Therefore, *conditioned on the weights*, the SNR at either mobile is proportional to the magnitude squared of a zero mean complex Gaussian random variable,

$$\mathrm{SNR}_j | \underline{w} \propto \chi_2^2. \tag{40}$$

Likewise, the conditional expected SNR at either mobile is

$$\mathrm{E}\{\mathrm{SNR}_j \mid \underline{w}\} = \mathrm{E}\left\{\rho_j |\underline{w}^\dagger \underline{h}_{jt}^{\mathrm{d}}|^2 \mid \underline{w}\right\} = \rho_j \underline{w}^\dagger K_j \underline{w}. \tag{41}$$

It follows that the adaptive algorithms can only achieve beamforming or generalized beamforming gains over nonadaptive weighting that involve increases in the expected SNR, but no significant change in the shape of its probability distribution.

5.3.1 Algorithm I

In Algorithm I there are no interferers and we solve

$$\hat{K}_0 \underline{w} = \lambda_{\max} \underline{w},$$

and apply $\underline{w}$ to the downlink. The SNR at the desired mobile is

$$\mathrm{SNR}_0 = \rho_0 |\underline{w}^\dagger \underline{h}_{0t}^{\mathrm{d}}|^2.$$

Perfect estimate of K_0

In this case, $\underline{w}$ satisfies $K_0\underline{w} = \lambda_{\max}\underline{w}$. Because the weight is deterministic we have $\mathrm{SNR}_0 \propto \chi^2_2$. The expected SNR is equal to the conditional expected SNR (41)

$$\mathrm{E}\left\{\mathrm{SNR}_0\right\} = \rho_0 \lambda_{\max}.$$

Now we assume that K_0 has rank L_0 with equal nonzero eigenvalues, so

$$\mathrm{E}\left\{\mathrm{SNR}_0\right\} = \frac{\rho_0 M}{L_0}. \tag{42}$$

Rank-one estimate of K_0

In this case, $\hat{K}_0 = \rho_0 \underline{h}^{\mathrm{u}}_{0t}\underline{h}^{\mathrm{u}\dagger}_{0t}$, and

$$\underline{w} = \frac{\underline{h}^{\mathrm{u}}_{0t}}{\|\underline{h}^{\mathrm{u}}_{0t}\|}.$$

We use the representation (24) and we assume that the nonzero eigenvalues are equal to obtain the SNR as

$$\begin{aligned}
\mathrm{SNR}_0 = \rho_0 |\underline{w}^{\dagger}\underline{h}^{\mathrm{d}}_{0t}|^2 &= \rho_0 \frac{|\underline{h}^{\mathrm{u}\dagger}_{0t}\underline{h}^{\mathrm{d}}_{0t}|^2}{\|\underline{h}^{\mathrm{u}}_{0t}\|^2} \\
&= \rho_0 \frac{|\underline{z}^{\mathrm{u}\dagger}_{0t}\Lambda_0\underline{z}^{\mathrm{d}}_{0t}|^2}{\underline{z}^{\mathrm{u}\dagger}_{0t}\Lambda_0\underline{z}^{\mathrm{u}}_{0t}} \\
&= \frac{\rho_0 M}{L_0}\frac{|\underline{z}^{\mathrm{u}\dagger}_{0t}\underline{z}^{\mathrm{d}}_{0t}|^2}{\underline{z}^{\mathrm{u}\dagger}_{0t}\underline{z}^{\mathrm{u}}_{0t}} \\
&= \frac{\rho_0 M}{L_0}|\underline{\theta}^{\dagger}\underline{z}^{\mathrm{d}}_{0t}|^2,
\end{aligned}$$

where $\underline{\theta} = \underline{z}^{\mathrm{u}}_{0t}/\|\underline{z}^{\mathrm{u}}_{0t}\|$ is isotropically distributed. As a consequence $\mathrm{SNR}_0 \propto \chi^2_2$ and

$$\mathrm{E}\left\{\mathrm{SNR}_0\right\} = \frac{\rho_0 M}{L_0}. \tag{43}$$

We have the surprising conclusion that, under the assumption of equal eigenvalues, weights based on the rank-one estimate of K_0 and weights based on the perfect estimate of K_0 perform equally well, in a statistical sense, on the downlink. For the rank-one case where $L_0 = 1$, Algorithm I performs as well as maximum ratio weighting. For any rank greater than one, Algorithm I is inferior to maximum ratio weighting, both with respect to expected SNR as well as the distribution of the instantaneous SNR.

5.3.2 Algorithm II

Algorithm II differs from Algorithm I by obtaining the covariance estimate $\hat{K}_0$ from an estimate for $\underline{h}_{0t}^{\mathrm{u}}$ that is relatively immune to the signal from the undesired mobile. With Algorithm II the SNR at the desired mobile should behave as if the undesired mobile were not present, i.e., according to the analysis of the previous subsection. In what follows, we again investigate the SNR at the undesired mobile under two different covariance models; the first model is the covariance structure (31), and the second is an arbitrary rank-one covariance matrices.

Perfect estimate of K_0

The weight vector $\underline{w}$ is the eigenvector of K_0 having the largest eigenvalue, but this is not unique because, by (22) and (31), the covariance K_0 has L_0 equal nonzero eigenvalues. Furthermore, K_0 shares L_{c} eigenvectors with K_1. Any linear combination of L_0 eigenvectors of K_0 is also an eigenvector of K_0, and depending on what combination happens to be chosen, the expected SNR at the undesired mobile can range from zero to $\rho_1 M/L_1$. To incorporate the fact that Algorithm II does not estimate K_1, and therefore does not know which of the L_0 eigenvectors to use as the weight, we set the weight equal to a linear combination of the eigenvectors,

$$\underline{w} = \Phi_0 \underline{\theta}, \tag{44}$$

where $\underline{\theta}$ is an L_0-component isotropically random unit vector. This weight gives the following SNR at the undesired mobile,

$$\mathrm{SNR}_1 = \rho_1 |\underline{\theta}^\dagger \Phi_0^\dagger \underline{h}_{1t}^{\mathrm{d}}|^2 = \frac{\rho_1 M}{L_1} \left| \underline{\theta}^\dagger \begin{bmatrix} I_{L_{\mathrm{c}}} & 0 \\ 0 & 0 \end{bmatrix} \underline{z}_{1t}^{\mathrm{d}} \right|^2 = \frac{\rho_1 M}{L_1} \left| \sum_{m=1}^{L_{\mathrm{c}}} [\underline{\theta}]_m^* [\underline{z}_{1t}]_m^{\mathrm{d}} \right|^2 . \tag{45}$$

Note that this expression is identical to the expression for SNR_1 when maximum ratio weighting is employed (32). We conclude that

$$\mathrm{E}\left\{\mathrm{SNR}_1\right\} = \frac{\rho_1 M L_{\mathrm{c}}}{L_0 L_1}. \tag{46}$$

When K_0 and K_1 are rank-one, Φ_0 and Φ_1 are steering vectors, $\underline{z}_{0t}^{\mathrm{u,d}}$ and $\underline{z}_{1t}^{\mathrm{u,d}}$ are scalars, and the choice of the weight vector is unique; we have

$$\mathrm{SNR}_1 \quad = \quad \rho_1 M \frac{\left| z_{0t}^{\mathrm{u}*} \Phi_0^\dagger \Phi_1 z_{1t}^{\mathrm{d}} \right|^2}{\left| z_{0t}^{\mathrm{u}} \right|^2} = \rho_1 M \left| \Phi_0^\dagger \Phi_1 \right|^2 \left| z_{1t}^{\mathrm{d}} \right|^2 ,$$

$$\mathrm{E}\left\{\mathrm{SNR}_1\right\} \quad = \quad \rho_1 M \left|\Phi_0^\dagger \Phi_1\right|^2,$$

which is identical to the expressions (36) and (37) for the maximum ratio weighting when K_0 and K_1 have rank one.

We therefore see that, as in maximum ratio weighting, a high SNR at the undesired mobile can be an unwelcome side effect if K_0 and K_1 have eigenvectors spanning similar subspaces.

Rank-one estimate of K_0

We focus exclusively on the covariance structure (31). (Recall that when the rank of K_0 is one, a stationary mobile can obtain a perfect estimate of K_0.) The SNR at the undesired mobile is

$$\mathrm{SNR}_1 = \frac{\rho_1 M}{L_1} \frac{\left| \underline{z}_{0t}^{\mathrm{u}\dagger} \begin{bmatrix} I_{L_\mathrm{c}} & 0 \\ 0 & 0 \end{bmatrix} \underline{z}_{1t}^{\mathrm{d}} \right|^2}{\underline{z}_{0t}^{\mathrm{u}\dagger} \underline{z}_{0t}^{\mathrm{u}}} = \frac{\rho_1 M}{L_1} \left| \underline{\theta}^\dagger \begin{bmatrix} I_{L_\mathrm{c}} & 0 \\ 0 & 0 \end{bmatrix} \underline{z}_{1t}^{\mathrm{d}} \right|^2, \tag{47}$$

where $\underline{\theta}$ is independent of $\underline{z}_{1t}^{\mathrm{d}}$ and isotropically distributed. This expression is equivalent to (45), so having a rank-one covariance estimate yields the same SNR at the undesired mobile as having a perfect estimate.

To summarize: with respect to the desired mobile, the performance of Algorithm II is the same as that of Algorithm I. For the undesired mobile, under a wide range of conditions, the SNR behavior of Algorithm II is identical to maximum ratio weighting, and the expected SNR can be undesirably high.

5.3.3 Algorithm III

We see that Algorithm II has the potential fault that it can boost the SNR to the undesired mobile if the subspaces spanned by the eigenvectors of the covariance matrices are similar. Algorithm III attempts to mitigate this effect by maximizing the ratio (16).

Perfect estimates of K_0 and K_1

Assume again the covariance structure (31). Because $L_\mathrm{c} \leq L_0$, there are only two cases to consider: first, $L_\mathrm{c} < L_0$, and second, $L_\mathrm{c} = L_0$. In the former case we can choose the weight to be one of $L_0 - L_\mathrm{c}$ eigenvectors of K_0 having eigenvalue M/L_0 that is also an eigenvector of K_1 having eigenvalue zero. This weight gives zero SNR at the undesired mobile, and an SNR at the desired mobile equivalent to that obtained from either Algorithm I or II,

$$\mathrm{SNR}_1 = 0, \qquad \mathrm{E}\left\{\mathrm{SNR}_0\right\} = \frac{\rho_0 M}{L_0}. \tag{48}$$

If $L_c = L_0$ (and therefore $L_1 \geq L_0$) then any eigenvector of K_0 that has a nonzero eigenvalue also is an eigenvector of K_1 having a nonzero eigenvalue, which yields

$$\mathrm{E}\{\mathrm{SNR}_1\} = \frac{\rho_1 M}{L_1}, \qquad \mathrm{E}\{\mathrm{SNR}_0\} = \frac{\rho_0 M}{L_0}. \tag{49}$$

Hence, the expected SNR gain-factor M/L_0 for the desired mobile is always at least as large as the gain-factor for the undesired mobile.

Alternatively we assume that both covariance matrices have rank one. The ratio (16) is maximized by choosing the weight as follows,

$$\underline{w} = \frac{\left[I - \Phi_1 \Phi_1^\dagger\right] \Phi_0}{\left\| \left[I - \Phi_1 \Phi_1^\dagger\right] \Phi_0 \right\|}. \tag{50}$$

This weight gives

$$\mathrm{E}\{\mathrm{SNR}_1\} = \mathrm{SNR}_1 = 0,$$

and

$$\mathrm{E}\{\mathrm{SNR}_0\} = \underline{w}^\dagger K_0 \underline{w} = \rho_0 M \Phi_0^\dagger (I - \Phi_1 \Phi_1^\dagger) \Phi_0 = \rho_0 M \left(1 - \left|\Phi_0^\dagger \Phi_1\right|^2\right). \tag{51}$$

For plane wavefronts, the SNR at the desired mobile is proportional to the square of the sine of the difference of the angles of arrival.

Rank-one estimates of K_0 and K_1

The analysis of the performance of Algorithm III for the covariance structure (31) is too complicated to pursue here. If the true covariance matrices have rank one, then as pointed out previously, a stationary mobile can obtain perfect estimates from high SNR measurements; this case has been treated above.

5.3.4 Algorithm IV

An analysis of Algorithm IV seems to be very complicated, and we do not attempt it.

6 Numerical Simulations

In this section, we display the results of computer simulations of the behaviors of Algorithms II and III in the presence of the desired mobile and one undesired mobile. In all cases, 1000 simulations are run, and the measures of performance that we plot are the cumulative distributions of the downlink SNR's at

the desired and undesired mobile. In all cases, we set $\rho_0 = \rho_1 = 15$ dB, to indicate that the desired and undesired mobile appear, on average, equally strong to the base station array. With no adaptive antenna array, the downlink average SNR at both mobiles would again be 15 dB. Therefore, with the antenna array, any downlink SNR improvements relative to 15 dB to the desired mobile, and SNR reductions relative to 15 dB to the undesired mobile can then be directly read from the plots.

6.1 Simulation details

In each case, a linear log-periodic array with a total of seven elements is employed. The array is configured to work at an uplink frequency of 900 MHz, and a downlink frequency of 945 MHz. The minimum spacing is $d_1 = \lambda^{\mathrm{d}}/2$, where $\lambda^{\mathrm{d}} = (3 \times 10^8 \text{ m/s})/(945 \text{ MHz})$, and the ratio between successive spacings is $d_{m+1}/d_m = 945/900$, for $m = 1, \ldots, 6$. The first six elements are used on the downlink and the last six elements are used on the uplink. The array is placed at the center of a cell, and rotated randomly on each run to include occasional boresite and endfire array effects relative to the mobiles. The mobiles have only one antenna each.

The desired mobile interacts with twelve scatterers that are placed randomly at a radius of 900–1100 m from the array in a region determined by the angle spread η that we specify. The mobile is placed at the center of this region at the beginning of each time slot. Each of the scatterers comprises five subscatterers contained within a 3 m mean-square radius of the scattering center. The reflections from the scatterers are therefore approximately Rayleigh distributed. The subscatterers of each scatterer are also equipped with a common randomly chosen log-normal shadow fading coefficient with variance $\sigma = 6$ dB; the mean is chosen so that the expected SNR at the base station is ρ_0. The mobile transmits random QPSK signals to the base station, and moves through the scatterers along a straight line away from the base station antenna array with a velocity v that we specify. The synthetic signal propagating between the mobile antenna and each element of the base station array is generated as the algebraic sum of the $12 \times 5 = 60$ signals reflected by the subscatterers. The phase-shift of each of these component signals depends on its path length.

The message sent by the desired mobile is assumed to be known at the base station—i.e., we set $\hat{s}^{\mathrm{u}}_{0t} = s^{\mathrm{u}}_{0t}$. But the mobile's channel response is estimated using the uplink algorithm described in Section 4.2. This assumption allows the simulations to be interpreted more easily.

The undesired mobile is simulated in a similar way and is assumed to travel with the same velocity away from the antenna array and have the same angle spread as the desired mobile, but it is given separate scattering centers and subscatterers. The signals from the desired and undesired mobiles therefore generally

have different paths to the base station. The position of the undesired mobile relative to the desired mobile (with respect to the base station) is given by the angle θ separating the undesired scattering region from the desired scattering region. In our most extreme case, $\theta = 0^{\rm o}$, meaning that the scattering regions overlap completely. The signal from the undesired mobile is treated at the base station as unknown out-of-cell interference.

The TDMA system we simulate corresponds to the IS-136 standard. We assume that the any mobile's time slot is composed of 162 successive samples with 41 μs between samples. Thus, on an uplink time slot the 162 samples are used to form the covariance estimates, from which the appropriate complex antenna-weights are derived and then used for the downlink during the next time slot. For every one of the 1000 simulations, all stochastic quantities change independently and, as a result, the true covariance matrices also change.

6.2 Simulation results

We plot four cases, each represented by two performance plots—one for the desired and one for the undesired mobile. The expected nonadaptive downlink SNR for both mobiles is 15 dB for all cases. Because we are using $M = 6$ transmitter antennas on the downlink, the expected ideal SNR for the desired mobile is 15 + $10 \log_{10} 6$ = 22.8 dB. In the first case, shown in Figures 3 and 4, both mobiles are stationary ($v = 0$ m/s), the desired and undesired mobile have angle-spreads of $\eta = 30^{\rm o}$, and are separated by $\theta = 0^{\rm o}$. We see that because the two mobiles are virtually overlapping, Algorithm II boosts the SNR to both mobiles, whereas Algorithm III boosts the SNR to the undesired mobile only a little less than to the desired mobile.

In the second case, shown in Figures 5 and 6, both mobiles again are stationary, and have angle-spreads of $\eta = 30^{\rm o}$, but are now separated by $\theta = 30^{\rm o}$. We see that because the two mobiles are well separated, Algorithm II boosts the SNR to the desired mobile more than the undesired mobile, whereas Algorithm III reduces the SNR to the undesired mobile while boosting the SNR to the desired mobile. This desirable performance is achieved despite the fact that both mobiles are stationary.

The third case, shown in Figures 7 and 8, is similar to the second except that both mobiles are now traveling with $v = 25$ m/s. We see that because the two mobiles are well separated and moving rapidly, Algorithm II boosts the SNR to the desired mobile more than the undesired mobile, whereas Algorithm III reduces the SNR to the undesired mobile while boosting the SNR to the desired mobile. Upon comparing Figures 5 and 7, we see that the adaptive algorithms perform equally well for the desired mobile, whether or not the mobile is actually moving. Figures 6 and 8 show that high speed improves the performance of

Case	Desired		Undesired	
	Alg. II	Alg. III	Alg. II	Alg. III
$v = 0$ m/s, $\eta = 30^{\circ}$, $\theta = 0^{\circ}$				
Mean	21.3	17.3	21.5	16.4
Median	18.9	13.5	19.2	12.6
$v = 0$ m/s, $\eta = 30^{\circ}$, $\theta = 30^{\circ}$				
Mean	22.0	20.1	19.0	11.9
Median	19.2	17.1	14.2	7.0
$v = 25$ m/s, $\eta = 30^{\circ}$, $\theta = 30^{\circ}$				
Mean	21.7	20.0	18.0	8.3
Median	19.3	17.3	13.7	2.8
$v = 0$ m/s, $\eta = 360^{\circ}$, $\theta =$N/A				
Mean	18.6	18.2	16.2	14.2
Median	15.3	15.3	13.0	11.3

Table 1: Mean and median SNR's (in dB) of Algorithms II and III for the four cases simulated. The nonadaptive mean SNR for the desired and undesired mobile is 15 dB.

Algorithm III for the undesired mobile.

In the fourth case, shown in Figures 9 and 10, $v = 0$ m/s, the angle-spread is $\eta = 360^{\circ}$. The separating angle θ is therefore irrelevant. We see that despite the enormous angle spreads for the two mobiles, that Algorithm III can exploit whatever clumping exists in the randomly placed scattering centers to reduce the SNR to the undesired mobile.

A summary of the means and medians of the distributions given in these figures is presented in Table 1.

7 Conclusions

We have shown that covariance-based adaptive antenna array techniques in TDMA cellular systems can successfully be employed on the downlink, even when only rank-one estimates of the downlink covariance are available. This is a rather surprising result since many adaptive array algorithms fail under similar circumstances.

We have also devised a log-periodic antenna array that makes the uplink and downlink covariances equal despite the different carrier frequencies used. This should make the implementation of covariance-based adaptive techniques more practical.

Acknowledgment

We are very grateful to Dennis Morgan at Bell Laboratories for a thorough reading and critique of

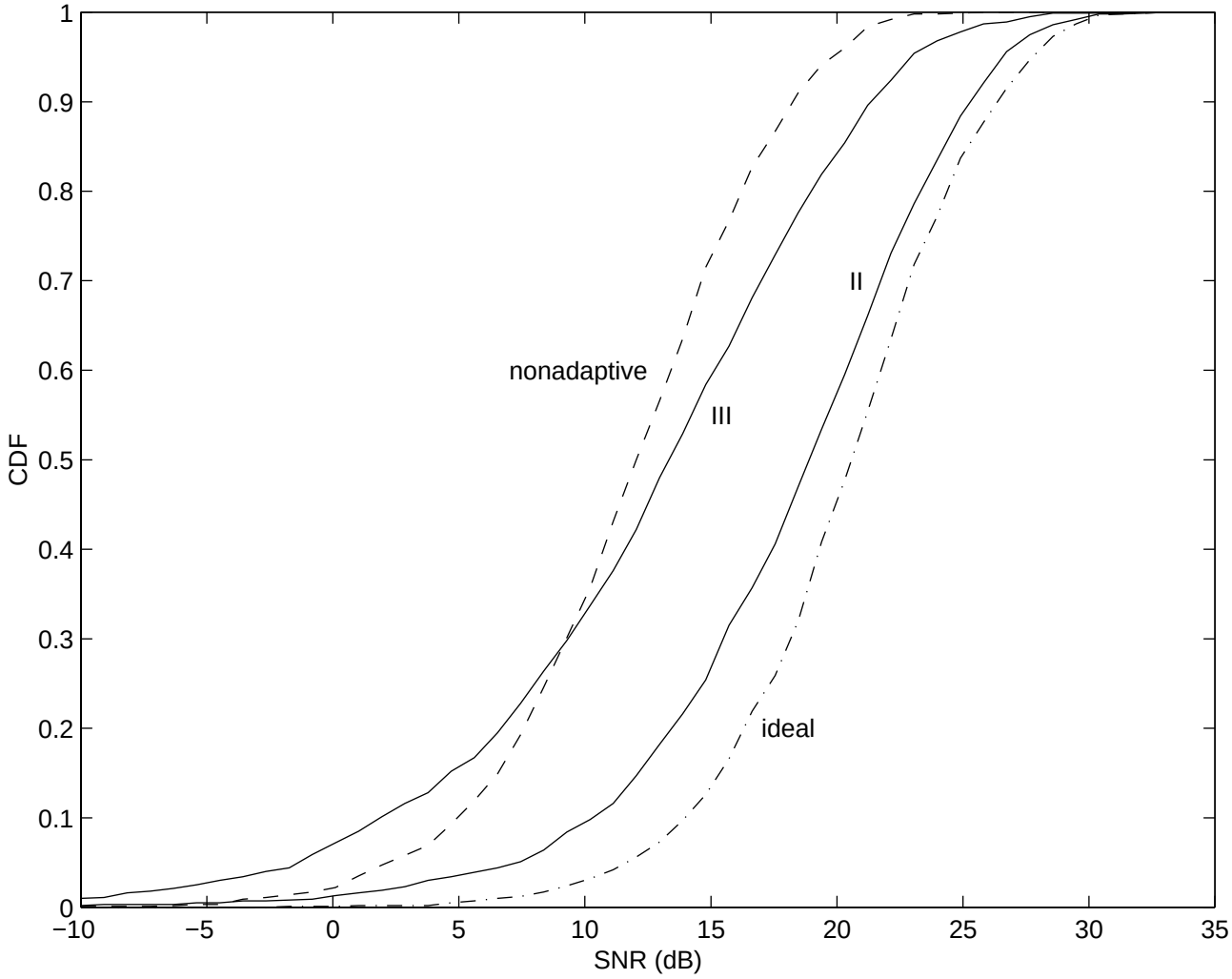


Figure 3: Cumulative distribution function of SNR at desired mobile for Algorithms II (right-most solid line) and III (left-most solid line). The dashed line is the nonadaptive SNR and the dash-dotted line is the ideal SNR obtained with maximum ratio weighting. The parameters are $\rho_0 = \rho_1 = 15$ dB, $v = 0$ m/s, $\eta = 30^{\rm o}$, and $\theta = 0^{\rm o}$. As compared with Algorithm II, Algorithm III has the detrimental effect of reducing power to the desired mobile, but also has the beneficial effect of reducing the power to the undesired mobile (see Figure 4).

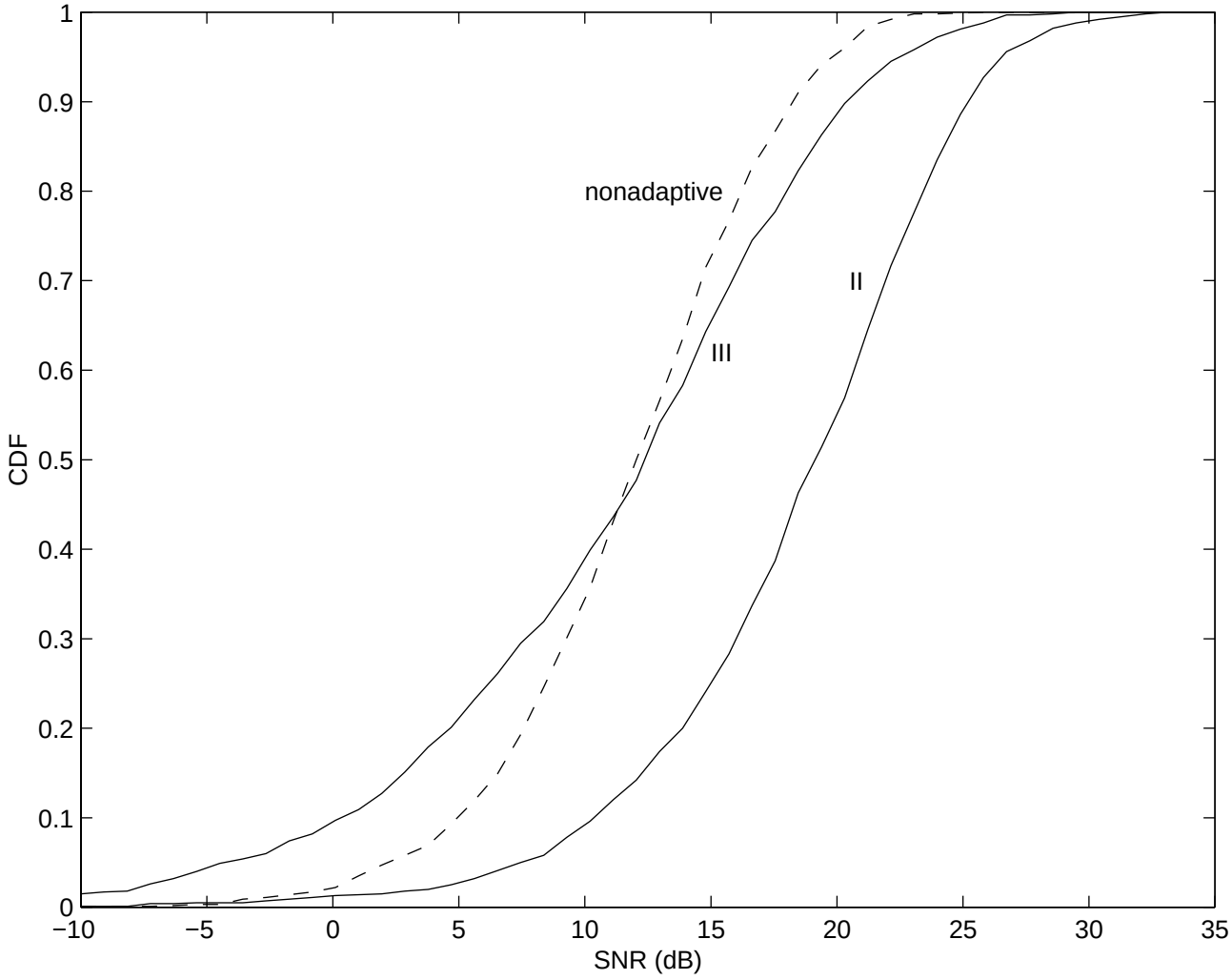


Figure 4: Cumulative distribution function of SNR at undesired mobile for Algorithms II (right-most solid line) and III (left-most solid line). The dashed line is the nonadaptive SNR. The parameters are $\rho_0 = \rho_1 = 15$ dB, $v = 0$ m/s, $\eta = 30^{\circ}$, and $\theta = 0^{\circ}$. As compared with Algorithm II, Algorithm III has the beneficial effect of reducing the power to the undesired mobile, but has the detrimental effect of reducing the power to the desired mobile (see Figure 3).

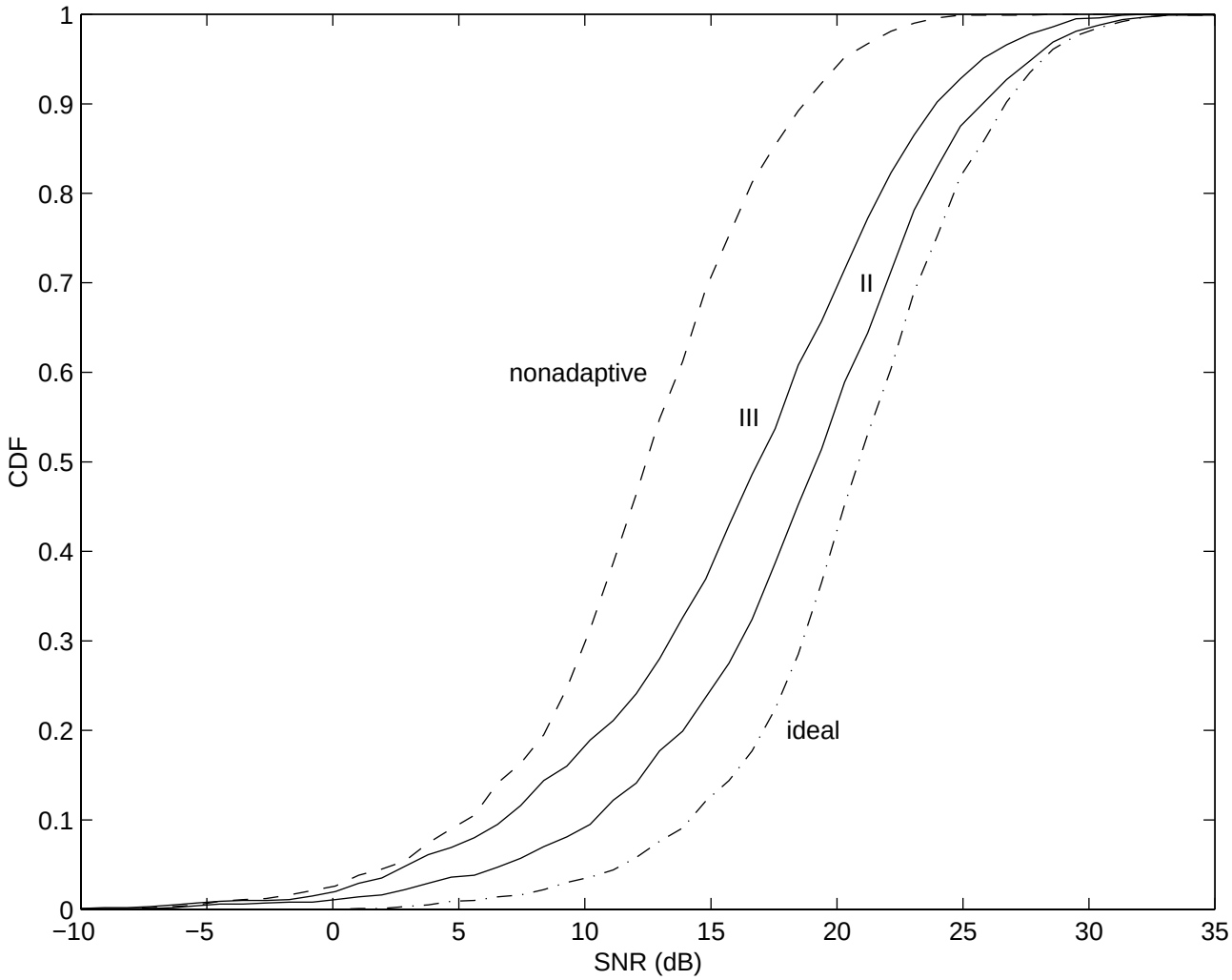


Figure 5: Cumulative distribution function of SNR at desired mobile for Algorithms II (right-most solid line) and III (left-most solid line). The dashed line is the nonadaptive SNR and the dash-dotted line is the ideal SNR obtained with maximum ratio weighting. The parameters are $\rho_0 = \rho_1 = 15$ dB, $v = 0$ m/s, $\eta = 30^{\circ}$, and $\theta = 30^{\circ}$.

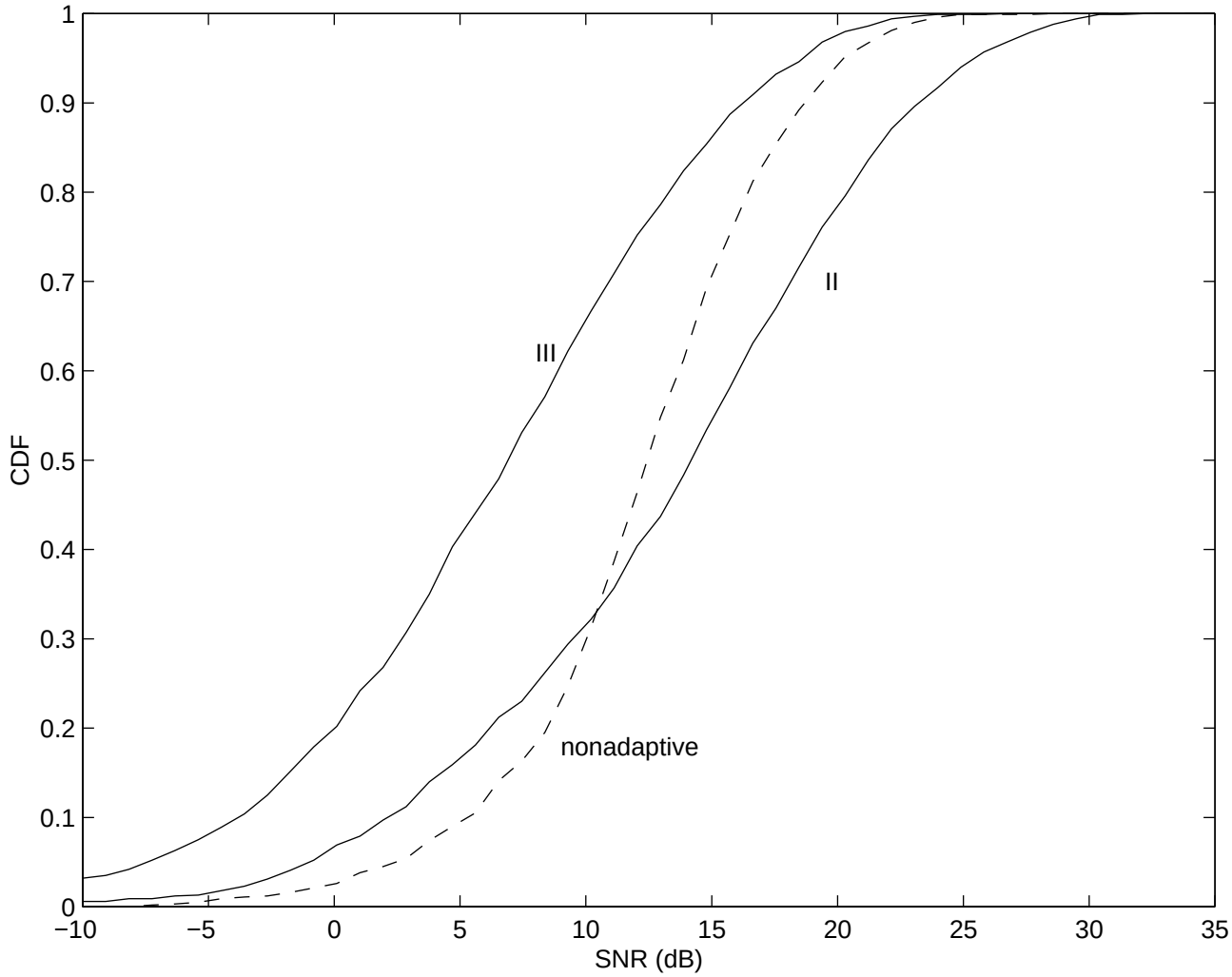


Figure 6: Cumulative distribution function of SNR at undesired mobile for Algorithms II (right-most solid line) and III (left-most solid line). The dashed line is the nonadaptive SNR. The parameters are $\rho_0 = \rho_1 = 15$ dB, $v = 0$ m/s, $\eta = 30^{\circ}$, and $\theta = 30^{\circ}$.

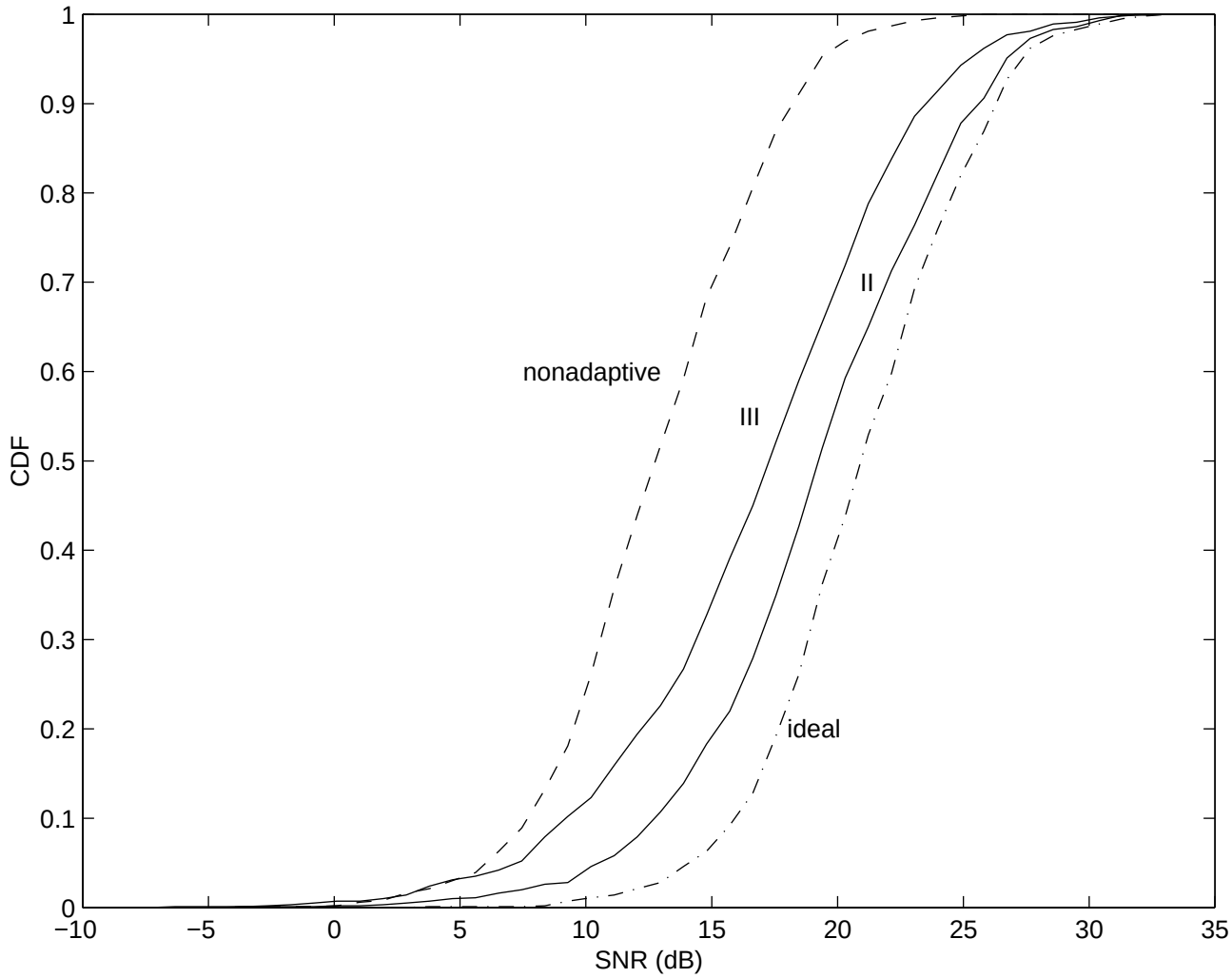


Figure 7: Cumulative distribution function of SNR at desired mobile for Algorithms II (right-most solid line) and III (left-most solid line). The dashed line is the nonadaptive SNR and the dash-dotted line is the ideal SNR obtained with maximum ratio weighting. The parameters are $\rho_0 = \rho_1 = 15$ dB, $v = 25$ m/s, $\eta = 30^{\rm o}$, and $\theta = 30^{\rm o}$.

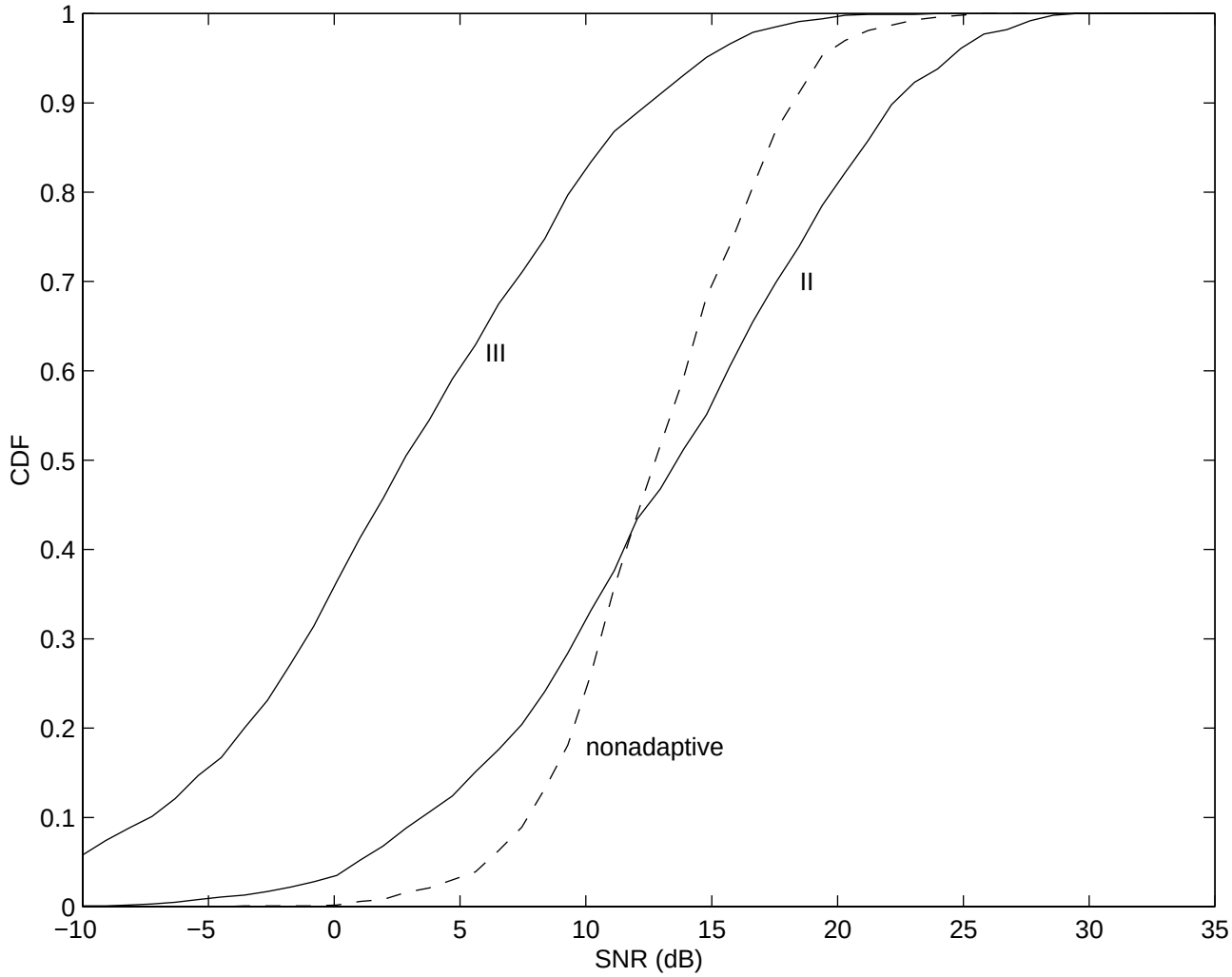


Figure 8: Cumulative distribution function of SNR at undesired mobile for Algorithms II (right-most solid line) and III (left-most solid line). The dashed line is the nonadaptive SNR. The parameters are $\rho_0 = \rho_1 = 15$ dB, $v = 25$ m/s, $\eta = 30^{\circ}$, and $\theta = 30^{\circ}$.

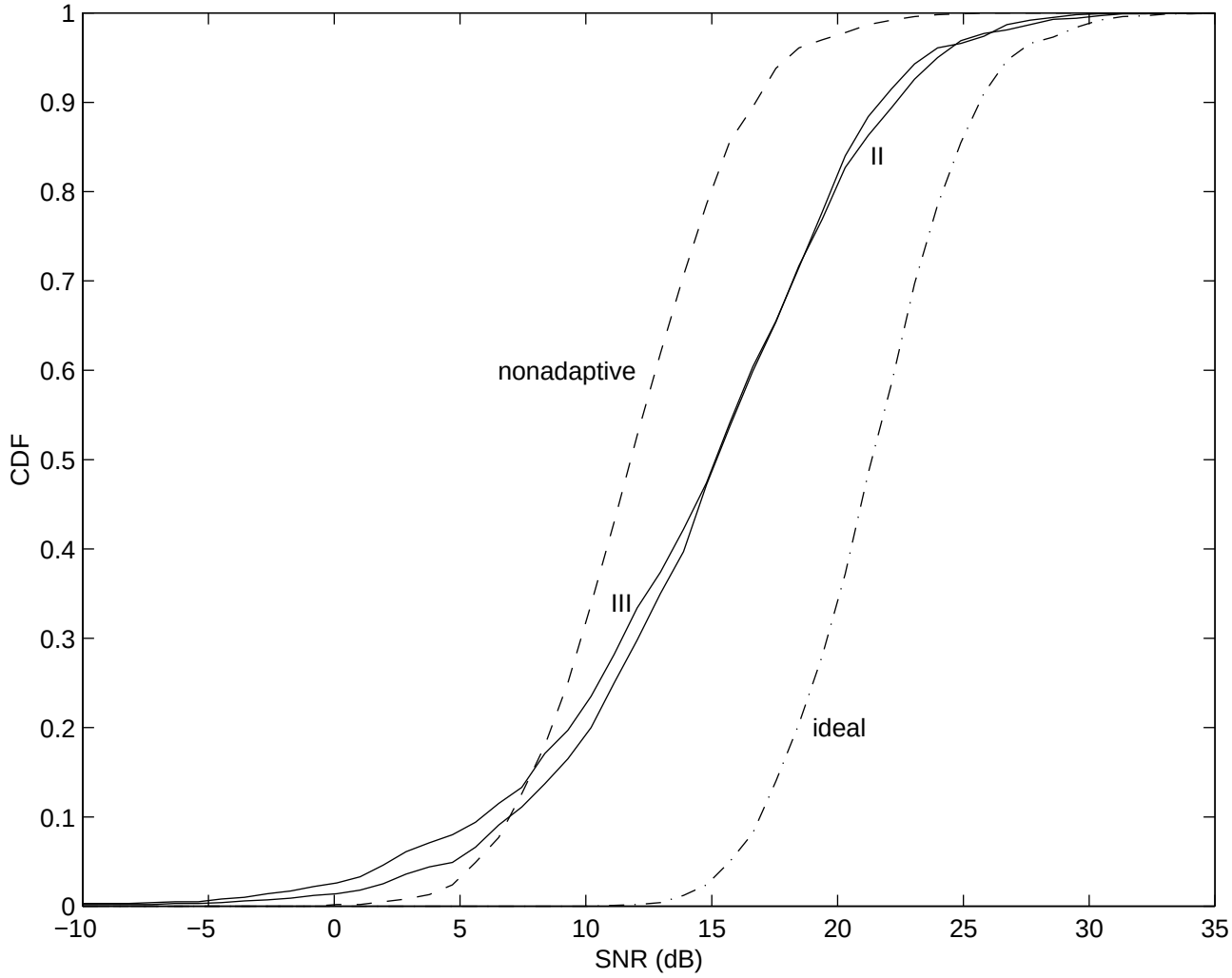


Figure 9: Cumulative distribution function of SNR at desired mobile for Algorithms II (right-most solid line) and III (left-most solid line). The dashed line is the nonadaptive SNR and the dash-dotted line is the ideal SNR obtained with maximum ratio weighting. The parameters are $\rho_0 = \rho_1 = 15$ dB, $v = 0$ m/s, $\eta = 360^{\circ}$, and θ=irrelevant. In this case, Algorithms II and III appear to behave virtually identically for the desired mobile.

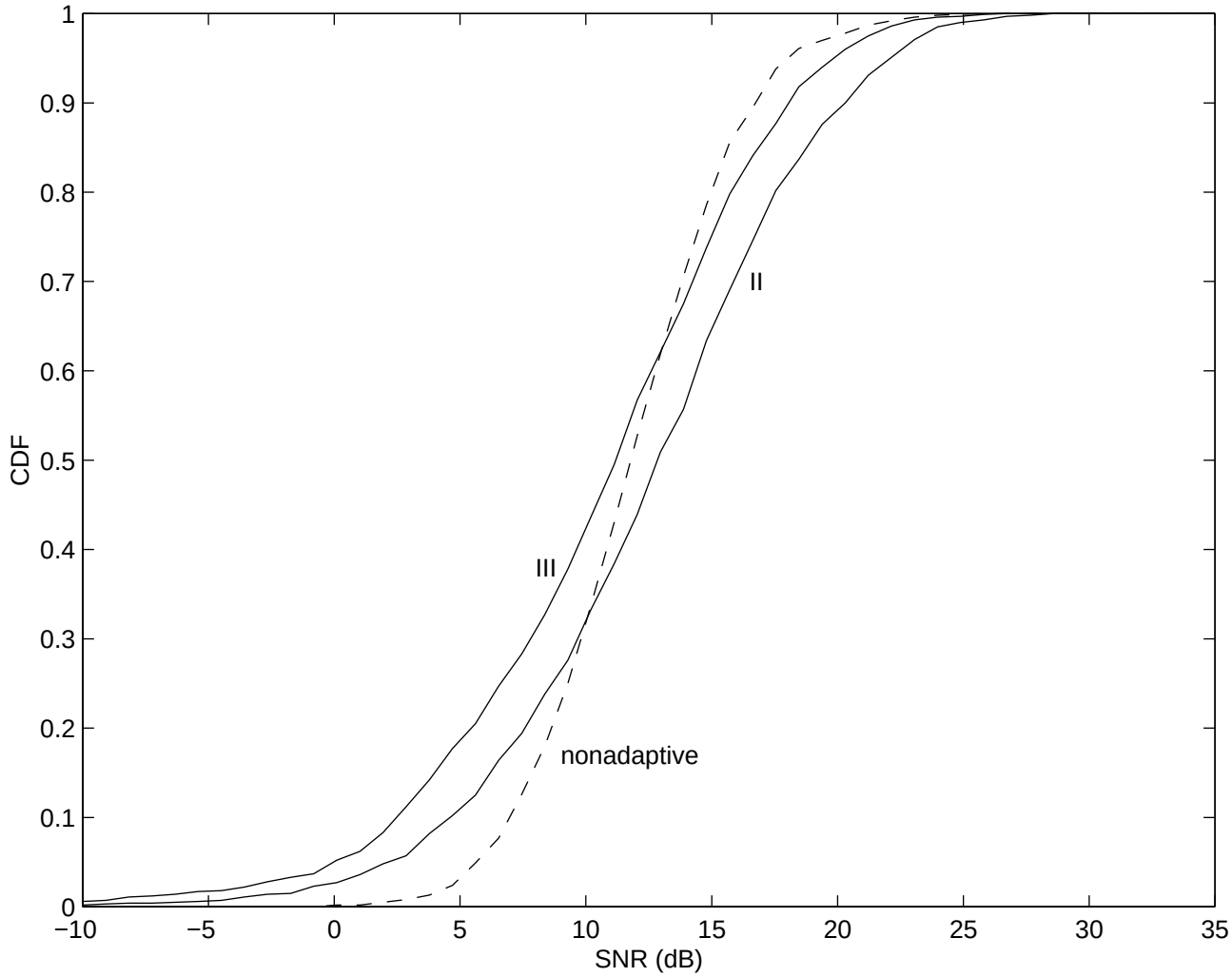


Figure 10: Cumulative distribution function of SNR at undesired mobile for Algorithms II (right-most solid line) and III (left-most solid line). The dashed line is the nonadaptive SNR. The parameters are $\rho_0 = \rho_1 = 15$ dB, $v = 0$ m/s, $\eta = 360^{\circ}$, and θ =irrelevant.

the entire manuscript, and to J. Mazo, R. Aiken, R. Benning, M. Solondz, and K. Wu for many fruitful discussions.

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